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Managing Multiple Information Sources for a Questioning Agenda

Abstract. In this paper, we consider how an agent may manage a questioning agenda in a situation where multiple information sources are available. We work within the framework of formal dialogue systems with the underpinning of Inferential Erotetic Logic. Firstly, we present the formal dialogue system $DL(IEL)_{\mathrm{mult}}$ for managing multi-agent information retrieval. Then, we extend the proposed system so that it is capable of representing group and individual levels for the question decomposition process. We also propose two measures for evaluating information sources: their cooperativeness and success levels; next, we analyse how the choice of agents may influence the way in which a solution for a given problem is reached.

Keywords: questions; formal dialogue system; inferential erotetic logic; erotetic search scenarios; problem decomposition; multi-agent environment; agents' rankings

1. Introduction

The main aim of this paper is to address the issue of how to model the process of problem solving *via* question decomposition in a situation where multiple information sources are available. As such, this paper may be seen as a continuation and extension of our previous work aimed at modelling questioning agendas of problem solving agents [see, e.g., 27].[1] Our main formal tool used in this work is Inferential Erotetic Logic

[1] For an alternative formal approach to research agendas, see, e.g., [4, 7], for argumentative contexts.

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(hereafter IEL) [32, 37]. This logic focuses on inferences whose premises and/or conclusions are questions (*erotetic inferences*). The main tool we use for modelling a questioning agenda for an agent is the erotetic search scenario (e-scenario) [35, 37]. E-scenario represents a certain map of possible courses of events in a given questioning process for the initial question, with respect to the initial knowledge. Each path of an e-scenario is one of the ways in which the process might go, depending on the answers obtained to queries. E-scenarios has proven to be a powerful logical tool for modelling cognitive goal-directed processes and problem solving.[2] E-scenarios explain the relevance of questions appearing in interactions between agents in the way proposed in [21].

Our approach also reaches for inspiration and aims at intuitions from the field of epistemic planning (however, with a different focus that is put mainly on questions and question processing). We aim (similarly like [17] or [25]) at a framework which is able to present how a problem solving may be planned and executed in a multi-agent environment. Within this environment, there is a clear distinction of common knowledge needed for achieving a common goal and private knowledge of agents that does not need to be revealed and shared.

In the works cited above, authors employ dynamic epistemic logic in order to express the information exchange process. In our case, we use the framework of formal dialogue systems, which is mainly inspired by [1, 8].[3] The $DL(IEL)_{\mathrm{mult}}$ system (firstly presented in [11]) is used here as an over-layer for IEL concepts and allows for expressing the dynamics of the information exchange. On the other hand, IEL is used as an underpinning of a formal dialogue system in order to check the validity (or justification) of certain dialogue moves. The system relies on the normative concepts of erotetic implication (e-implication for short) and e-scenario developed within IEL.

In order to account for the multi-agent interaction (involving agents with different informational resources at hand), we employ the *blackboard architecture* [see, e.g., 2].

[2] See [35, 27, 10, 11, 29], and the overview in Section 2.2.

[3] It is worth mentioning that [15] presents an early approach to the epistemic version of e-scenarios (for one questioning agent). Dynamic Epistemic Logic is used as a point of departure in the presented research. Epistemic erotetic search scenarios (e-e-scenarios) are constructed dynamically *via* information gathering and is relative to the initial knowledge-base of an agent. The first step is the (so called) basic e-e-scenario, which is then expanded by deductive and erotetic moves.

The paper is structured as follows. We start with short outlines of our formal frameworks. In Section 2, Inferential Erotetic Logic's basics are presented, including erotetic implication and erotetic search scenarios. In Section 3, $DL(IEL)_{\mathrm{mult}}$ is introduced. We present in detail locution types allowed in the system, interaction rules, and commitment store rules. We discuss how the solution for a given problem will depend on the information sources involved into to the problem-solving process. We also explain how IEL underpins $DL(IEL)_{\mathrm{mult}}$. Towards the end of this section, we propose a simple measure which may be used for construing information sources' rankings. This measure is based on an agent's performance in a given information exchange, and its underlying intuition is that it represents the agent's level of cooperativeness. In Section 4, we refer to the concept of the blackboard architecture. We discuss how it may be combined with a pragmatic interpretation of e-scenarios in order to express two levels of problem solving: a group level and an individual level of initial problem decomposition. In this section, we also introduce the second measure which may be used to evaluate agents, this time, the measure refers to certain characteristics of their individual questioning agendas. Finally, we discuss how the two presented measures may be combined in order to optimise choice of information sources.

2. Erotetic inferences and search scenarios

Firstly — following [10] — we introduce the formal language $\mathcal{L}^{?}_{\mathbf{K3}}$, which will be used further on in this paper. This language allows us to construe questions with three possible answers: 'yes', 'no', or 'I don't know', thus representing polar questions and possible gaps in the agent's knowledge. For the purpose of expressing the third answer, Kleene's strong three-valued logic **K3** is used [see 31]. This choice is motivated by an intuitive epistemic interpretation of the third value in this logic. As the starting point, we take the language $\mathcal{L}_{\mathbf{K3}}$, which is an extension of the usual Kleene's logic language. It contains the following primitive connectives: $\neg$ (negation), $\rightarrow$ (implication), $\vee$ (disjunction), $\wedge$ (conjunction), $\leftrightarrow$ (equivalence), and additionally unary connectives: $\boxminus$ and $\boxplus$. The concept of a *well-formed formula* (wff for short) is defined in a traditional manner. We use p, q, r, s, p_1, ... for propositional variables. The connectives are defined by the following truth-tables:

	$\neg$
1	0
$1/2$	$1/2$
0	1

$\wedge$	1	$1/2$	0
1	1	$1/2$	0
$1/2$	$1/2$	$1/2$	0
0	0	0	0

$\vee$	1	$1/2$	0
1	1	1	1
$1/2$	1	$1/2$	$1/2$
0	1	$1/2$	0

$\rightarrow$	1	$1/2$	0
1	1	$1/2$	0
$1/2$	1	$1/2$	$1/2$
0	1	1	1

$\leftrightarrow$	1	$1/2$	0
1	1	$1/2$	0
$1/2$	$1/2$	$1/2$	$1/2$
0	0	$1/2$	1

A	$\boxminus A$	$\boxplus A$
1	0	1
$1/2$	1	0
0	0	1

The intended reading of the unary connectives $\boxminus$ and $\boxplus$ is the following:

- $\boxminus A$: it *cannot* be decided if it is the case that A.
- $\boxplus A$: it *can* be decided if it is the case that A.

In the next step, we will define the language $\mathcal{L}^?_{\mathbf{K3}}$, which is built upon $\mathcal{L}_{\mathbf{K3}}$, and in which questions may be formed. The vocabulary of $\mathcal{L}^?_{\mathbf{K3}}$ contains the vocabulary of $\mathcal{L}_{\mathbf{K3}}$ (and thus also the connectives $\boxplus$, $\boxminus$) and the signs: $?,\{,\}$.

Questions of $\mathcal{L}^?_{\mathbf{K3}}$ are expressions of the form:

$$?\{A_1, \ldots, A_n\}$$

where $n > 1$ and $A_1, \ldots, A_n$ are nonequiform (i.e., pairwise syntactically distinct) declarative, well-formed formulas of $\mathcal{L}_{\mathbf{K3}}$. If $?\{A_1, \ldots, A_n\}$ is a question, then each of the d-wffs $A_1, \ldots, A_n$ is called a *direct answer* to the question.[4] If Q is a question of $\mathcal{L}^?_{\mathbf{K3}}$, then $\mathbf{d}Q$ denotes the set of direct answers to Q.

We are interested in a specific category of *ternary* questions, which may be viewed as the counterparts to polar questions, extended with the epistemically motivated third possible answer "it is not known whether". A ternary question will be represented in language $\mathcal{L}^?_{\mathbf{K3}}$ as follows:

$$?\{A, \neg A, \boxminus A\}$$

[4] Thus, questions in $\mathcal{L}^?_{\mathbf{K3}}$ are construed according to the set-of-answers methodology [see 6, 18].

In what follows, we adopt the following notation:

$$?\pm\boxminus|A,B|$$

it refers to a question of the form:

$$?\{A\wedge B, A\wedge\neg B, A\wedge\boxminus B, \neg A\wedge B, \neg A\wedge\neg B, \neg A\wedge\boxminus B, \boxminus A\wedge B, \boxminus A\wedge\neg B, \boxminus A\wedge\boxminus B\}$$

It is a ternary counterpart to the binary conjunctive questions [see 26].

2.1. Erotetic implication

In IEL, erotetic inferences of two kinds are analysed:

(i) *Erotetic inferences of the first kind*, where a set of premises consists of declarative sentence(s) only, and an agent passes from it to a question — validity of such inferences is modelled by the concept of *question evocation* [see 37, Ch. 6].

(ii) *Erotetic inferences of the second kind*, where a set of premises consists of a question and possibly some declarative sentence(s), and an agent passes from it to another question — validity of which is modelled in terms of *erotetic implication*.

In this paper, we are only interested in the erotetic inferences of the second kind. Erotetic implication (e-implication) is a semantic relation between a question, Q, a (possibly empty) set of declarative well-formed formulas, X, and a question, Q_1. It is an ordered triple $\langle Q, X, Q_1\rangle$, where Q is called an *interrogative premise*, or simply *initial question*, the elements of X are *declarative premises* and the question Q_1 is the *conclusion*, or the *implied question* [see 37, pp. 51–52].

The intuition behind e-implication might be expressed as follows. Let us imagine an agent who is trying to solve a certain (possibly) complex problem. The problem is expressed by his/her initial question (Q). We assume that the agent does not have resources to answer the initial question on his/her own. Thus, the initial question has to be processed/decomposed. This decomposition is aimed at replacing the initial question with a simpler auxiliary question — Q_1. The auxiliary question obtained as a result of the decomposition process should have certain characteristics. First of all, it should stay on the main topic. In other words, no random questions should appear here. However, the main characteristic that we are aiming at here is that the answer provided to the auxiliary question should be at least a partial answer to the initial question — i.e., it should narrow down the set of direct answers to the initial question [see 37, p. 43]. In summary, we can perceive the discussed process of replacing one question with another (simpler) one as a well-motivated step from the problem-solving perspective.

A slightly artificial, but nevertheless instructive, example may be in order here [30]. Suppose that you face the following problem:

Q_1 Who stole the tarts?

and also, that you managed to gather the evidence:

E_1 It is one of the courtiers of the Queen of Hearts attending the afternoon tea-party that stole the tarts.

Thus, it makes perfect sense to ask the following question:

Q_2 Which of the Queen of Hearts' courtiers attended the afternoon tea-party?

Two conditions are met in such inference from Q_1 and E_1 to Q_2. First, if the initial question (Q_1, question-premise) may be truthfully answered (we shall call such questions *sound*), and if the evidence gathered (E_1) is true, then the resulting question (Q_2, question-conclusion) also may be answered truthfully (that is, it is sound as well). Second, answering the question-conclusion is useful (in the sense outlined above) in answering the question-premise: each answer to the question-conclusion, in view of the available evidence, narrows down the set of possibilities offered by the question-premise.[5] For the very same reason, in order to answer Q_1, it makes perfect sense to ask Q_3, in view of E_1 and E_2:

E_2 Queen of Hearts invites for a tea-party only these courtiers who made her laugh the previous day.

Q_3 Which courtiers made the Queen of Hearts laugh the previous day?

[5] What counts as direct answers to Q_2 are lists of courtiers who attended the event. Thus, if the complete payroll of the Queen's courtiers is $A_1, A_2, \ldots, A_n$, and the list of attendees comprises $A_1, A_2, \ldots, A_k$ $(k < n)$, then in view of the fact that 'It is one of the courtiers of the Queen of Hearts attending the afternoon tea-party that stole the tarts.' Q_2 quite intuitively narrows down the set of possibilities offered by Q_1: 'Who stole the tarts?'. Naturally, there are different answers to Q_2 possible, identifying different lists of attendees (e.g., partial or indirect ones). However, to fulfill the first condition of erotetic implication (the transmission of truth/soundness into soundness) we need to stick to the direct answers only.

Before providing formal definition of e-implication, we will introduce the necessary concepts.

Definition 2.1 (Partition of $\mathcal{L}_{\mathbf{K3}}$). Let $\mathcal{D}_{\mathcal{L}^?_{\mathbf{K3}}}$ be the set of declarative well-formed formulas of language $\mathcal{L}^?_{\mathbf{K3}}$. By a *partition of language* $\mathcal{L}^?_{\mathbf{K3}}$ we mean an ordered pair $\mathsf{P} = \langle \mathsf{T}_\mathsf{P}, \mathsf{U}_\mathsf{P} \rangle$ such that:

(i) $\mathsf{T}_\mathsf{P} \cap \mathsf{U}_\mathsf{P} = \emptyset$
(ii) $\mathsf{T}_\mathsf{P} \cup \mathsf{U}_\mathsf{P} = \mathcal{D}_{\mathcal{L}^?_{\mathbf{K3}}}$.

By a *partition of the set* $\mathcal{D}_{\mathcal{L}^?_{\mathbf{K3}}}$, we mean a partition of language $\mathcal{L}^?_{\mathbf{K3}}$. If for a certain partition P and a d-wff A, $A \in \mathsf{T}_\mathsf{P}$, then we say that A is *true in partition* P; otherwise, A is *untrue in* P. What is essential for the semantics of $\mathcal{L}^?_{\mathbf{K3}}$ is the notion of a **K3**-admissible partition. Therefore, we first define the notion of a **K3**-*assignment* as a function VAR $\to \{0, {}^1\!/_2, 1\}$. Next, we extend **K3**-assignments to **K3**-*valuations* according to the truth-tables of **K3**. Now we are ready to present:

Definition 2.2 (Admissible partition of $\mathcal{L}_{\mathbf{K3}}$). We will say that partition P is **K3**-*admissible* provided that for a **K3**-valuation V, the set T_P consists of formulas true[6] under V and the set U_P consists of formulas which are not true under V.

Now we can provide definitions of sound and safe questions.

Definition 2.3 (Soundness). A question Q is called *sound under a partition* P provided that some direct answer to Q is true in P.

Definition 2.4 (Safety). We will call a question Q *safe*, if Q is sound under each **K3**-admissible partition.

We will make use of the notion of a multiple-conclusion entailment [see 22, 23] that denotes a relation between sets of declarative well-formed formulas generalising the standard relation of entailment.

Definition 2.5 (Multiple-conclusion entailment in $\mathcal{L}^?_{\mathbf{K3}}$). Let X and Y be sets of declarative well-formed formulas of language $\mathcal{L}^?_{\mathbf{K3}}$. We say that X *mc-entails* Y in $\mathcal{L}^?_{\mathbf{K3}}$, in symbols $X \mathrel{||\!\!=}_{\mathcal{L}^?_{\mathbf{K3}}} Y$, if there is no **K3**-admissible partition $\mathsf{P} = \langle \mathsf{T}_\mathsf{P}, \mathsf{U}_\mathsf{P} \rangle$ of $\mathcal{L}^?_{\mathbf{K3}}$ such that $X \subseteq \mathsf{T}_\mathsf{P}$ and $Y \subseteq \mathsf{U}_\mathsf{P}$.

Let us now introduce the notion of erotetic implication in $\mathcal{L}^?_{\mathbf{K3}}$.

[6] Out of the three values, we count 1 as 'true' and the remaining ones as 'untrue' in the standard way.

Definition 2.6 (Erotetic implication in $\mathcal{L}^?_{\mathbf{K3}}$). Let Q and Q^* stand for questions of $\mathcal{L}^?_{\mathbf{K3}}$ and let X be a set of d-wffs of $\mathcal{L}^?_{\mathbf{K3}}$. We will say that Q $\mathcal{L}^?_{\mathbf{K3}}$-*implies* Q^* *on the basis of* X, in symbols $\mathbf{Im}_{\mathcal{L}^?_{\mathbf{K3}}}(Q, X, Q^*)$, if

1. for each $A \in \mathbf{d}Q$, $X \cup \{A\} \Vert\!\!= _{\mathcal{L}^?_{\mathbf{K3}}} \mathbf{d}Q^*$, and
2. for each $B \in \mathbf{d}Q^*$, there is a non-empty proper subset Y of $\mathbf{d}Q$ such that $X \cup \{B\} \Vert\!\!= _{\mathcal{L}^?_{\mathbf{K3}}} Y$.

The first clause of the above definition warrants the *transmission of soundness* (of the implying question Q) *and truth* (of the declarative premises in X) *into soundness* (of the implied question Q^*). The second clause expresses the property of "open-minded cognitive usefulness" of e-implication; that is, the fact that each answer to the implied question Q^* narrows down the set of direct answers to the implying question Q.

Definition 2.7 (Pure erotetic implication). A question Q implies a question Q_1 (in symbols, $\mathbf{Im}_{\mathcal{L}^?_{\mathbf{K3}}}(Q, Q_1)$) iff:

1. for each $A \in \mathsf{d}Q$: $A \Vert\!\!= _{\mathcal{L}^?_{\mathbf{K3}}} \mathsf{d}Q_1$, and
2. for each $B \in \mathsf{d}Q_1$ there exists a non-empty proper subset Y of $\mathsf{d}Q$ such that $B \Vert\!\!= _{\mathcal{L}^?_{\mathbf{K3}}} Y$.

Let us consider examples of e-implication in $\mathcal{L}^?_{\mathbf{K3}}$ (more examples may be found in [10]).

$$\mathbf{Im}_{\mathcal{L}^?_{\mathbf{K3}}}(?\{A * B, \neg(A * B), \boxminus(A * B)\}, ? \pm \boxminus|A, B|)$$

(where $*$ stands for any of the $\wedge, \vee, \rightarrow, \leftrightarrow$)

$$\mathbf{Im}_{\mathcal{L}^?_{\mathbf{K3}}}(? \pm \boxminus|A, B|, ?\{A, \neg A, \boxminus A\})$$

$$\mathbf{Im}_{\mathcal{L}^?_{\mathbf{K3}}}(?\{A, \neg A, \boxminus A\}, \{A \leftrightarrow B \wedge C, \boxminus B\}, ?\{C, \neg C, \boxminus C\}) \qquad (\star)$$

2.2. Erotetic search scenarios

When one thinks about e-implication used for decomposing questions as described above, it is easy to imagine that it might be repetitively applied while solving a particularly complex problem. As a result, we shall obtain a sequence of questions such that answers to these questions, gathered consecutively, are instrumental in answering the initial question. The intuition behind such a process is perfectly grasped under:

> (Erotetic Decomposition Principle) Transform a principal question into auxiliary questions in such a way that: (a) consecutive auxiliary questions are dependent upon the previous questions and, possibly, answers

to previous auxiliary questions, and (b) once auxiliary questions are resolved, the principal question is resolved as well. [37, p. 103]

This leads us to the notion of an *erotetic search scenario*. As the name suggests, it is a scenario for solving a problem expressed in the form of a question. The pragmatic intuition behind the e-scenario is that it:

> [...] provides information about possible ways of solving the problem expressed by its principal question: it shows what additional data should be collected if needed and when they should be collected. What is important, an e-scenario provides the appropriate instruction for every possible and just-sufficient, i.e., direct answer to a query: there are no "dead ends". [36, p. 110]

In this paper — modelled after [37] — we will present the e-scenario as a family of interconnected sequences of the so-called erotetic derivations.[7] Erotetic derivation is defined as follows [37, pp. 110–111]:

DEFINITION 2.8 (Erotetic derivation). A finite sequence $\mathtt{s} = \mathtt{s}_1, \ldots, \mathtt{s}_n$ of wffs is an erotetic derivation (e-derivation for short) of a direct answer A to question Q on the basis of a set of d-wffs X iff $\mathtt{s}_1 = Q$, $\mathtt{s}_n = A$, and the following conditions hold:

(1) for each question $\mathtt{s}_k$ of $\mathtt{s}$ such that $k > 1$:
  (a) $\mathbf{d}\mathtt{s}_k \neq \mathbf{d}Q$,
  (b) $\mathtt{s}_k$ is implied by a certain question $\mathtt{s}_j$ that precedes $\mathtt{s}_k$ in $\mathtt{s}$ on the basis of the empty set, or on the basis of a non-empty set of d-wffs such that each element of this set precedes $\mathtt{s}_k$ in $\mathtt{s}$, and
  (c) $\mathtt{s}_{k+1}$ is either a direct answer to $\mathtt{s}_k$ or a question;
(2) for each d-wff $\mathtt{s}_i$ of $\mathtt{s}$:
  (a) $\mathtt{s}_i \in X$, or
  (b) $\mathtt{s}_i$ is a direct answer to $\mathtt{s}_{i-1}$, where $\mathtt{s}_{i-1} \neq Q$, or
  (c) $\mathtt{s}_i$ is entailed by a certain non-empty set of d-wffs such that each element of this set precedes $\mathtt{s}_i$ in $\mathtt{s}$.

An e-derivation is *goal-directed*: it leads from an initial question Q to a direct answer to this question. Clause (1a) of the above definition requires that an auxiliary question (i.e., a question of an e-derivation

[7] See [34, 35], where the idea of e-scenarios has been presented for the first time. It is worth mentioning that e-scenarios can also be viewed as labelled trees [see 9].

different from Q) appearing in an e-derivation should have different direct answers than the initial question Q. Clause (1b) amounts to the requirement that each question of the e-derivation which is different from the initial question Q must be e-implied by some earlier item(s) of the e-derivation. Clause (1c) requires that an immediate successor of an auxiliary question in the e-derivation must be a direct answer to that question or a further auxiliary question. Clause (2) enumerates reasons for which a d-wff may enter an e-derivation. Such a d-wff may be: (2a) an element of a set of d-wffs X; (2b) a direct answer to an auxiliary question; (2c) a consequence of earlier d-wffs.

Definition 2.9 (Erotetic search scenario). A finite family Σ of sequences of wffs is an erotetic search scenario (e-scenario for short) for a question Q relative to a set of d-wffs X iff each element of Σ is an e-derivation of a direct answer to Q on the basis of X and the following conditions hold:

(1) $\mathbf{d}Q \cap X = \emptyset$;
(2) Σ contains at least two elements;
(3) for each element $\mathtt{s} = \mathtt{s}_1, \ldots, \mathtt{s}_n$ of Σ, for each index k, where $1 \leqslant k < n$:
   (a) if $\mathtt{s}_k$ is a question and $\mathtt{s}_{k+1}$ is a direct answer to $\mathtt{s}_k$, then for each direct answer B to $\mathtt{s}_k$: the family Σ contains a certain e-derivation $\mathtt{s}^* = \mathtt{s}^*_1, \mathtt{s}^*_2, \ldots, \mathtt{s}^*_m$ such that $\mathtt{s}_j = \mathtt{s}^*_j$ for $j = 1, \ldots, k$, and $\mathtt{s}^*_{k+1} = B$;
   (b) if $\mathtt{s}_k$ is a d-wff, or $\mathtt{s}_k$ is a question and $\mathtt{s}_{k+1}$ is not a direct answer to $\mathtt{s}_k$, then for each e-derivation $\mathtt{s}^* = \mathtt{s}^*_1, \mathtt{s}^*_2, \ldots, \mathtt{s}^*_m$ in Σ such that $\mathtt{s}_j = \mathtt{s}^*_j$ for $j = 1, \ldots, k$ we have $\mathtt{s}_{k+1} = \mathtt{s}^*_{k+1}$.

The e-scenario has a tree-like structure, with the main question as the root and direct answers to it as leaves. Other questions are auxiliary. An auxiliary question has another question as the immediate successor or it has all the direct answers to it as the immediate successors. In the latter case, the immediate successors represent the possible ways in which the relevant request for information can be satisfied, and the structure of the e-scenario shows what further information requests (if any) are to be satisfied in order to arrive at an answer to the main question. Schema of an exemplary e-scenario of $\mathcal{L}^?_{\mathbf{K3}}$ is presented in Figure 1 ([30] offer some more natural language examples). E-derivations being elements of an e-scenario will be called *paths* of this e-scenario. If an auxiliary question is a 'branching point' of an e-scenario, it is called a *query* of the e-scenario

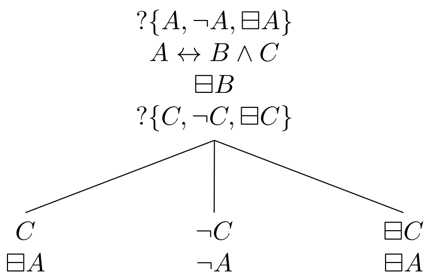


Figure 1. Schema of an e-scenario for the question $?\{A, \neg A, \boxminus A\}$ relative to the set of premises $A \leftrightarrow B \wedge C, \boxminus B$ (notice that the lack of knowledge about B is expressed in the premises)

[37, pp. 112–113]. Among auxiliary questions, only queries are to be asked; the remaining auxiliary questions serve as erotetic premises only. In the scenario presented in Figure 1 there is only one auxiliary question, $?\{C, \neg C, \boxminus C\}$, and it is a query — see erotetic implication example ($\star$) on p. 8. Auxiliary questions which are not queries are required in erotetic search scenarios because erotetic implication is not transitive.

An introduction of ternary questions and e-scenarios for them allows us to express certain pragmatic features of the e-scenarios. Let us imagine that an agent wants to establish whether A is the case. The agent knows that: $A \leftrightarrow B \wedge C$, but knows nothing about B. We may now imagine that the agent solves his/her problem according to the e-scenario presented in Figure 1 (as can be observed, the agent's premise and the fact that $\boxminus B$ are incorporated in the initial premises of the e-scenario).

In this example, $\boxminus B$ might be treated as an information gap. However, sometimes we do not need a complete information concerning the involved states of affairs in order to solve a complex problem. It can be observed that there is one possible course of events that will lead to the answer to the initial question despite a lack of knowledge about B — namely the case where the answer to the question $?\{C, \neg C, \boxminus C\}$ is negative (then the answer to the initial question is also negative). We may say that the proposed e-scenario offers three cognitive situations (from most to least preferable) — for a more detailed discussion, see [10]:

1. A 'maximal' cognitive situation is represented by the path going through the answer $\neg C$, because it leads to $\neg A$, i.e., a definite answer to the initial question.

2. A ‘minimal’ cognitive situation is reflected by the path which goes through the answer C, as in this situation, the questioning process ends up with some knowledge gains (despite the fact that we did not manage to solve the initial problem, we know C).
3. A ‘zero knowledge’ cognitive situation is represented by the third path going through $\boxminus C$, because it finishes the questioning process without any knowledge gains.

We shall consider only situation No. 1. as offering a successful solution to the initial problem: it is the only e-derivation in the scenario — or a path of it — leading to an answer which decides an answer for the initial problem. Further on we shall call a path of a scenario *successful* iff it does not end with an answer of the form $\boxminus A$ to the initial question.

E-scenario as a questioning agenda. E-scenarios are a convenient tool for representing questioning agendas for information seeking contexts. First of all, their formal properties are well suited for this task. E-scenarios are intuitive because the e-implication and e-scenarios address the idea of goal-directness. Due to the concept of the Minimal Erotetic Semantics [37], e-scenarios are also universal (it is possible to extend a given logic with questions and erotetic concepts as long as it meets certain requirements). E-scenarios have also the Golden Path property [see 35], which guarantees that when the requirements are met, e-scenario will lead us to at least one solution to the initial problem. What is more, [37] introduces certain well-defined operations on e-scenarios which allows the merging of two e-scenarios or to contract a given part of e-scenario. They prove also useful as a normative yardstick for empirical research (like the one based on the corpus study data [11, 12, 13]; Games With a Purpose [14, 16]; and the psychometric tool named Erotetic Reasoning Test [28, 29]).

3. Dialogue Erotetic Games: $\boldsymbol{DL(IEL)_{\mathrm{mult}}}$

When one thinks about e-scenarios involved in the group problem solving, two intuitive stages of the process can be distinguished.

1. *Preparation*: e-scenario generation. This step might be viewed as a preparatory step to the actual problem solving. At this level, the initial problem is analysed and decomposed into simpler sub-problems. As a result, an agent is designing an e-scenario for the main question.

2. E-scenario *execution*. This step is the actual problem solving process. The agent will ask questions, gain answers, and put the collected information together to reach the solution. The execution stage might be viewed as the activation of one of the e-scenario's paths. After the execution is finished, we are left with one path of our initial e-scenario leading from the initial question, through auxiliary ones and their answers, to the answer to the main question (i.e., a solution to the main problem).

After [11, Ch. 4] we shall introduce our framework for dialogue games: $DL(IEL)_{\mathrm{mult}}$. This framework allows to describe the process of e-scenario execution. We should stress here that the language in which the dialogue game is conducted is a metalanguage for the language $\mathcal{L}^{?}_{\mathbf{K3}}$ introduced in Section 2. The questioning agendas used by the game participants (formulated as e-scenarios) are expressed in $\mathcal{L}^{?}_{\mathbf{K3}}$ and $DL(IEL)_{\mathrm{mult}}$ allows for grasping the interaction of agents when they execute these scenarios.

3.1. Locutions, interaction rules and commitment store rules

A dialogue is a k-step finite game between Bob and a finite number of agents (information sources) $\mathsf{IS}_1, \ldots, \mathsf{IS}_n$. Each move is done by a locution event performed by one of the players (done one-by-one).

Bob is playing a game accordingly to a previously prepared questioning agenda (e-scenario). Bob starts the game with the first auxiliary question of this e-scenario which is followed by direct answers to it (query). Then he moves down the tree structure in accordance to the gathered information. As Bob uses the e-scenario to decompose the main question he never asks the main question directly to the information sources [see, e.g., discussion in 27].

We accept certain assumptions concerning agents involved in a game:

(i) As the game is conducted in a metalanguage for the language $\mathcal{L}^{?}_{\mathbf{K3}}$ the agents have the capacity to jump into a metalanguage.

(ii) Each party in the game is assumed to be honest. Bob is obliged to provide correct logical statements during the game and information sources are expected to provide information according to the best of their knowledge.[8]

[8] Such assumptions are usually made when IEL framework is used to model information seeking procedures (see, e.g., [11, Chs. 3 and 4], [12] or [15], see also for the empirical study of e-scenarios based strategies [14]).

As for the interactions in the game we assume that:

(a) There is no other communication between information sources.
(b) Bob asks questions, and the information sources provide answers.
(c) Bob always asks only one information source at a time.

Each move in the game is presented as follows $\langle n, \phi, \psi, U \rangle$, where:

- n ($1 \leqslant n \leqslant k$) is a number of the step of the dialogue;
- ϕ is an agent producing the utterance;
- ψ is an addressee;
- U is the locution of the agent ϕ.

The following types of locution are allowed in $DL(IEL)_{\mathrm{mult}}$:

Categorical statement: A, $\neg A$, $A \wedge B$, $A \leftrightarrow B$, $A \leftrightarrow B$, and $\boxminus A$.
Question: $?\{A_1, A_2, ..., A_n\}$. Questions asked by Bob.
Logical statement: Justifications provided by Bob, stating that an e-implication holds between certain question and question being attacked by an agent who is an IS.
Challenge: An IS attack on a question asked by Bob.
Withdrawal: An IS statement, that she does not want to answer a question.

In $DL(IEL)_{\mathrm{mult}}$ interaction between players is governed by the following rules. Let us remind that Bob plays the game with an e-scenario in the background. The e-scenario is executed by Bob from top to bottom. This provides the enumeration of questions appearing in the game as Bob reads them going downwards the tree like structure.

(In0) (`GameStart`) The game starts with Bob asking the first query from his e-scenario to the first information source. The following is introduced into the game at hand: $\langle 1, Bob, \mathsf{IS}_1, Q_1 \rangle$. (As we discuss in Section 3.3 Bob is free in ordering the information sources.)
(In1) (`Repstat`) No locution may occur if it is in the commitment store of any of the participants (see commitment store rules below).
(In2) (`Q-response`) When $\langle n, Bob, \mathsf{IS_i}, Q_i \rangle$ is contained in the game at hand, then:
1. $\langle n+1, \mathsf{IS_i}, Bob, ans(Q_i) \rangle$ is introduced to the game; or
2. $\langle n+1, \mathsf{IS_i}, Bob, CH(Q_i) \rangle$ is introduced to the game; or
3. $\langle n+1, \mathsf{IS_i}, Bob, WD(Q_i) \rangle$ is introduced to the game at hand.

The rule states that when Bob asks a certain query from his e-scenario, an information source involved in the game at a given time

($\mathsf{IS_i}$) may react by (1) simply providing a direct answer to this question (which is one of the immediate successors of the query in Bob's e-scenario); (2) by challenging the question (i.e., demanding an erotetic justification for the question: "Why are you asking this question? Provide a reason for this."); or (3) by withdrawal — i.e., informing Bob that he/she does not want to answer the question. Observe that reaction (1) allows Bob to move downwards his background e-scenario for the game, and leads him to (In3). Reaction (2) leads Bob to rule (In5) and requires him to provide a logical statement (established on the basis of an appropriate path of the background e-scenario). Reaction (3) does not require Bob to check the underlying e-scenario but it is treated as a form of declaration concerning the cooperativeness level of a given information source, and leads Bob to (In7).

(In3) (`Q-answer`) When $\langle n, \mathsf{IS_i}, Bob, ans(Q_i)\rangle$ appears in the game at hand, and $ans(Q_i) \neq \boxminus A$, then in the next step $\langle n+1, Bob, \mathsf{IS_i}, Q_{i+1}\rangle$ is introduced to the game.
When a direct answer to Q_i is provided by the information source Bob using his background e-scenario to identify another question to be asked Q_{i+1} (by going downwards the tree-like structure).

(In4) (`IgnoranceResp`) When $\langle n, \mathsf{IS_i}, Bob, ans(Q_i)\rangle$ is contained in the game at hand, and $ans(Q_i) = \boxminus A$, then:
1. $\langle n+1, Bob, \mathsf{IS_i}, Q_{i+1}\rangle$ is introduced to the game under the condition that Bob identifies a successful path (see page 12) of his background e-scenario.
2. $\langle n+1, Bob, \mathsf{IS_{i+1}}, ?Q_i\rangle$ is introduced to the game if Bob does not identify a successful path in his background e-scenario; in that case Bob starts a sub-game with next information source.

In the first case Bob may still play the game with a given information source as there is still the possibility (pointed by his background e-scenario) that a solution to the main question will be reached despite answer of the form $\boxminus A$ provided. Thus In4.1. results in next question from the e-scenario asked to the information source (Bob is going downwards his e-scenario). If Bob's e-scenario does not point such a possibility, Bob is changing the information source and asking the same question again.

(In5) (`Q-challenge`) When $\langle n, \mathsf{IS_i}, Bob, CH(Q_i)\rangle$ appears in the game at hand, then in the next step $\langle n+1, Bob, \mathsf{IS_i}, LS(Q_i)\rangle$ is introduced to the game.
This regulates Bob's reaction to $\mathsf{IS_i}$ challenging the question asked.

He is obliged to provide a logical statement for that question (in our case it should be a statement of the form: $\mathbf{Im}_{\mathcal{L}^?_{\mathbf{K3}}}(Q, X, Q_i)$ or $\mathbf{Im}_{\mathcal{L}^?_{\mathbf{K3}}}(Q, Q_i)$, where Q_i is the challenged question). The intuition is that Bob will state that the challenged question is e-implied (or purely e-implied) by one of the previous questions (possibly on the basis of Bob's declarative premises and answers already provided by $\mathsf{IS_i}$). Such a statement is easily obtainable by Bob on the basis of his e-scenario for the game.[9]

(In6) (`Q-ChallengeResp`) When $\langle n, Bob, \mathsf{IS_i}, LS(Q_i)\rangle$ appeared in the game at hand, then:
1. $\langle n+1, \mathsf{IS_i}, Bob, ans(Q_i)\rangle$ is introduced to the game; or
2. $\langle n+1, \mathsf{IS_i}, Bob, WD(Q_i)\rangle$ is introduced to the game.

After the logical statement for the challenged question is provided by Bob, $\mathsf{IS_i}$ may (1) provide the answer or (2) withdraw. We assume that both $\mathsf{IS_i}$ and Bob accept the normative yardstick for erotetic reasoning provided by e-implication. Because of this, we may say that $\mathsf{IS_i}$, provided with the logical statement for the challenged question of the form proposed in (In5), will accept the relevance of the justified question. The acceptance leads to providing a direct answer to the challenged question (and just like in rule In2.1. it allows Bob to move downwards the background e-scenario). However, despite the logical reasons, $\mathsf{IS_i}$ is still allowed to act non-cooperatively and withdraw from providing any information.

(In7) (`NoSol`) When $\langle n, \mathsf{IS_i}, Bob, WD(Q_i)\rangle$is contained in the game at hand, then: $\langle n+1, Bob, \mathsf{IS_{i+1}}, ?Q_i\rangle$ is introduced to the game. Bob starts a sub-game with next information source as a consequence of non-cooperative move of $\mathsf{IS_i}$.

(In8) (`SubGameEnd`) rule to end a sub-game. Ending a sub-game depends on the purpose for which the sub-game was initiated. The distinction here is aimed at grasping the cooperativeness of an information source. With the honesty assumption the withdrawal is the only move of an information source in our approach which is linked

[9] Let us for example imagine that at certain step of a game Bob is asking a question $?\{q, \neg q, \boxminus q\}$. Bob is playing this game on the basis of the e-scenario presented in Figure 2. IS challenges the question asked. Bob localise $?\{q, \neg q, \boxminus q\}$ in the e-scenario and goes upwards the tree. The LS that he will return in the game will be the following $\mathbf{Im}_{\mathcal{L}^?_{\mathbf{K3}}}(?\pm\boxminus|p, q|, p, ?\{q, \neg q, \boxminus q\})$. Note that Bob knows all the e-implications used in this e-scenario as he used them during the e-scenario generation phase.

to his/her 'decision' not to reveal information to Bob.

1. If a sub-game is a result of (`IgnoranceResp`), then after obtaining a response to a given question, Bob ends the instance of a sub-game with a new information source and gets back to the main play with the first agent.
2. If a sub-game is a result of (`NoSol`), then Bob continues the sub-game with a new information source $\mathsf{IS}_{\mathsf{i+1}}$.

(In9) (`GameEnd`) The game ends when Bob executed the entire background e-scenario. At the leaf of the executed path Bob reads the answer to the initial question of his e-scenario (SOL). The following is introduced to the game: $\langle k, Bob, \mathsf{IS}_\mathsf{i}, SOL\rangle$. Bob's categorical statement is the answer to the initial question of his e-scenario. The addressee is the last information source involved in the dialogue.

In $DL(IEL)_{\mathrm{mult}}$, we assume that Bob is communicating with information sources in a private way. This means that the exchange of information is not public — as a consequence, only commitment stores of players engaged in a certain part of a dialogue will change respectively. The table below presents six rules used for commitment stores management during a game. In Bob's commitment store, we use ordered pairs in which the second element points at the information source involved in that part of a game. After a formula is introduced to the game on the basis of one of the rules above it is added (+) or removed (−) from the commitment store of a game participant.

Rule	**Locution**	**Bob's ComSt**	IS_i **ComSt**
(CS1)	Q_i	$+\langle Q_i, \mathsf{IS}_\mathsf{i}\rangle$	$+Q_i$
(CS2)	$ans(Q_i)$	$+\langle ans(Q_i), \mathsf{IS}_\mathsf{i}\rangle$	$+ans(Q_i)$
(CS3)	$CH(Q_i)$	$-\langle Q_i, \mathsf{IS}_\mathsf{i}\rangle$ $+\langle CH(Q_i), \mathsf{IS}_\mathsf{i}\rangle$	$-Q_i$ $+CH(Q_i)$
(CS4)	$LS(Q_i)$	$+\langle Q_i, \mathsf{IS}_\mathsf{i}\rangle$ $-\langle CH(Q_i), \mathsf{IS}_\mathsf{i}\rangle$	$+Q_i$ $+LS(Q_i)$
(CS5)	$WD(Q_i)$	$+\langle WD(Q_i), \mathsf{IS}_\mathsf{i}\rangle$	$+WD(Q_i)$
(CS6)	SOL	$+\langle SOL, \mathsf{IS}_\mathsf{i}\rangle$	$+SOL$

The first rule states that when a question appears in the game (asked by Bob on the basis of the background e-scenario) then it is added to the Bob's commitment store as well as to the commitment store of the addressee (i.e., a given information source).

The second rule states that the formula provided as an answer to Bob's question is added to commitment stores of Bob and the involved information source.

When a given question is challenged by the information source it has to be removed from ComSt of both participants and the information about the challenge is introduced there (CS3).

After Bob introduce the logical sequence for the challenged question the information about this is added to the ComSt of the information source and the information about the challenge is removed from Bob's ComSt. Observe that $CH(Q_i)$ is not removed from IS_i ComSt. Due to the In1 (`Repstat`) this information source will not be allowed to challenge the same question again. Additionally the challenged question is back again to ComsSt of both participants (CS4).

Rule five regulates the withdrawal situation — information about withdrawal is added to Bob's and IS's ComSt.

When the solution is communicated by Bob, CS6, it is added to ComSt of both participants.

It is important that for each described case Bob keeps track of the IS related to a given formula appearing in his ComSt.

3.2. $DL(IEL)_{\mathrm{mult}}$ Example

Let us consider an example of a $DL(IEL)_{\mathrm{mult}}$ game [modelled after 11, pp. 81–83]. Bob's questioning strategy is represented by the same e-scenario in Figure 2. The scenario starts with Bob's main question ($?\{p \wedge q, \neg(p \wedge q), \boxminus(p \wedge q)\}$) and ends with direct answers to it. It also points out which questions should be asked to information sources (branching points of the tree-like structure) and in which order it should be done. Bob starts the game (In0) with the first information source IS_1. Bob starts the game by asking the first query from his e-scenario, namely $?\{p, \neg p, \boxminus p\}$ (see the first branching point of the e-scenario presented in Figure 2).

In the second step of the game, IS_1 withdraws from answering Bob's question. As a consequence, Bob starts a sub-game with another information source IS_2 according to rule (In2.3).

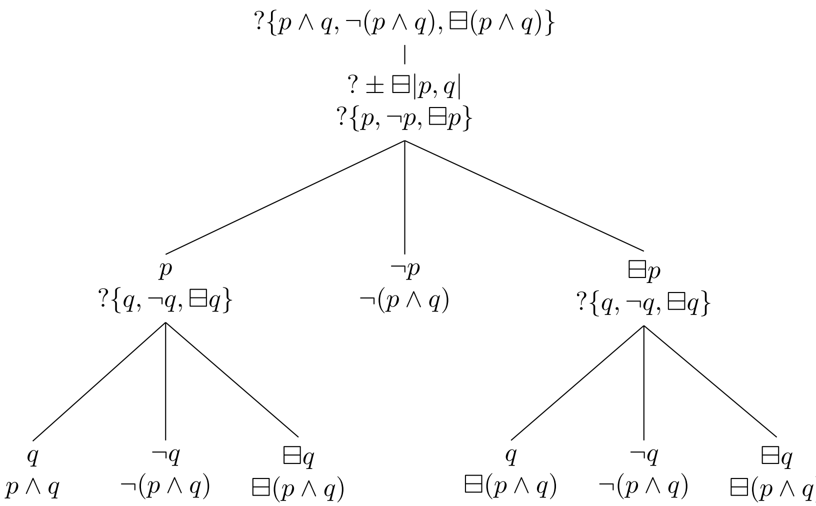


Figure 2. Questioners' strategy used in the presented $DL(IEL)_{\text{mult}}$ game example

Move	**Bob's ComSt**	IS_1 **ComSt**	**IR**
$\langle 1, Bob, \mathsf{IS}_1 ?\{p, \neg p, \boxminus p\}\rangle$	$+\langle ?\{p, \neg p, \boxminus p\}, \mathsf{IS}_1\rangle$	$+?\{p, \neg p, \boxminus p\}$	0
$\langle 2, \mathsf{IS}_1, WD(?\{p, \neg p, \boxminus p\})\rangle$	$+\langle WD(?\{p, \neg p, \boxminus p\}), \mathsf{IS}_1\rangle$	$+WD(?\{p, \neg p, \boxminus p\})$	2.3
$\vdots$	$\vdots$	$\vdots$	$\vdots$

In this sub-game with IS_2, Bob gathers information about p in the 4th step. On the basis of In3 Bob can now introduce another question accordingly to the underlying e-scenario (by going downwards the tree-structure). As it may be observed in Figure 2, after retrieving p, Bob should ask the question about q: $?\{q, \neg q, \boxminus q\}$. It is done in step 5. Unfortunately, in the 6th step, IS_2 declares a lack of knowledge with respect to q (i.e., $\boxminus q$); at this point Bob has no successful path in his e-scenario. Interaction rule (In4) states that when $\langle n, \mathsf{IS}_{\mathsf{i}}, Bob, ans(Q_i)\rangle$ and $ans(Q_i) = \boxminus A$, then Bob checks his strategy whether there is a successful path of his e-scenario. If not, then $\langle n+1, Bob, \mathsf{IS}_{\mathsf{i+1}}, ?Q_i\rangle$ is introduced to the game; Bob starts a sub-game with another information source.

In our example the only move left for Bob leads from the formula $\boxminus q$ to $\boxminus(p \wedge q)$, thus it is unsuccessful. That is why he needs to start a new sub-game with yet another information source. Observe that Bob will not get back to the initial source because the reason for quitting the game with IS_1 was a withdrawal (see `SubGameEnd`; In8.1). What is more — according to the same rule (In8.2) it is possible that Bob will get back to the IS_2 because the reason for leaving this game is only a lack of necessary information to solve the problem. Due to (In4.2) Bob starts another sub-game with IS_3 asking the question concerning q again.

Move	**Bob's ComSt**	IS_2 **ComSt**	**IR**
⋮	⋮	⋮	⋮
$\langle 3, Bob, \mathsf{IS}_2, ?\{p, \neg p, \boxminus p\}\rangle$	$+\langle ?\{p, \neg p, \boxminus p\}, \mathsf{IS}_2\rangle$	$+?\{p, \neg p, \boxminus p\}$	7
$\langle 4, \mathsf{IS}_2, Bob, p\rangle$	$+\langle p, \mathsf{IS}_2\rangle$	$+p$	2.1
$\langle 5, Bob, \mathsf{IS}_2, ?\{q, \neg q, \boxminus q\}\rangle$	$+\langle ?\{q, \neg q, \boxminus q\}, \mathsf{IS}_2\rangle$	$+?\{q, \neg q, \boxminus q\}$	3
$\langle 6, \mathsf{IS}_2, Bob, \boxminus q\rangle$	$+\langle \boxminus q, \mathsf{IS}_2\rangle$	$+\boxminus q$	2.1
⋮	⋮	⋮	⋮

Move	**Bob's ComSt**	IS_3 **ComSt**	**IR**
⋮	⋮	⋮	⋮
$\langle 7, Bob, \mathsf{IS}_3, ?\{q, \neg q, \boxminus q\}\rangle$	$+\langle ?\{q, \neg q, \boxminus q\}, \mathsf{IS}_3\rangle$	$+?\{q, \neg q, \boxminus q\}$	4.2
$\langle 8, \mathsf{IS}_3, Bob, q\rangle$	$+\langle q, \mathsf{IS}_3\rangle$	$+q$	2.1
⋮	⋮	⋮	⋮

In the 8th step IS_3 provides the answer to the question about q. All the necessary information is collected at this point in the game, and Bob may reach the solution in his e-scenario. That is the reason why there is no need to come back to the sub-game with IS_2. The last thing to do

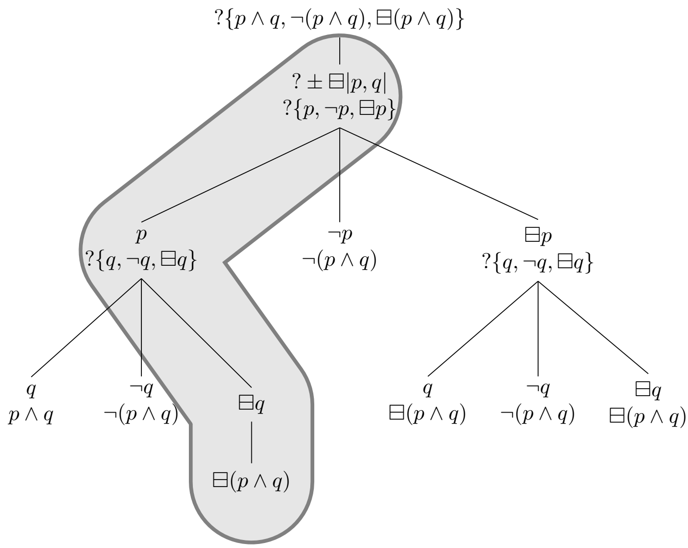


Figure 3. Questioners' strategy check in the 6th step of the game

is to communicate the solution to all the the last agent involved in the information gaining — see rule (In9) — i.e., IS_3.

$\vdots$	$\vdots$	$\vdots$	$\vdots$
$\langle 9, Bob, (\mathsf{IS}_3,$ $SOL = p \wedge q\rangle$	$+\langle (p \wedge q), \mathsf{IS}_3\rangle$	$+p \wedge q$	In9

When we review Bob's ComSt we learn that the obtained solution is relative to the following information sources: IS_1, IS_2, IS_3. One may observe that a simple accounting of all agents involved in a dialogue with Bob (visible in Bob's ComSt) results in counting IS_1. This might not be an intuitive solution, because this agent actually withdrew from providing the answer to Bob's question. In order to avoid a situation like this, we may only take into account indices of the declarative formulas from Bob's ComSt. This solution will exclude agents that did not contribute to the solution of the initial problem — see ComSt rules for $DL(IEL)_{\text{mult}}$. If we analyse the final declarative part of Bob's ComSt we have:

Declarative formulas	Step in the game
$\langle p, \mathsf{IS}_2\rangle$	$\langle 4, \mathsf{IS}_2, Bob, p\rangle$
$\langle \boxminus q, \mathsf{IS}_2\rangle$	$\langle 6, \mathsf{IS}_2, Bob, \boxminus q\rangle$
$\langle q, \mathsf{IS}_3\rangle$	$\langle 8, \mathsf{IS}_3, Bob, q\rangle$

In this case, the solution $p \wedge q$ is relative only to agents $\mathsf{IS}_2, \mathsf{IS}_3$, who contributed to providing this solution to the initial problem.

3.3. Information sources' ranking based on the game performance

In the presented example of a $DL(IEL)_{\text{mult}}$ game, the questioner does not use any information sources' rankings for the sub-games. The rules of $DL(IEL)_{\text{mult}}$ by default allow Bob for any ordering of information sources. However, it is possible to supplement the sub-game starting rules (In4.2, In7 and In8) with such a ranking in order to introduce reasons for selecting particular information sources. There are a couple of possible criteria to be considered. The $DL(IEL)_{\text{mult}}$ games format offers a straightforward measure focusing on the level of cooperativeness of the agents, which is based on IS's performance during an instance of a game. The information about agents involved in reaching the solution to the initial problem allows us to obtain a simple ranking of information sources. We may refer to it as for the *ranking of cooperativeness*. To build this ranking, we will assign cooperativeness points for each declarative formula provided by an agent during the game. A simple (and arbitrary) point system may be the following:

(i) a formula of a form different than $\boxminus A$ earns the information source 1 point;
(ii) a formula of the form $\boxminus A$ earns the information source 0.5 point.

The output ranking is dynamic in the sense that it is updated after each game. This simple measure allows the questioner to choose information sources for appropriate sub-games on the basis of their previous cooperativeness. Thus the aforementioned game rules (In4.2, In7 and In8) would not refer to "another information source" but the information source with the highest position in the IS ranking.[10]

We may imagine that for the first game the questioner will choose a random order of players with 0 points of cooperativeness assigned to

[10] We thank the anonymous reviewer for pointing this out.

them (thus Bob is using the original rules for the first game). Then, after each game, the points are assigned and the order is adjusted accordingly to points (and for these games information sources are chosen on the basis of the ranking, so Bob is using modified rules).

For our exemplary game we would have (where $\mathsf{IS}^{\text{cooperativeness level}}_{\text{agent's number}}$):

Before the game	**After the game**
1. IS^0_1	1. $\mathsf{IS}^{1.5}_2$
2. IS^0_2	2. IS^1_3
3. IS^0_3	3. IS^0_1

Such a simple hierarchy helps the questioner in deciding which IS should be chosen for a given sub-game, based on the history of interactions with them. With time, such an approach will promote cooperative agents from a group of information sources.[11]

Notice that the proposed measure may also be interpreted along the lines of the agents being well-informed to different degrees, as an agent may offer the $\boxminus A$ answer due to non-cooperativeness as well as a simple lack of certain information. We could also refine the system by adding further points for agents who contributed to obtaining certain solutions to the initial problem. In what follows, we propose how this simple measure may be enriched with yet another assessment of an information source — this time related to his/her questioning agenda.

4. Group and individual question decomposition

In this section, we propose a way to enrich the simple measure of IS introduced above. We will also describe how a strategy of questioning may be divided into two levels: group and individual level. This approach is mainly inspired by the epistemic interpretation of IEL proposed in [15, 18, 19, 20, 24], however, we still use only formal tools introduced

[11] As pointed by the anonymous reviewer purely positive point system may lead to the situation when it is harder and harder to ask any previously unconsulted information source even though some of those that have been consulted withdraw in a high percentage of cases.

earlier in the current paper. For this purpose, we will start by introducing the idea of the *blackboard architecture*.[12]

4.1. Blackboard architecture and question decomposition

The following quotation summarises the main idea of the blackboard architecture.

> Imagine a group of human specialists seated next to a large blackboard. The specialists are working cooperatively to solve a problem, using the blackboard as the workplace for developing the solution. Problem solving begins when the problem and initial data are written onto the blackboard. The specialists watch the blackboard, looking for an opportunity to apply their expertise to the developing solution. When a specialist finds sufficient information to make a contribution, she records the contribution on the blackboard, hopefully enabling other specialists to apply their expertise. This process of adding contributions to the blackboard continues until the problem has been solved. [2, p. 1]

In the proposed model, we are able to express a situation in which a group of agents is solving a complex problem in a cooperative manner. The central element is the blackboard visible for all the agents. We have one main agent, called the Writer (who writes down questions and information on the blackboard), and other agents we will refer to as a, b, c, etc.) — see Figure 4. As for agents from the group, we assume that they have different knowledge and different credibility/expertise levels.

Let us imagine that the group attacks a complex question which cannot be resolved by any agent from the group by him/her alone — we illustrate this situation in Figure 4: Q represents this complex question and Γ the common knowledge. This question is then written down on the blackboard, along with the common knowledge of the group (which is available for, and shared by each group member). Afterwards, the initial group question is decomposed on a group level into a series of simpler questions (using the aforementioned common knowledge). This is illustrated in the schema by a double-lined arrow leading to Q^*. As the main question Q cannot be solved only with the use of Γ, this first group-level question decomposition process will not lead to an answer to Q.

In what follows, this simpler question (Q^*) is analysed by group members separately with respect to their own knowledge (Figure 4 presents

[12] The authors would like to thank Michal Peliš for suggesting this useful framework.

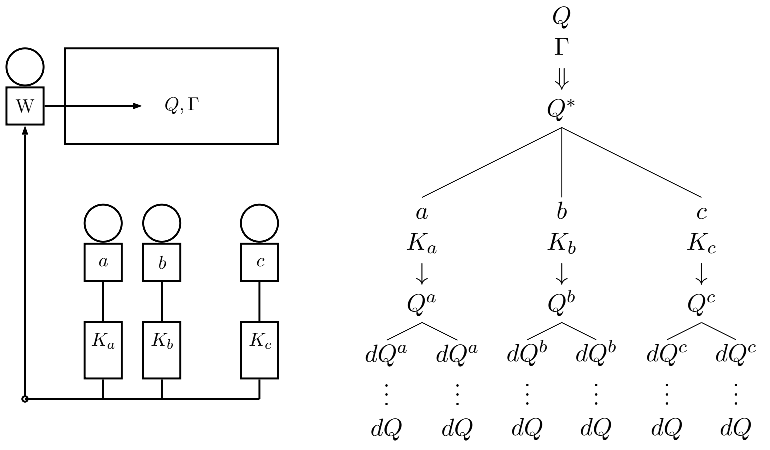


Figure 4. The blackboard architecture schema for the cooperative problem solving with distributed information sources. The right a schematic representation of a meta-search-scenario illustrating the idea of group and individual levels of the decomposition of the initial question

three agents a, b and c). At this level, e-scenarios for each agent are introduced (Q^a, Q^b and Q^c respectively). This initiates question decomposition on the level of agents, which is represented by a single-lined arrow in the schema. The last step is collecting the solutions to these auxiliary questions by the Writer and establishing the answer to the initial question.

Let us consider an example of such a complex decomposition — see Figure 5 for a schematic representation. The initial question for a group of agents consisting of a questioner and three information sources is $?\{A, \neg A, \boxminus A\}$. Let us assume that the blackboard contains two declarative premises: $A \leftrightarrow (C \wedge B)$ and $\boxminus B$. This information allows for group decomposition of the initial question and leads to the auxiliary questions $?\{C, \neg C, \boxminus C\}$ and $?\{B, \neg B, \boxminus B\}$. However, an answer to the question about B cannot be reached at the group level. Thus, this question is decomposed on the individual level of IS_1, IS_2, and IS_3, with the use of their private knowledge and their own e-scenarios. In Figure 5, this transition is indicated by dashed lines. IS_1, IS_2, and IS_3 develop e-scenarios according to their premises related to B. Here, we can observe that their scenarios will differ. Each leaf from the individual agendas is then connected to the solution to the initial question (again this transition to a group level

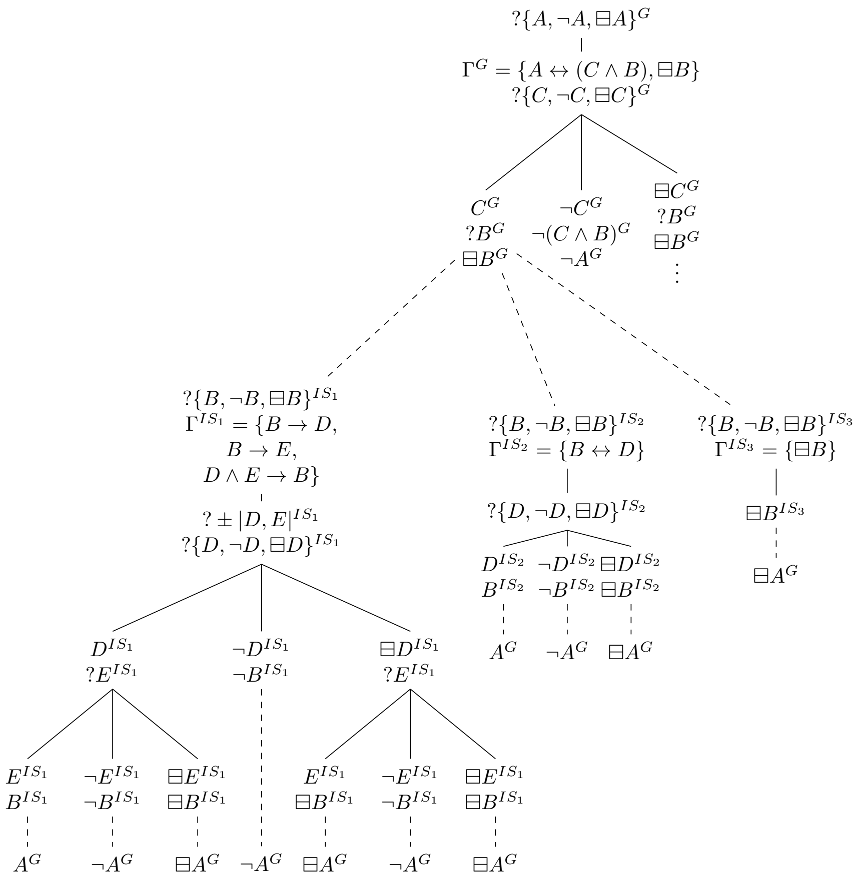


Figure 5. The example of a group and an individual question decomposition

is indicated by a dashed line). We should point here that the schema presented in Figure 5 does not constitute an e-scenario. Parts of the schema are representing schematic representations of the structures which may be developed into full-fledged e-scenarios described in Section 2.2.

4.2. Information sources' ranking based on their individual questioning agendas

As it is visible in the presented example, introducing the individual level of question decomposition results in different questioning strategies available for different agents. On this basis, we may propose another measure for information sources — this time related to their e-scenarios. A simple measure will use the number of successful paths (meaning: leading to a definite solution to the initial problem, that is, no $\boxminus$; see page 11). Such a measure is intuitive for the proposed approach. Let us remember that the initial question is first decomposed on a group level.

At this point, a questioning agenda is public for all the agents involved in the process. When it comes to the second level, i.e., the individual decomposition, our agents do not want to reveal their individual knowledge. Thus, it seems reasonable to assume that their agendas will not be known by other members of the group. There is no problem, however, in the questioner asking about certain features of information sources' questioning agendas. Here, we may use the number of premises used for the question decomposition, the number of queries for a given agenda, the depth of a search tree, or the number of successful paths of a given agenda. In our opinion, the last measure is straightforward and easy to establish, and it is also useful for our purpose. The individual level of decomposition appears at the point where group decomposition stops because certain auxiliary questions cannot be resolved. Consequently, agents develop their individual questioning agendas for this auxiliary question. For each agent we may establish the ration: number of successful paths/number of paths. The higher the number for such an agenda, the more likely that this particular search will end with success.

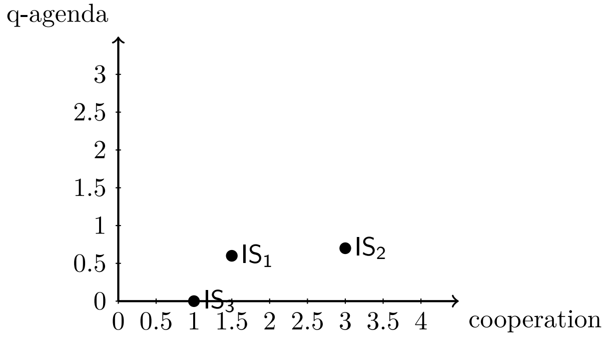
For our example presented in Figure 5, we have the following measures. An agent with a higher number would be a more preferred choice.

Inf. source	**No of succ. paths**	**No of paths**	**Ratio**
IS_1	4	7	0.6
IS_2	2	3	0.7
IS_3	0	1	0.0

We can now combine two of the proposed measures. Let us assume that cooperativeness level of the agents based on the previous steps is the following:

Information source	**Cooperativeness level**
IS_1	1.5
IS_2	3.0
IS_3	1.0

We can represent two introduced measures using the following graph.



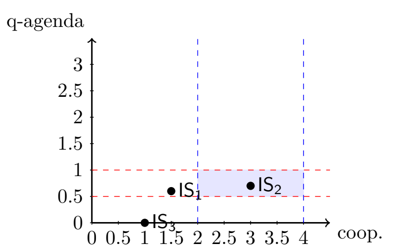
We may also use the previous convention and simply label each information source as follows:

$$\mathsf{IS}^{\text{(q-agenda measure, cooperativeness level)}}_{\text{agent's number}}$$

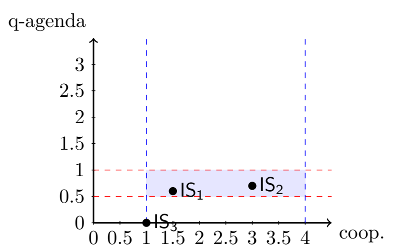
For our exemplary case, we would have: $\mathsf{IS}_1^{(0.6,1.5)}$; $\mathsf{IS}_3^{(0.7,3)}$; $\mathsf{IS}_3^{(0,1)}$.

By combining the cooperativeness and q-agenda measures for an information source, the questioner may optimise his/her choice of the information source at a given stage of a game.

Let us for example assume that the questioner is interested in information sources with a cooperativeness level ranging from 2 to 4 and with a questioning agenda measure between 0.5 and 1. As it is visible in the graph (A) below, such parameters will point to IS_2.



(A)



(B)

It may happen that the method will point out two agents. This would be the case when the questioner prefers the exemplary measures presented in graph (B).

In such a case, the decision about the order may be:

1. random;
2. simply based on information sources' initial numerals;
3. based on a measure which is preferred by the questioner, e.g., IS_2 has a slightly better score when it comes to q-agenda, thus it will be preferred over IS_1 (for the q-agenda measure).

This way we do not aggregate the evaluation criteria, and we avoid introducing trade-offs between them. As a result, what we are looking for is, in fact, a Pareto-optimal decision concerning information sources [3]. With just two criteria in play, this is relatively easy. Employing more criteria may lead to more complex cases, in which additional preferences may need to be added (like No. 3 above).

5. Summary

In this paper, a formal dialogue system $DL(IEL)_{\text{mult}}$ was presented. The underpinning of the system are concepts from IEL. Dialogue logic overly allows for expressing dynamics of interactions between agents. One of the system's advantages is its modularity. Modifications on the level of interaction rules may be introduced to obtain the desired behaviours of our agents — e.g., to generate cooperative behaviours in a dialogue, see [11, 12]. One may also adopt modified underlying normative concepts — e.g., use the e-scenario based on the notion of weak e-implication [29], falsificationist [5] or epistemic e-implication [15].

On the basis of the rules of $DL(IEL)_{\text{mult}}$, we proposed a measure of agents cooperation in dialogue. This allows us to generate agents' ranking based on their performance (willingness to contribute information).

We also proposed an extension of the framework based on the blackboard system. This step allows for differentiation between group and individual levels of question decomposition. As a consequence, we can introduce another measure for agent's ranking — this time based on certain properties of their e-scenarios (questioning agendas). In what follows, we illustrated how these two measures may be used by the questioner to optimise his/her choice of information sources.

Future works will cover the following:

- Including the rule for an information source choice based on the presented measures into $DL(IEL)_{\text{mult}}$.

- Handling inconsistencies between information provided by information sources.
- Developing more sophisticated measures based on a given information source questioning agenda.
- Enabling interactions between information sources engaged in the problem-solving process.

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