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## COMPOSITIONS OF FUZZY RELATIONS APPLIED TO VERIFYIFICATION LEARNING OUTCOMES ON THE EXAMPLE OF THE MAJOR "GEODESY AND CARTOGRAPHY"

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**Abstract** The paper presents discussion about using mathematical functions in order to help academic teachers to verify acquirement of learning outcomes by students on the example of the major "geodesy and cartography". It is relatively easy to build fuzzy relation describing levels of realization and validation learning outcomes during subject examinations and the fuzzy relation with students' grades is already built by teachers, the problem is to combine these two relations to get one which describes the level of acquiring learning outcomes by students. There are two main requirements facing this combinations and the paper shows that the best combination according to these requirements is algebraic composition.

**Keywords:** learning outcome, fuzzy relation, algebraic composition.

### *Introduction*

In 1999 Polish Under Secretary of State of National Education signed the Bologna Declaration<sup>1</sup> and began the reform of education, including the higher education in order to create A Europe of Knowledge and in the future the European Higher Education Area (EHEA).

<sup>1</sup> The Bologna Declaration of 19 June

1999 [http://www.ekspercibolonscy.pl/sites/ekspercibolonscy.org.pl/files/1999\\_EN\\_Bologna\\_Declaration.pdf](http://www.ekspercibolonscy.pl/sites/ekspercibolonscy.org.pl/files/1999_EN_Bologna_Declaration.pdf),  
2.04.2015.

One of important elements of the EHEA is the capacity of comparing educational achievements of graduates of different HEIs<sup>2</sup> in different countries. The solution of this problem was the European Qualification Framework (EQF), which “is a translation tool that helps communication and comparison between qualifications systems in Europe<sup>3</sup>”.

In Poland, the Polish Qualification Framework was described according to the EQF. The Ministry of Science and Higher Education issued a regulation with the description of the Polish Qualification Framework for Higher Education<sup>4</sup>. According to this regulation all HEIs had to describe the set of learning outcomes for each major in the range of knowledge, skills and competences.

### *Matrices of learning outcomes*

Learning outcomes, which were described for each major, have to be acquired by students and properly validated by academic teachers. In tab. 1, a few sample learning outcomes for the major “geodesy and cartography” are presented.

Tab. 1 Sample learning outcomes described for the major „geodesy and cartography”

<b>Symbol</b>	<b>Learning outcomes</b>
<b>Knowledge</b>	
K01	Graduate has knowledge of mathematics, physics and chemistry used in formulating and solving simple problems in the field of geodesy and cartography
K02	Graduate knows basic methods, techniques, tools and materials used in solving simple engineering tasks in the field of geodesy and cartography
K03	Graduate has basic knowledge of kinematics, electromagnetism, wave and geometric optics, acoustics and physics of the atmosphere
K04	Graduate has a basic knowledge of the management of the institution / company, including quality management
<b>Skills</b>	
S01	Graduate plans and carries out computer simulations, interprets the results and draws conclusions
S02	Graduate edits and develops basic maps and their derivatives
S03	Graduate presents in the Polish language oral presentation on specific issues in the field of geodesy and cartography
S04	Graduate obtains information from the literature, databases and other carefully selected sources in Polish and English; integrates the obtained information, makes its interpretation, draws conclusions, formulates and justifies opinions
<b>Competence</b>	
C01	Graduate understands the need for learning lifelong
C02	Graduate is able to identify properly the priorities for implementing tasks specified by themselves or other people

Source: the sample set of learning outcomes.

After describing the set of learning outcomes for the major, academic teachers have to check accordance of this set with the set of learning outcomes described in the Regulation<sup>5</sup>.

The next step, which can be done by experts-academic teachers, is preparing the matrix which gives the level of realization and validation of each learning outcome while studying each subject of the curriculum. Moreover, while preparing the matrix academic teachers check whether each subject is needed for realization learning outcomes and whether each learning outcome is properly taught by the set of subjects.

<sup>2</sup> Higher Education Institutions.

<sup>3</sup> [http://ec.europa.eu/ploteus/search/site?f%5B0%5D=im\\_field\\_entity\\_type%3A97](http://ec.europa.eu/ploteus/search/site?f%5B0%5D=im_field_entity_type%3A97), 22.04.2015.

<sup>4</sup> ROZPORZĄDZENIE MINISTRA NAUKI I SZKOLNICTWA WYŻSZEGO z dnia 2 listopada 2011 r. w sprawie Krajowych Ram Kwalifikacji dla Szkolnictwa Wyższego (D.U. z roku 2011 Nr 253 Poz. 1520).

<sup>5</sup> Ibidem.

The good tool for checking whether each subject is needed for realizing the set of learning outcomes is the matrix which rows contain learning outcomes and columns – subjects and each element of this matrix shows the level of realization of the learning outcome during studying the subject (comp. tab. 2).

Tab. 2 Matrix describing the level of realization of learning outcomes while studying subjects of the major „geodesy and cartography”

Subject \ Learning outcome	Mathematics (MAT)	Basic course in geodesy (BCG)	Geodesy and satellite navigation(GSN)	Adjustment calculus (ADC)	Physics (PH)	Management and land management (MLM)	Construction techniques (CT)	Geodesy and information technology(GIT)	C-GEO	Engineering graphics (EG)	Field exercises (FE)	Diploma thesis (DT)	Land information systems (LIS)	Geographic information systems (GIS)	Basics of earth and soil science (BESS)
K01	1.0														
K02		0.4	0.4	0.4											
K03					1										
K04						0.5	0.5								
S01								0.5	0.5	0.5					
S02													0.6	0.6	0.4
S03											0.5	1.0			
S04												1.0			
C01			0.5					0.5				0.6	0.5	0.5	
C02											0.5	0.5			

Source: the sample matrix.

After building the matrices which describe levels of realization and validation of the set of learning outcomes while studying subjects from the curriculum “geodesy and cartography”, the academic teachers have to consider rules and methods which let them establish whether students have acquired each learning outcome and whether acquiring of each of the learning outcomes is properly validated. For example, the teachers have to ask themselves whether they are able to positively answer to this question: whether students who have got positive grades on the examination of Mathematics, has acquired the learning outcome K01 and it has been properly validated.

According to tab. 3, it can be noticed that learning outcome K01 is validated by only one subject, Mathematics. It is more difficult to validate the acquiring of the learning outcome by a few subjects (for example K02), especially in the case if students get different grades during examinations of different subjects.

### ***Matrix of grades***

Academic teachers have to prepare the matrix with grades which students get during examinations. All universities have described the grading scale in their study regulations and

for most of the Polish universities the grades are positive integers from 1 to 5, from 1 to 6 or from 2 to 5.

Let us assume that values of the matrix of grades should belong to the interval  $[0,1]$ , so there has to be defined an increasing function<sup>6</sup> that transforms the students' grades to values belonging to the interval  $[0,1]$  (comp. tab.3).

Tab. 3 Grades of students of the major „geodesy and cartography” transformed in such a way they belong to the interval  $[0,1]$

Student \ Subject	Paweł Nowak	Igor Zabłocki	Anna Nowik	Barbara Siwak	Piotr Tracki	Zofia Pawlak	Jolanta Zobicka	Renata Wartoś
MAT	0.2	0.4	0.8	0.2	0.4	0.8	0.6	1
BCG	1	0.2	1	0.2	0.8	1	0.6	1
GSN	0.6	0.2	0.6	0.2	0.6	0.6	0.6	0.8
AJ	0.4	0.4	0.8	0.2	0.2	0.8	0.8	1
PH	0.4	0.2	1	0.2	0.2	0.8	0.8	1
MLM	0.6	0.2	0.6	0.2	0.6	0.6	0.6	1
CT	0.2	0.2	0.6	0.6	0.2	1	0.8	1
GIT	1	0.8	1	0.2	1	1	1	1
C-GEO	0.8	0.8	0.8	0.2	0.6	0.8	0.8	0.8
EG	0.4	0.2	0.8	0.2	1	0.8	0.4	1
FE	0.6	0.2	0.6	0.6	0.6	0.6	0.8	1
DT	0.6	0.6	0.6	0.2	0.6	0.6	0.6	0.6
LIS	0.4	0.2	0.8	0.2	0.6	0.8	0.8	1
GIS	0.8	0.8	1	0.2	0.8	0.8	0.8	1
BESS	1	0.2	1	0.2	0.8	0.8	1	1

Source: the sample matrix.

### *Fuzzy relations*

While communications, people use imprecise expressions, for example teachers say that “they got good students”, “students were poorly prepared for the examination” or “student X is very good at maths”. Thus it is important to find out the way of speaking and writing about achievements of students or about acquiring learning outcomes.

Classical logic during answering questions says “yes” or “no” and uses numbers 0 or 1. For example, for the problem “has student X acquired learning effect Y?”, classical logic can only say “yes” or “no”.

Since fuzzy logic uses imprecise expressions and for the problem “has the student X acquired the learning effect Y?”, fuzzy logic can say “student X have acquired learning effect poorly”, so the level of acquirement of this effect is equal to 0.2.

Let us quote the definitions<sup>7</sup>. Fuzzy set  $(X, \mu_x)$  is a pair of the set  $X$  and the membership  $\mu_x : X \rightarrow [0,1]$ , which for element  $x \in X$  describes the level of belonging of  $x$  to the set  $X$ . Moreover, fuzzy relation  $R$  between sets  $X$  and  $Y$  is the set of pairs  $(x, y) \in X \times Y$  with the membership function  $\mu_R : X \times Y \rightarrow [0,1]$ .

Let  $R_1$  be a fuzzy relation between learning outcomes and subjects of the major “geodesy and cartography”, which membership values are set in the tab. 3 and denote the level of acquiring of these learning outcomes by these subjects. Let  $R_2$  be a fuzzy relation between students and subjects which membership values are put to the tab. 4 and denote the

<sup>6</sup> A function is increasing if  $x_1 < x_2$ , then  $f(x_1) < f(x_2)$ . For example, the function  $f(x) = 2x + 3$  is increasing.

<sup>7</sup> L. Rutkowski, Metody i techniki sztucznej inteligencji, PWN, Warszawa, 2009.

marks of these subjects which these students have got during examinations transformed to the interval [0,1].

### ***Operations of fuzzy relations***

Since the problem that is discussed in the paper is acquiring the learning outcomes by students, these two matrices are not adequate to solve this problem. There have to be defined the next matrix-fuzzy relation  $R_3$  between learning outcomes and students which membership values describes the level of acquiring learning outcomes by students. This fuzzy relation could be defined as the composition of fuzzy relations  $R_1$  and  $R_2$ .

Let us quote some definitions<sup>8</sup>. Let be done two fuzzy sets  $(X, \mu_A), (X, \mu_B)$  defined on the same set  $X$ . The product of these sets is a pair  $(X, \mu_{A \cap B})$  which membership function is defined as follows:

$$\mu_{A \cap B}(x) = T(\mu_A(x), \mu_B(x)),$$

where  $a_1, a_2 \in X$  and  $T$  is one of the functions:

$$T_M(a_1, a_2) = \min\{a_1, a_2\}, \quad T_L(a_1, a_2) = \max\{a_1 + a_2 - 1, 0\}, \quad T_P(a_1, a_2) = a_1 a_2.$$

The union of these two sets is a pair  $(X, \mu_{A \cup B})$  which membership function is defined as follows:

$$\mu_{A \cup B}(x) = S(\mu_A(x), \mu_B(x)),$$

where  $a_1, a_2 \in X$  and  $S$  is one of the functions:

$$S_M(a_1, a_2) = \max\{a_1, a_2\}, \quad S_L(a_1, a_2) = \min\{a_1 + a_2, 1\}, \quad S_P(a_1, a_2) = a_1 + a_2 - a_1 a_2.$$

Let be done two fuzzy relations  $R = \{(x, y), \mu_R(x, y)\}$  and  $P = \{(y, z), \mu_P(y, z)\}$ . The composition type  $S-T$  of fuzzy relations  $R \subset X \times Y$  and  $P \subset Y \times Z$  is a fuzzy relations  $R \circ P \subset X \times Z$  with membership function defined in the following way:

$$\mu_{R \circ P}(x, z) = S_{y \in Y}(T(\mu_R(x, y), \mu_P(y, z))).$$

### ***Requirements from the composition of fuzzy relations***

The composition  $R_3$  of fuzzy relations has to fulfill a few basic requirements:

1. Values of the membership function belong to the interval [0,1], so it is natural to accept that if the value of the membership function of the fuzzy relation  $R_3$  between learning outcomes and students is equal to zero, namely

$$R_3(LO\_n, student\_m) = 0,$$

There can be said that *student\_m* did not acquire the learning outcome *LO\_n*.

2. If the learning outcome is taught on the few subjects which are planned in consecutive semesters, there should be increase in acquiring of this learning outcome, so values of the membership function should be bigger.

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<sup>8</sup> Ibidem.

### *Composition $R_3$ type max-min*

In the case of the composition type max-min, functions  $T$  and  $S$  are defined in the following way:

1) for two elements  $a_1, a_2$ :

$$T_M(a_1, a_2) = \min\{a_1, a_2\},$$

$$S_M(a_1, a_2) = \max\{a_1, a_2\},$$

2) for  $n$  elements  $a_1, a_2, \dots, a_n$ :

$$T_M(a_1, a_2, \dots, a_n) = \min_{i=1,2,\dots,n}\{a_i\},$$

$$S_M(a_1, a_2, \dots, a_n) = \max_{i=1,2,\dots,n}\{a_i\}.$$

When these functions are used, the composition of fuzzy relations  $R_1$  and  $R_2$  (tab.2 and tab. 3), the fuzzy relation  $R_3$ , is defined as follows:

$$R_3(K, P) = \max_{m=1,2,\dots,M}\{\min\{R_1(K, m); R_2(m, P)\}\}$$

Using this formula, values of the membership functions of  $R_3$ , can be calculated. In tab. 4 there are presented values of levels of acquiring learning outcome K02 by students of the major “geodesy and cartography” after studying one (Basic course in geodesy), two (Basic course in geodesy and Adjustment calculus) and three subjects (Basic course in geodesy, Adjustment calculus and Geodesy and satellite navigation).

Tab. 4 Levels of acquiring learning outcome K02 by the students of the major „geodesy and cartography” with max-min composition after studying one, two and three subjects

Students Subjects	Paweł Nowak	Igor Zabłocki	Anna Nowik	Barbara Siwak	Piotr Tracki	Zofia Pawlak	Jolanta Zobicka	Renata Wartoś
BCG	0.4	0.2	0.4	0.2	0.4	0.4	0.4	0.4
BCG, ADC	0.4	0.2	0.4	0.2	0.4	0.4	0.4	0.4
BCG, ADC, GSN	0.4	0.4	0.4	0.2	0.4	0.4	0.4	0.4

Source: data calculated on the basis of data from tab. 3 and tab. 3

Let us calculate the level of acquiring learning outcome K02 by student Paweł Nowak. This learning outcome has been taught on the three subjects: Basic course in geodesy, Adjustment calculus and Geodesy and satellite navigation. After studying first subject Basic course in geodesy, the level of acquiring K02 is equal to

$$\max\{\min\{0.4;1.0\}\} = 0.4.$$

After studying two subjects: Basic course in geodesy and Adjustment calculus, the level of acquiring K02 is equal to

$$\max\{\min\{0.4;1.0\}; \min\{0.4;0.6\}\} = 0.4,$$

and finally after studying these three subjects, the level of acquiring K02 is equal to

$$\max\{\min\{0.4;1.0\}; \min\{0.4;0.6\}; \min\{0.4;0.4\}; 0; \dots; 0\} = 0.4.$$

Thus in this case there is no increase in the level of acquiring learning outcome K01 by Paweł Nowak. Notice that only in the case of student Igor Zabłocki, there is an increase in the level of acquiring learning outcome K01.

Note that the second requirement is not fulfilled in the case of the max-min composition.

Summarizing, the max-min composition is not appropriate for our purpose because in the case of a few subjects which realize and validate the same learning outcome, there could be no increase in the level of acquiring learning outcomes by students. Thus generally the second requirement is not fulfilled.

### **Composition $R_3$ type Łukasiewicz**

In the case of the composition type Łukasiewicz, functions  $T$  and  $S$  are defined as follows:

- 1) for two elements  $a_1, a_2$ :

$$T_L(a_1, a_2) = \max\{a_1 + a_2 - 1, 0\},$$

$$S_L(a_1, a_2) = \min\{a_1 + a_2, 1\},$$

- 2) for  $n$  elements  $a_1, K, a_n$ :

$$T_L(a_1, a_2, \dots, a_n) = \max\left\{\sum_{i=1}^n a_i - (n-1), 0\right\}$$

$$S_L(a_1, a_2, \dots, a_n) = \min\left\{\sum_{i=1}^n a_i, 1\right\}$$

As in the case of max-min composition, after applying Łukasiewicz type composition, the membership function is calculated using the formula

$$R_3(K, P) = \min\left\{\sum_{m=1}^M \max\{R_1(K, m) + R_2(m, P) - 1, 0\}, 1\right\}.$$

Then levels of acquiring learning outcome K02 by the students of the major “geodesy and cartography” after passing one, two and three subjects are put to the table 5.

Tab. 5 Levels of acquiring learning outcome K02 by the students of the major „geodesy and cartography” with Łukasiewicz composition after studying one, two and three subjects

Students Subjects	Paweł Nowak	Igor Zabłocki	Anna Nowik	Barbara Siwak	Piotr Tracki	Zofia Pawlak	Jolanta Zobicka	Renata Wartoś
BCG	0.4	0	0.4	0	0.2	0.4	0	0.4
BCG, ADC	0.4	0	0.4	0	0.2	0.4	0	0.6
BCG, ADC, GSN	0.4	0	0.6	0	0.2	0.6	0.2	1

Source: data calculated on the basis of data from tab. 2 and tab. 3

Let us calculate the level of acquiring learning outcome K02 by student Paweł Nowak. After passing first subject Basic course in geodesy, the level of acquiring K02 is equal to

$$\min\{\max\{0.4 + 1 - 1, 0\}, 1\} = 0.4.$$

After passing two subjects: Basic course in geodesy and Adjustment calculus, the level of acquiring K02 is equal to

$$\min\{\max\{0.4+1-1,0\} + \max\{0.4+0.6-1,0\},1\} = 0.4,$$

and finally after passing these three subjects, the level of acquiring K02 is equal to

$$\min\{\max\{0.4+1-1,0\} + \max\{0.4+0.6-1,0\} + \max\{0.4+0.4-1,0\},1\} = 0.4.$$

Thus in this case there is no increase in the level of acquiring learning outcome K02 by Paweł Nowak.

Notice that in the case of student Barbara Siwak, the level of acquiring K02 after studying one, two or even three subject is still equal to 0. The levels of validating K02 in the case of these three subjects were equal to positive numbers and her grades were positive numbers as well, so the level of acquiring K02 by Barbara Siwak should not be equal to zero.

Summarizing, Łukasiewicz-type composition  $R_3$  of fuzzy relations  $R_1$  and  $R_3$  do not fulfill the first and second requirements.

### *Composition $R_3$ type algebraic*

In the case of the composition type algebraic, functions  $T$  and  $S$  are defined in the following way:

1) for two elements  $a_1, a_2$ :

$$T_p(a_1, a_2) = a_1 a_2,$$

$$S_p(a_1, a_2) = a_1 + a_2 - a_1 a_2,$$

2) for  $n$  elements  $a_1, a_2, \dots, a_n$ :

$$T_p(a_1, a_2, \dots, a_n) = \prod_{i=1}^n a_i,$$

$$S_p(a_1, a_2, \dots, a_n) = 1 - \prod_{i=1}^n (1 - a_i).$$

Similarly like before, we construct the composition type algebraic, which is equal to

$$R_3(K, P) = 1 - \prod_{m=1}^M (1 - R_1(K, m) \cdot R_2(m, P)),$$

calculate the values of relation  $R_3$  for learning outcome K02 after passing these three subjects for each students which are put them to the tab. 6.

Tab. 6 Levels of acquiring learning outcome K02 by the students of the major „geodesy and cartography” with algebraic composition after studying one, two and three subjects

Students Subjects	Paweł Nowak	Igor Zabłocki	Anna Nowik	Barbara Siwak	Piotr Tracki	Zofia Pawlak	Jolanta Zobicka	Renata Wartoś
BCG	0.40	0.08	0.40	0.08	0.32	0.40	0.24	0.40
BCG, ADC	0.54	0.15	0.54	0.15	0.48	0.54	0.42	0.59
BCG, ADC, GSN	0.62	0.29	0.69	0.22	0.52	0.69	0.61	0.76

Source: data calculated on the basis of data from tab. 2 and tab. 3

Now, we calculate the level of acquiring learning effect K02 by student Paweł Nowak. After passing the first subject, the level of acquiring K02 is equal to

$$1 - (1 - 0.4 \cdot 1) = 0.4.$$



After passing two subjects, the level of acquiring learning effect K02 is equal to

$$1 - (1 - 0.4 \cdot 1) \cdot (1 - 0.4 \cdot 0.6) = 0.54.$$

And finally after passing these three subjects, the level of acquiring K02 is equal to

$$1 - (1 - 0.4 \cdot 1) \cdot (1 - 0.4 \cdot 0.6) \cdot (1 - 0.4 \cdot 0.4) = 0.62.$$

Notice that for all students the level of acquiring learning outcome K02 is higher when they have passed more subjects (realizing this learning outcome).

Analyzing the data put in tab. 6, there can be noticed that all values are positive, so if students' grades and the level of validating learning outcomes are positive then levels of acquiring learning outcomes are also positive.

**Theorem** For each student, the more subjects have been validating the learning outcome, the higher level of acquiring this learning outcome is.

**Proof.** Let  $S$  denote the student and let  $K$  denote a learning outcome.

Let  $r_n$  be a level of acquiring learning outcome  $K$  after studying  $n$  subjects, where  $n = 1, 2, \dots, N$  and  $N$  is the number of subject which validate learning outcome  $K$ . Let us denote by  $a_n$  the level of validating  $K$  by the subject  $n$  and let  $b_n$  be the grade of this subject. Assume that  $a_n, b_n > 0$  for each  $n = 1, 2, \dots, N$ .

If there is one subject, then let  $r_1$  be a level of acquiring  $K$ , so  $r_1 > 0$ . Let  $n < N$ . According to the definition of algebraic composition:

$$r_n = 1 - (1 - a_1 b_1) \cdot (1 - a_2 b_2) \cdot \dots \cdot (1 - a_n b_n).$$

Let

$$r_{n+1} = 1 - (1 - a_1 b_1) \cdot (1 - a_2 b_2) \cdot \dots \cdot (1 - a_n b_n) \cdot (1 - a_{n+1} b_{n+1}).$$

Since  $a_{n+1}, b_{n+1} > 0$ ,  $a_{n+1} b_{n+1} > 0$ , so  $1 - a_{n+1} b_{n+1} < 1$ . Then

$$(1 - a_1 b_1) \cdot (1 - a_2 b_2) \cdot \dots \cdot (1 - a_n b_n) > (1 - a_1 b_1) \cdot (1 - a_2 b_2) \cdot \dots \cdot (1 - a_n b_n) \cdot (1 - a_{n+1} b_{n+1}).$$

Thus

$$1 - (1 - a_1 b_1) \cdot (1 - a_2 b_2) \cdot \dots \cdot (1 - a_n b_n) < 1 - (1 - a_1 b_1) \cdot (1 - a_2 b_2) \cdot \dots \cdot (1 - a_n b_n) \cdot (1 - a_{n+1} b_{n+1}),$$

hence

$$r_n < r_{n+1}.$$

Therefore, by the principle of mathematical induction, the more subjects have been validating the learning outcome, the higher level of acquiring this learning outcome is.

In tab. 7 there are put levels of learning outcomes of the students of the major "geodesy and cartography" with the application of algebraic composition.

Tab. 7 Levels of acquiring learning outcome K02 by the students of the major „geodesy and cartography” with algebraic composition

Students Subjects	Paweł Nowak	Igor Zabłocki	Anna Nowik	Barbara Siwak	Piotr Tracki	Zofia Pawlak	Jolanta Zobicka	Renata Wartoś
K01	0.20	0.40	0.80	0.20	0.40	0.80	0.60	1.00
K02	0.62	0.29	0.69	0.22	0.52	0.69	0.61	0.76
K03	0.40	0.20	1.00	0.20	0.20	0.80	0.80	1.00
K04	0.37	0.19	0.51	0.37	0.37	0.65	0.58	0.75
S01	0.76	0.68	0.82	0.27	0.83	0.82	0.76	0.85
S02	0.76	0.58	0.88	0.29	0.77	0.82	0.84	0.90
S03	0.72	0.64	0.72	0.44	0.72	0.72	0.76	0.80
S04	0.60	0.60	0.60	0.20	0.60	0.60	0.60	0.60
C01	0.89	0.81	0.93	0.42	0.91	0.92	0.92	0.95
C02	0.51	0.37	0.51	0.37	0.51	0.51	0.58	0.65

Source: data calculated on the basis of data from tab. 2 and tab. 3

Notice that students can achieve the maximal value of acquiring learning outcomes, for example student Anna Nowik achieved the level of acquiring learning outcome K03 equal to 1.

### *Conclusions*

Nowadays it is required that teachers should verify acquiring learning outcomes by students. Since the number of learning outcomes, which are described for each HEI' major, belongs in most cases to the interval (50, 80) and their acquiring are validated on different subjects, therefore this process is not easy.

One of the way to help teachers could be the computer system which calculates levels of acquiring learning outcomes by students on the data given by teachers: levels of verifying learning outcomes during subjects' examinations and students' grades.

To achieve this goal, it is better to use fuzzy than classical logic so fuzzy relations are used to store data. The paper presents discussion which type of composition of fuzzy relations is the best one for considered purpose according to two requirements.

Only the algebraic composition fulfills these two requirements, namely if students got positive grades, the levels of acquirement of learning outcomes are positive and the more subjects students realize and pass, the higher levels of acquirement they achieve.

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