

# A Mathematical Borderscape in Eratosthenes' *Geographika* North-South Distances of the *Oikoumene*

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**Abstract:** In the Hellenistic world, landscapes were often tied to the borders of kingship. This paper challenges the prevailing geopolitical framework by showing that Eratosthenes defined the *oikoumene* not by shifting imperial frontiers, but through a mathematical conception of its outermost edges. It examines how his *Geographika* constructs a borderscape shaped by intellectual inquiry and mathematical precision, drawing on geographical evidence through textual analysis, empirical observation, and scientific reasoning. His engagement with mathematics moved his research beyond the Library of Alexandria into the natural world. This interplay reveals an alternative mode of boundary-making: one rooted in scientific inquiry. To examine these borderscapes, this study employs GIS to reconstruct Eratosthenes' spatial framework, focusing on the southern connections between the Near East and North Africa to show how boundaries and landscapes were theorised in the Hellenistic world.

**Keywords:** Eratosthenes, third century BC, Near East, ancient Libya, borderscapes, ancient geography, ancient science, ancient mathematics

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While borderscapes have been investigated by scholars like Dina Krichker, Julian Degen, Robert Rollinger, Luis Moreno, Chiara Brambilla, Jussi Laine, and Gianluca Bocchi, one thing remains – the concept of borderscapes and its various definitions lack a single interpretation; the study of borderscapes encompasses many elements, including a variety of disciplines relating to borders, landscape, and the people within them.<sup>1</sup> In the third century BC, territories, landscapes, and borders were ever more important to the successor

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<sup>1</sup> Moreno 1999; Brambilla *et al.* (Eds) 2015; Laine, Tervonen 2015; Krichker 2019: 1, 5; Rollinger, Degen 2021.

kingdoms.<sup>2</sup> The Seleukid and Ptolemaic empires constantly sought connections to Alexander, by claiming the entire *oikoumene*.<sup>3</sup> Seleukid and Ptolemaic borderscapes were geopolitical, with ever-constant and changing territories;<sup>4</sup> Eratosthenes' *Geographika* and his connections to the Ptolemaic Empire, as well as other ancient geographical texts from this period, have been regarded as representative of political undertones, promoting one empire over another.<sup>5</sup> Although, of course, there is abundant evidence of propaganda promoting 'universal kingship' and 'boundaries of kingship' in antiquity,<sup>6</sup> this paper sets out to highlight that viewing Eratosthenes and his geographical work within this imperial framework may not be quite satisfactory; it is not the only way to examine Eratosthenes' *Geographika*. Eratosthenes' division of the *oikoumene* not only established physical and mathematical boundaries, but also contributed to an intellectual borderscape – a conceptual geography that shaped perceptions of the inhabited world. It will be shown that Eratosthenes, a geographer, examined boundaries, borders, and invisible borderscapes differently.

Natural borders, such as mountains, rivers, seas, straits – from the Pillars of Herakles to India and from the most northern regions to the equator – were instrumental in how Eratosthenes defined borders and boundaries of the *oikoumene*. In thirteen fragments, he records the borders (or furthest known points) of the inhabited world, and in no less than 31 times in 22 fragments<sup>7</sup> he uses the words borders, boundaries, furthest point, and end (ὄρος, ὄριον, πέρας, τερμόνων, σημείον, ὄμορος, *finis*, and ὁρίζω).<sup>8</sup> Although there are numerous fragments that address borders and borderscapes in Eratosthenes' *Geographika*,<sup>9</sup> this paper will focus on only four (Fragments 38, 35, 34, and 60), which are central to the most northern and southern distances of the inhabited world.<sup>10</sup> Although many additional fragments reinforce

<sup>2</sup> Kosmin 2014: 4–7, 18–19, 25–26, 32; 2018: 91–93; Strootman 2014a: 4, 5; 2014b: 38, 40, 47; 2020: 135, 137.

<sup>3</sup> See above footnote 2.

<sup>4</sup> App., *Syr.* 55; Bagnall 1976: 81, 143–146; Boehm 2018: 29, 34, 86; Diod. Sic. 18.68–73, 18.74–75, 19.12–35, 19.37–44; Habicht 1956: 122f.; Hölbl 2001: 42–44, 46–47, 57, 60, 356–359; Heckel 2006: 246–248; Grainger 2014: 108–110; *IG* XII,3 466/1390; *IG* XII,3 467/1391; Kosmin 2018: 91; Paus. 3.24.2; Rey-Coquais 1978: 313–325; Roller 2015: 100, 242; *SEG* 1:343; Eratosth. F62 2.1.36, F63 Str. 2.1.29, F66 Str. 2.1.22, F69 Str. 15.1.11, F77–80 Str. 15.2.1, 15.2.8–9, 2.1.22, 2.1.28, F82 Str. 2.1.31, F83 Str. 2.1.23–6, F108 Str. 11.8.8–9; Walbank *et al.* (Eds) 1989: 307, 319.

<sup>5</sup> Strootman 2014a: 8, 12, 14; 2020: 117, 135, 137, 139; Kosmin 2014: 85–86, 93–94; Bianchetti 2016: 138–139; Krichker 2019: 1.

<sup>6</sup> See above footnotes 4–5. See also Strootman 2014a; 2014b; 2020; Grainger 2014; Kosmin 2014; 2018; Erskine, Llewellyn-Jones, Wallace (Eds) 2017; Rollinger, Degen 2021.

<sup>7</sup> Roller 2010; Eratosth. F30 Str. 2.5.5–6, F33 Str. 1.4.6–8, F34 Str. 2.5.7–9, F48 Str. 11.12.4–5, F49 Str. 2.1.31, F53 Str. 2.5.14, F58 Str. 2.2.2, F64 Str. 2.1.34, F69 Str. 15.1.10–11, F70 Plin., *HN* 6.56, F77 Str. 15.2.1, F78 Str. 15.2.8–9, F83–86 Str. 2.1.23–6, 2.1.27, 2.1.34, Str. 15.3.1, respectively, F88 Str. 14.2.29, F98 Str. 17.1.1–2, F100 Str. 17.3.1–2, F104 Str. 2.5.20, F108 Str. 11.8.8–9, F132 Str. 2.4.2.

<sup>8</sup> Eratosth. F30 Str. 2.5.5–6, F49 Str. 2.1.31 as περιπίζω, F69 Str. 15.1.10–11 as περιπίζω, F77 Str. 15.2.1, F78 Str. 15.2.8–9, F70 Plin., *HN* 6.56 uses *finis*.

<sup>9</sup> See above footnote 7.

<sup>10</sup> For a continuation of borderscapes, examples from Eratosthenes, and his spatial approaches to the *oikoumene*, please refer to papers in preparation: Sink, S., A Border Within and Without Bounds – Eratosthenes

the argument, these four are presented as case studies within the space this paper affords. Along with the textual sources that will be examined, this paper adopts an interdisciplinary investigation, which includes a convergence of mathematical, scientific, and theoretical practices, as well as the physical landscape. Instead of examining borderscapes as political boundaries, this investigation seeks to re-think and re-evaluate the placement of borders<sup>11</sup> as recorded by Eratosthenes' *Geographika*, not as political constructs promoting any single kingship over another, but as mathematical constructs. To undertake such a study, this paper implements the modern concept of borderscapes to study the ancient past. The exploration and research stress that border studies are interdisciplinary, and they can demonstrate non-political configurations, especially when applied to Hellenistic geography. Thus, more succinctly, mathematical borderscapes within the works of Eratosthenes – and in the context of the mid-third century BC – will be demonstrated through his geographical fragments.<sup>12</sup>

Eratosthenes was interested in empirical explorations through mathematical applications. In this way, Eratosthenes left the confines of the Library to conduct research outside. He calculated equinoctial measurements with the use of gnomons and sundials. With these combined methods of ancient science, Eratosthenes focused on geographic connections from a spatial and *oikoumenic* perspective. By taking his text 'off the page', the upcoming figures and tables contribute to under-theorised and underrepresented areas of ancient geography, which allows the reader to understand Eratosthenes' visualisation of borders. Such reconstructions will illustrate mathematical geography, calculations of distances, and north-south boundaries of the inhabited world. The GIS reconstructions will demonstrate mathematical modelling and Eratosthenes' conceptualisation of borders and boundaries. Ultimately, this interdisciplinary approach highlights the connection between mathematical methods and the charting of borders as an ancient scientific endeavour and as an under-explored aspect of ancient geography.

Naturally occurring geographical landmarks and measurements to and from places were created and documented by Eratosthenes, detailing how he described and defined the inhabited world from the east to the west and from the north to the south. Fragments 38, 35, 34, and 60 are used to reveal his spatial and *oikoumenic* perspective, as well as his scientific approach to mathematically define the furthest known borders. These fragments reference stadia distances that were calculated by blending empirical observations with available records, such as equinoctial calculations, land journeys, and *periploi* itineraries. It will be demonstrated that these borderscapes are not easily grasped as descriptive, verbose passages until they are illustrated through mathematical modelling, where physical geography, cultural perceptions, and theoretical concepts intersect.<sup>13</sup>

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Fragment 33, [in:] Gatto, M.C. *et al.*, *Borderscapes in Archaeology: A Comparative Perspective*; Sink, S., *An Interconnectedness from the Pillars to India – A Spatial Approach of Eratosthenes Geography*, [in:] Nuttall, C., *Spatial Approaches to Ancient Greek and Cypriot Landscapes*, Athens, Swedish Institute at Athens and Rome Series.

<sup>11</sup> Krichker 2019: 2.

<sup>12</sup> Krichker 2019: 4.

<sup>13</sup> Krichker 2019: 2, 7–9.

## MATHEMATICAL AND CONTEXTUAL EXPLANATIONS

In order to make the geographical descriptions presented in Eratosthenes' *Geographika* more accessible to a modern audience, this research provides contextual explanations for the ancient place names and regions Eratosthenes references. This includes identifying their approximate modern equivalents, along with geographic coordinates, when possible. Such information serves to situate Eratosthenes' descriptions within the contemporary world map and to better visualise the distances and spatial relationships he described.

Since Eratosthenes' work survives only in fragmentary form, and because his geographical system relies on mathematical reasoning and astronomical observations, translating his ancient measurements and coordinates into modern reference points is not simply an exercise in comparison – it is essential for understanding his intellectual framework. My aim is to illuminate how Eratosthenes imagined and measured the boundaries of the inhabited world, and how he used empirical data to construct what I refer to as 'mathematical borderscapes'.

This approach allows for a clearer understanding of the precision with which Eratosthenes worked, and it highlights the sophistication of his geographical thought. Moreover, connecting ancient place names to modern topography allows these fragments not only to become more intelligible but also more compelling, especially for readers who may be less familiar with the complexities of ancient geography or the scholarly debates surrounding its interpretation.

While the term 'mathematical borderscapes' may be novel, it is used here to describe modelling or conceptualisation of borders (and boundaries)<sup>14</sup> within a mathematical, spatial, and geographical context. Its main methodology involves quantitative analysis and mathematical modelling, and it considers theoretical frameworks that include applied mathematics. Combining these, 'mathematical borderscapes' directly concerns the mathematical representation and mathematical analysis of borders, border regions, and boundaries. Applications of mathematical borderscapes can include fractal geometry, network theory, and GIS too. These applications utilise mathematical models to represent and analyse spatial data. Eratosthenes' work laid the foundation for mathematically conceptualising geographical spaces, including borders. His methods involved quantifying distances and defining and describing the inhabited world; these are essential aspects of what 'mathematical borderscapes' encompass.

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<sup>14</sup> Rooted in an ancient mathematical context, Euclid described boundaries as, Ὁρος ἐστίν, ὃ τινός ἐστι πέρας. Σχήμα ἐστὶ τὸ ὑπὸ τινος ἢ τινῶν ὁρῶν περιεχόμενον ('A boundary is that which is the limit of anything. A figure is that which is bounded by certain boundaries') (translated by author). To understand the current use of the term, see Parker 2006: 77–100. Often, borders and boundaries are used interchangeably. Boundaries 'indicate the bounds or limits of anything' and 'they are unspecified divisions that indicate the limits of various kinds' (Parker 2006: 79).

## THE DATA

While Eratosthenes' use of naturally occurring landscape features, such as mountains, rivers, seas, straits, and promontories, was emphasised elsewhere,<sup>15</sup> this paper takes a different approach. By examining the north-south distances of the *oikoumene*, these fragments instead reflect a distinctly mathematical borderscape, where spatial boundaries are constructed through equinoctial measurement, geometric reasoning, astronomical observations, and by sailing and land journeys, rather than highlighting physical landmark features.

This is not a new method for Eratosthenes; he used it widely. His *Geographika* includes many mathematical and geometric shapes to define borders. The emphasis here in these fragments is that Eratosthenes is focused primarily on determining the greatest width and length of the known world (with the data he had available, knowing full well other places existed beyond the knowledge he acquired).<sup>16</sup> This paper discusses the mathematical borders from the north-south, with primary focus given to the southern borderscape connections that involve the Near East and North Africa. While the north-south boundaries demonstrate applied mathematics, they also reveal a visible pattern – about the prime location of the Library of Alexandria, in which Eratosthenes' work took place. Moreover, the landscape he was a part of was extremely instrumental in assisting him when constructing the *oikoumene*. His proximity to the equator allowed him to calculate such distances within ancient Libya and Egypt with mathematical accuracy. Moreover, the Ptolemaic connections to the rest of the Mediterranean allowed him access to position the furthest known locales in the north and the east with tangible knowledge. Let us look at the following fragments.

Fragment 38, 'Measurement of the Entire Inhabited Earth' 1 in Mullerus 1885: 424; translated by the author.

'It is necessary to know that the length of all of the earth was 252,000 stadia. The length of our inhabited world from the mouth of the Ganges to the Gades is 83,800 stadia. The width from the Aithiopian Sea to the Tanaïs River is 35,000 stadia. [...] Eratosthenes, one of the most studious [scholars] of antiquity made this measurement'.

In Fragment 38, the total distance from the Aithiopian Sea (Arabian Sea) to the Tanaïs River (Don) is 35,000 stadia.

Fragment 35. Strabo, *Geography* 1.4.2; translated by Roller 2010.

'In determining the width of the inhabited world, he [Eratosthenes] says that from Meroë it is 10,000 stadia along its meridian to Alexandria, and from there to the Hellespont about 8,100, and then 5,000 to Borysthenes, and then to the parallel that runs through Thoule (which Pytheas says is six days' sail north of Brettanike and is near the frozen sea) an additional 11,500. Moreover, if we add 3,400 more beyond Meroë, so that we include the Egyptian island, the Kinnamomophoroi, and Taprobane, we have 38,000 stadia'.

<sup>15</sup> See forthcoming publications by the author (see above footnote 10).

<sup>16</sup> Roller 2010; Eratosth. F30 Str. 2.5.5–6, F33 Str. 1.4.6–8, F34 Str. 2.5.7–9, F37 Str. 1.4.5, F39 Str. 1.1.8–9, F53 Str. 2.5.14, F61, F69 Str. 15.1.10–11, F72 Arr., *Ind.* 3.1–5, F95 Str. 16.4.2–4, F100 Str. 17.3.1–2, F108 Str. 11.8.8–9.

In Fragment 35, the total distance from Thoule (Iceland?)<sup>17</sup> to the Kinnamomophoroi (Horn of Africa) is 38,000 stadia.

Fragment 34. Strabo, *Geography* 2.5.7, 9; translated by the author.

(7) '[...] Being exact to Eratosthenes, "the celestial equator is 252,000 stadia, a fourth part [of that] would be 63,000 – for this is the distance from the equator to the pole [which is] about fifteen sixtieths of sixty.<sup>18</sup> From the equator to the Tropic of Cancer<sup>19</sup> is four sixtieths. For this is the description of the parallel line of latitude through Syene. Each distance is computed from known measurements. The tropic happens to be established at Syene, because there at the summer solstice the gnomon is shadowless in the middle of the day [i.e., noon]. But the meridian through Syene is certainly in line with the course of the Nile River from Meroë to Alexandria. The stadia measurement is about 10,000. It happens that Syene lies in the middle of that distance. So that from there to Meroë is 5,000 stadia. Going in a straight line as much as 3,000 stadia to the south, it is no longer habitable due to the burning heat; thus, the parallel through these regions – the same one as through the Kinnamomophoroi – establishes the boundary and beginning of our inhabited world near the south. Therefore, since it is 5,000 stadia from Syene to Meroë, and adding the other 3,000 stadia, the entire total would be 8,000 stadia to the southern boundary of the *oikoumene*. But from Syene to the equator is 16,800 stadia – this is as large as four sixtieths, having placed each parallel 4,200 stadia [apart]. Thus, the remaining [distance] would be 8,800 stadia from the boundary of the inhabited world to the equator, and 21,800 stadia from Alexandria".

Again, all agree that the voyage from Alexandria to Rhodes is in a straight line with the course of the Nile. Hence, the voyage [continues] along the coastline of Karia and Ionia as far as the Troad and Byzantion, and Borysthenes. Therefore, having taken the known distance and [covered by] sailing across, [the geographers] examine [the places] beyond the boundary line [of the Borysthenes] as to how far it is inhabitable, and as to how far the northern parts within the *oikoumene* have its limits. The Roxolanians, the farthest of the known Skythians, dwell beyond the Borysthenes. The Roxolanians are farther south than those known [to dwell] at the most extreme border, in Britain. But [the regions] beyond [the Roxolanians] are uninhabitable [due to] the cold weather – but further south of them, the Samaritans [dwell] beyond Lake Maeotis, and the Skythians too as far as the eastern Skythians'.

(9) '... For the length is said to be at least 70,000 stadia. This is from the west to the east, from the farthest point in Iberia to the farthest point in India, [which was] carefully measured both by road journeys and by sailing'.

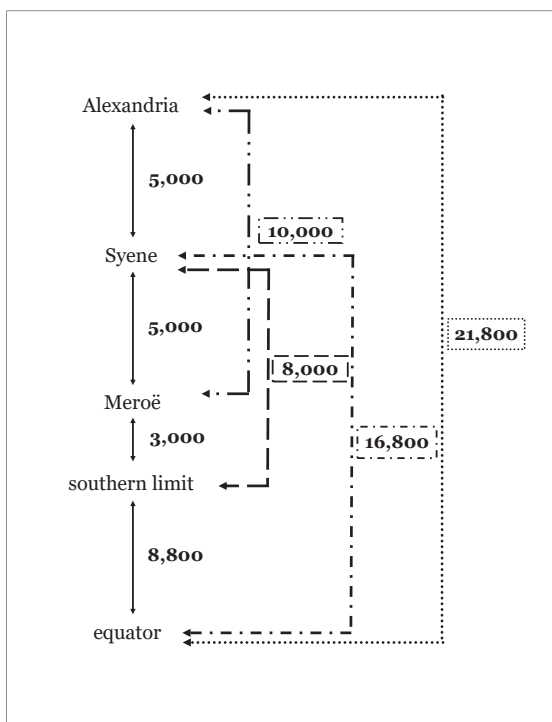
In sum, the distances from 'the northern region beyond the Borysthenes' (Dnieper River) to 'the southern limit of the inhabited world' is 30,000 stadia, and Alexandria to the equator is 21,800 stadia (**Fig. 1**).

<sup>17</sup> Cartwright 2001: 109–110; Roller 2006: 77–79; McPhail 2014: 252.

<sup>18</sup> This equals 15.

<sup>19</sup> The Tropic of Cancer is located at 36° 26' 22", which is 23.4394° north of the equator.

1. The positioning of toponyms and their respective stadia distances (as given in Fragment 34) from Alexandria to Syene to Meroë to the southern limit, where the Kinnamomophoroi live, and onwards beyond the southern limit to the equator (Elaborated: S. Sink).



## THE NORTH-SOUTH MATHEMATICAL BORDERSCAPE CALCULATIONS

Several of the toponyms referenced in Eratosthenes' *Geographika* cannot be confidently identified today; additionally, ancient place names often do not correspond directly to modern cities. Although modern toponyms could be selected based on available archaeological, historical, and geographical scholarship, the following figures focus on Eratosthenes' calculations to demonstrate his mathematical methodology and the landscape within.

Providing demonstrations of the ancient distances offers a quantitative framework for analysing Eratosthenes' geographical reasoning. It allows for comparisons between equinoctial hours, conversions to latitudinal data, and stadia distances; such contextuality helps to assess accuracy, methodology, and preconceived notions regarding his calculations and how ancient borderscapes might have been conceptualised by Greek geographers like Eratosthenes. Providing these conversions contributes to our understanding of his geographic approach, and they also reveal the practical use of how Eratosthenes mathematically defined the *oikoumene*. The use of measurements is not just illustrative – it is analytical. His recorded distances and systematised grid of parallels and meridians illustrated his application of mathematics, which revealed his analytical and methodological approach to defining such borderscapes (Tables 1–3).

Table 1. Eratosthenes’ systematised grid of parallels; alphanumeric labelling of parallels by the author; prime parallel was indicated by Eratosthenes (Elaborated: S. Sink)

| Parallel | Recorded toponyms per parallel   |
|----------|--|
| A        | Thoule   |
| B        | Mouth of the Borysthenes   |
| C        | Lysimacheia, Mysia, Paphlagonia, Sinope, Hyrkania, Baktra, Byzantion, Tauros Mountain Range (northern) |
| Prime    | Pillars of Herakles, Rhodes, Tauros Mountain Range (southern)  |
| E        | Karchedon, Alexandria, Palimbothra   |
| F        | Syene  |
| G        | Meroë  |
| H        | Kinnamomophoroi, southern limit of the inhabited world, Taprobane                                      |
| I        | Equator  |

Table 2. Eratosthenes’ systematised grid of meridians; alphanumeric labelling of meridians by the author; prime meridian was indicated by Eratosthenes (Elaborated: S. Sink)

| Meridian | Recorded toponyms per meridian  |
|----------|---|
| A        | Sacred Promontory   |
| B        | Pillars of Herakles   |
| C        | Rome and Karchedon  |
| Prime    | Meroë, Syene, Alexandria, Rhodes, Byzantion, mouth of the Borysthenes River |
| E        | Thapsakos   |
| F        | Kaspian Gates   |
| G        | Indos River   |

Table 3. Eratosthenes’ recorded distances and respective methodology for each one; only the recorded distances from Fragments 34 and 60 are displayed (Elaborated: S. Sink)

| Location   | Equinoctial hours    | Stadia | Method of measurement   | Notes  |
|--|----------------------|--------|---|--|
| Alexandria to equator                                | longest day = 14 hrs | 21,800 | latitudinal calculations                                      | Arcturus at zenith; gnomon-shadow ratio 5:3 at Alexandria, adds Alexandria to Syene and Syene to equator |
| Meroë to ‘the southern limit of the inhabited world’ | –                    | 3,000  | explorers’/travellers’ reports                                | coincides with the Kinnamomophoroi latitude  |
| Syene to Meroë                                       | –                    | 5,000  | latitudinal calculations                                      | –  |
| Syene to ‘the southern limit of the inhabited world’ | –                    | 8,000  | bematists + travellers’ reports, and latitudinal calculations | –  |



| Location   | Equinoctial hours       | Stadia         | Method of measurement   | Notes  |
|--|-------------------------|----------------|---|--|
| 'the southern limit of the inhabited world' to the equator | –                       | 8,800          | bematists + travellers' reports, and latitudinal calculations   | calculates the difference between Syene to equator and Syene to 'the southern limit of the inhabited world'                  |
| equator to Syene   | day length = 13.5 hrs   | 16,800         | latitudinal calculations  | four sixtieths of the circumference; each sixtieth = 4,200 stadia; calculated from the noon sun at zenith at summer solstice |
| equator to pole  | day length = 18 hrs     | 46,300         | astronomical + latitudinal calculations                         | –  |
| Rhodes to Borysthenes                                      | –                       | 8,700 or 9,000 | sailing route + latitudinal calculation                         | –  |
| Rhodes to equator  | longest day = 14.5 hrs  | 25,800         | sailing + latitudinal calculation                               | –  |
| Hellespont to equator                                      | longest day = 15 hrs    | 29,800         | latitudinal calculations  | –  |
| Byzantion to equator                                       | longest day = 15.25 hrs | 30,700         | sailing + latitudinal calculations                              | ratio 120:42 minus 1/5   |
| mid-Pontos (north of Byzantion) to equator                 | day length = 15.5 hrs   | 32,300         | latitudinal calculations  | Arctic circle at zenith; equidistant from pole and equator   |
| Borysthenes to northern regions                            | –                       | 4,000          | travellers' reports   | –  |
| north to south distance of inhabited world                 | –                       | <30,000        | combination of land + sea journeys, and latitudinal calculation | sum of Rhodes to 'the southern limit of the inhabited world' + Rhodes to the northern regions beyond the Borysthenes         |
| west to east distance of the inhabited world               | –                       | ~70,000        | combination of land + sea journeys                              | sum of the 'narrowest point of India' to Hieron  |

Following his renowned calculation of the Earth's circumference as 252,000 stadia, Eratosthenes set out to determine arc distances between locations, both north-south and east-west – using the sun's position to measure variations in daylight.<sup>20</sup> **Table 4** provides Eratosthenes' recorded equinoctial hours for several locations, which constructs a north-south mathematical borderscape of the *oikoumene*. Equinoctial hours (expressed as the length of the longest day) can be converted into latitudinal degrees based on his calculation for the circumference of the earth.<sup>21</sup> This process reveals his broader methodological approach of dividing and shaping the Earth.<sup>22</sup>

<sup>20</sup> There are twenty-six examples of gnomon measurements taken in his *Geographika* within a handful of fragments: Eratosth. F34 Str. 2.5.7–9, F40 Str. 2.1.20, F41 Plin., *HN* 2.183, F42 Plin., *HN* 6.171, F47 Str. 2.1.3, F53 Str. 2.5.14, F54 Str. 2.1.35, F59 Str. 2.5.36, F60 Str. 2.5.38–41, F128 Str. 2.5.24.

<sup>21</sup> Diller 1934: 263.

<sup>22</sup> Diller 1934: 262–263.

In antiquity, such calculations would have been done by trigonometric chords (τὼν ἐν κύκλῳ ἐϋθειῶν).<sup>23</sup> Such mathematical propositions were used by the Pythagorean school, Hippocrates of Chios, Euclid, and Archimedes.<sup>24</sup> Currently, the spherical trigonometry formula to solve for the latitudinal degree is:  $\sin \frac{1}{2} (a-12) \times 15^\circ = \tan \phi \tan \omega$ , where  $a$  is the number of hours in the longest day,  $\phi$  the latitude of the place and  $\omega$  the latitude of the tropic.<sup>25</sup> However, Eratosthenes’ practice assumed that every hour was equivalent to  $15^\circ$  of latitude.<sup>26</sup> To convert time (equinoctial hours) into degrees of latitude, the standard daylight hours per day is 12, and each hour corresponds to  $15^\circ$  of latitude. Thus, if Syene has 13.5 hours of daylight on the summer solstice, then it has 1.5 hours of sunlight more than the standard. Thus,  $13.5 - 12 = 1.5$  and  $1.5 \text{ hours} \times 15 = 22.5^\circ$ . Therefore, Syene is  $22.5^\circ$  N of the equator. This same formula is applied to the following lengths of day to calculate the remaining latitudinal coordinates for the north-south locations (Table 4).

Table 4. Conversion of length of day (equinoctial hours as recorded in Fragments 34, 59, and 60) to lines of latitude (Elaborated: S. Sink)

| Location   | Ancient equinoctial hours | Latitude conversion of ancient equinoctial ours |
|------------|---------------------------|---|
| Meroë      | 13 hours                  | $15^\circ$ N                                    |
| Syene      | 13.5 hours                | $22.5^\circ$ N                                  |
| Alexandria | 14 hours                  | $30^\circ$ N                                    |
| Rhodes     | 14.5 hours                | $37.5^\circ$ N                                  |
| Hellespont | 15 hours                  | $45^\circ$ N                                    |
| Byzantium  | 15.25 hours               | $48.75^\circ$ N                                 |
| Mid-Pontus | 15.5 hours                | $52.5^\circ$ N                                  |

The measurements demonstrate Eratosthenes’ systematic approach to defining borders and boundaries that include and extend to the furthest known edges. Eratosthenes established a precedent. Scholars such as Duane Roller, Klaus Geus, and others have adopted similar methods, providing equivalents as interpretive tools rather than claims of precision.<sup>27</sup> In this way, the use of distances and mathematical calculations serve as a realistic and pragmatic approach to contextualising ancient geographical claims within a modern framework.

<sup>23</sup> Although no early examples exist, the work of Archimedes would have involved the concept of chords for his studies of circles and spheres in his mathematical propositions, *On the Measurement of a Circle* and *The Quadrature of the Parabola*. An early form of calculus and trigonometry required work by hand to find angles and arc lengths. This would have been done through the usage of chords in Toomer 1974: 6; Thomas (Ed.) 1939: 98, 234, 346, 486–502, respectively.

<sup>24</sup> Thomas (Ed.) 1939.

<sup>25</sup> Diller 1934: 266. Based on this math, the modern formula would calculate a latitude of  $16.7^\circ$  N.

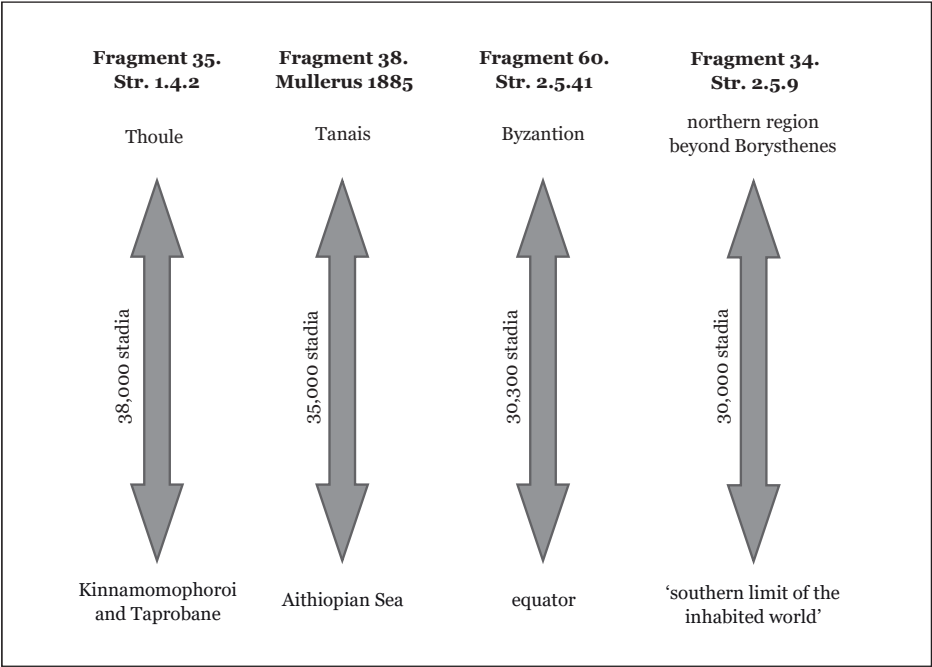
<sup>26</sup>  $360^\circ \div 24 \text{ hours} = 15^\circ$  per hour.

<sup>27</sup> Geus 2002; Roller 2010: 263–267; 2020; Dueck, Brodersen 2012; Talbert 2012; 2017; Leidwanger 2013; Rihll 2020.

The north-south mathematical borderscapes and scientific methods of Eratosthenes continue. In Fragments 34 and 60, Eratosthenes also provides stadia distances for many of these same north-south places (**Fig. 2**). The total distances are provided more clearly in **Table 5**.

Table 5. Ancient stadia distances taken via a gnomon as recorded within Fragments 34 and 60 (Elaborated: S. Sink)

| Location                   | Eratosthenes’ stadia |
|----------------------------|----------------------|
| Kinnamomophoroi to equator | 8,800                |
| Meroë to equator           | 11,800               |
| Syene to equator           | 16,800               |
| Alexandria to equator      | 21,800               |
| Hellespont to equator      | 29,800               |
| Byzantium to equator       | 30,700               |
| Borysthenes to equator     | 34,800               |
| Thoule to equator          | 46,300               |



2. The north-south variances found in the fragments of Eratosthenes. Thoule to the Kinnamomophoroi is 38,000, Tanais to the Aithiopian Sea is 35,000, Byzantion to the equator is 30,300, and the ‘northern region’ to the ‘southern limit of the inhabited world’ is 30,000 stadia (Elaborated: S. Sink).

Furthermore, ancient stadia distances based on Eratosthenes' mathematical circumference of the Earth can thus be converted into latitudinal degrees.<sup>28</sup> As an example of this conversion, Borysthenes is 34,800 stadia from the equator (**Table 5**); every 700 stadia is 1° of latitude.<sup>29</sup> As a result,  $34,800 \div 700 = 49.71^\circ$ . Therefore, Borysthenes, based on Eratosthenes' method, is  $49.71^\circ$  N of the equator (**Table 6**). This exercise is undertaken to provide a comparison to demonstrate the differences in latitudinal degrees between equinoctial hours and ancient stadia distances given by Eratosthenes.<sup>30</sup>

Lastly, **Table 7** provides a comparison between two different mathematical practices of Eratosthenes: 1. Conversions from ancient equinoctial hours → degrees of latitude → ancient stadia and 2. Given ancient stadia → degrees of latitude. To calculate the converted ancient stadia from latitudes based on equinoctial hours, the mathematical measurement of Syene is as follows,  $22.5^\circ \div 6^\circ = 3.75^\circ$ . Every  $1^\circ = 4,200$  stadia.<sup>31</sup> Thus,  $3.75^\circ \times 4,200$  stadia = 15,750 stadia.

Table 6. Ancient stadia distances converted to lines of latitude, based on Eratosthenes' math, which equates that every 700 stadia = 1° of latitude (Elaborated: S. Sink)

| Location    | Eratosthenes' stadia distances | Ancient stadia distances to latitudes |
|-------------|--------------------------------|---------------------------------------|
| Meroë       | 11,800                         | $16.86^\circ$ N                       |
| Syene       | 16,800                         | $24^\circ$ N                          |
| Alexandria  | 21,800                         | $31.14^\circ$ N                       |
| Rhodes      | 25,800                         | $36.86^\circ$ N                       |
| Hellespont  | 29,800                         | $42.57^\circ$ N                       |
| Byzantium   | 30,700                         | $43.86^\circ$ N                       |
| Mid-Pontus  | 32,300                         | $46.14^\circ$ N                       |
| Borysthenes | 34,800                         | $49.71^\circ$ N                       |
| Thoule      | 46,300                         | $66.14^\circ$ N                       |

<sup>28</sup> His work rarely provides latitudinal data, but stadia distances are always provided.

<sup>29</sup> The math decomposition is as follows:  $360^\circ \div 60$  meridians =  $6^\circ$ . Each meridian is  $6^\circ$  apart. The Earth's circumference is 252,000 stadia (31,500 Roman miles).  $252,000$  stadia  $\div 60$  meridians = 4,200 stadia. Every  $6^\circ = 1$  meridian = 4,200 stadia. Each degree is 700 stadia. Author's own calculations that can also be confirmed in Tozer 1935: 176 and confirmed by primary sources, Str. 2.5.34, Hipp. F39.

<sup>30</sup> However, that is not the main emphasis of this paper here. The lack of accuracy is at no fault to Eratosthenes.

<sup>31</sup> Eratosth. M1 *Geminus*, Introduction to Astronomy 16.6–9; Eratosth. M2 *Macrobius*, Commentary on the Dream of Scipio 2.6.2–5; Eratosth. F34 Str. 2.5.7–9.

Table 7. Comparison of the latitude conversions based on ancient equinoctial hours and based on given stadia distances against the modern known latitudes (Elaborated: S. Sink)

| Location    | Ancient equinoctial hours | Ancient equinoctial hours to latitudes | Converted ancient stadia | Given ancient stadia | Ancient stadia distances to latitudes |
|-------------|---------------------------|--|--------------------------|----------------------|---------------------------------------|
| Meroë       | 13 hours                  | 15° N                                  | 10,500                   | 11,800               | 16.86° N                              |
| Syene       | 13.5 hours                | 22.5° N                                | 15,750                   | 16,800               | 24° N                                 |
| Alexandria  | 14 hours                  | 30° N                                  | 21,000                   | 21,800               | 31.14° N                              |
| Rhodes      | 14.5 hours                | 37.5° N                                | 26,250                   | 25,800               | 36.86° N                              |
| Hellespont  | 15 hours                  | 45° N                                  | 31,500                   | 29,800               | 42.57° N                              |
| Byzantium   | 15.25 hours               | 48.75° N                               | 34,125                   | 30,700               | 43.86° N                              |
| Mid-Pontus  | 15.5 hours                | 52.5° N                                | 36,750                   | 32,300               | 46.14° N                              |
| Borysthenes | N/A                       | N/A                                    | N/A                      | 34,800               | 49.71° N                              |
| Thoule      | N/A                       | N/A                                    | N/A                      | 46,300               | 66.14° N                              |

**Table 7** compares degrees of latitudes and ancient stadia distances based on ancient equinoctial hours and given stadia distances. The figure reflects that the stadia distances for Meroë, Syene, Alexandria, and Rhodes, based on both forms of Eratosthenes' math, were most similar to one another. Starting from the Hellespont to the equator and for further northern locations, the comparable latitudinal degrees and ancient stadia conversions start to become extremely distant from one another. The closer Eratosthenes is to the source, that is, ancient African and Egyptian locations that are closer to the equator, the more consistent the borderscape is. Discrepancies are vastly noticeable and become more distorted the further from the equator.<sup>32</sup> For instance, based on the equinoctial hour method, the location of Borysthenes presents an error of 10.29° in comparison, and well beyond the given north-south distance provided in the four fragments, with the furthest north-south *oikoumenic* distance recorded as 38,000 stadia in Fragment 35.

These equinoctial hours and stadia distances reveal that Eratosthenes' southern borderscape mathematically contributes to the discrepancies between ancient and modern spatial knowledge – especially in the application of GIS. The equinoctial hours, latitudinal degrees, and ancient stadia distances in his *Geographika* underscores the very issues of geographical representation and knowledge-making that Eratosthenes himself was grappling with. Thus, reiterating and demonstrating his mathematical practices does not just clarify his ancient thought; it draws attention to the historical contingency of geographical knowledge.

<sup>32</sup> Inaccuracies are not the focus of this paper.

## DISCUSSION AND ANALYSIS

Eratosthenes provided four east-west and north-south distances of the entire inhabited world. The four east-west distances include two occurrences of 70,000,<sup>33</sup> and additional measurements of 73,800,<sup>34</sup> and 83,800 stadia.<sup>35</sup> The four north-south distances are recorded as 30,000,<sup>36</sup> 30,300,<sup>37</sup> 35,000,<sup>38</sup> and 38,000 stadia.<sup>39</sup> The variances are due to the placements of the starting and end locations for each measurement. For example, Fragment 38 records a north-south distance from the Tanaïs River to the Aithiopian Sea,<sup>40</sup> while Fragment 34 records a north-south distance from the ‘northern region beyond the Borysthenes’ down to the ‘southern limit of the inhabited world’ (Fig. 3). The variances reveal that the north-south extent of the *oikoumene* exceeds the 30,000 stadia distance consistently cited by Strabo – who is notably the only ancient source to repeatedly assert this measurement.<sup>41</sup> A similar pattern emerges in the east-west borderscape, where the *oikoumene* aligns more closely with the commonly accepted figure of 70,000 stadia (Fig. 4). As both Fig. 2 and Fig. 4 illustrate, the north-south variances and the east-west variances are dependent on the locations of each measurement. Fragment 35 begins at Thoule and ends at the region of the Kinnamomophoroi, which is on the same latitude as Taprobane (Sri Lanka). Fragment 60 begins at Byzantion (Constantinople) and ends at the equator. The beginning and end locations for both Fragments 38 and 34 have already been previously mentioned.

To aid modern readers in following Eratosthenes’ descriptions of the world, this paper provides modern place names for the ancient toponyms he mentions, and approximate coordinates are also provided when applicable. This helps ground Eratosthenes’ measurements and shows how his geography – established more than two millennia ago – is regularly consistent, precise, and mathematical. Eratosthenes did not just provide topographical information; he used such data to define borders and map the known world. By translating his ideas into modern terms, we can better understand how ancient thinkers viewed geography not just as a list of places, but as a scientific way of defining and understanding space. Fig. 2 and Fig. 4 reconstruct Eratosthenes’ geographical approach. These fragments, with their accounts of distance from the furthest edges of the *oikoumene*, reflect a mathematical borderscape from the north to the south and from the east to the west.

<sup>33</sup> Eratosth. F34 Str. 2.5.7–9, 2.4.3.

<sup>34</sup> Eratosth. F37 Str. 1.4.5.

<sup>35</sup> Eratosth. F38 ‘Measurement of the Entire Inhabited Earth 1’ (Mullerus 1885: 424); Eratosth. F14 Str. 2.4.1–2, F132 Str. 2.4.2, F133 Str. 2.4.4. Str. 2.4.3 does not quote Eratosthenes, but Eratosth. F14 Str. 2.4.1–2 provides details of a north-south distance from Thoule.

<sup>36</sup> Eratosth. F34 Str. 2.5.7–9.

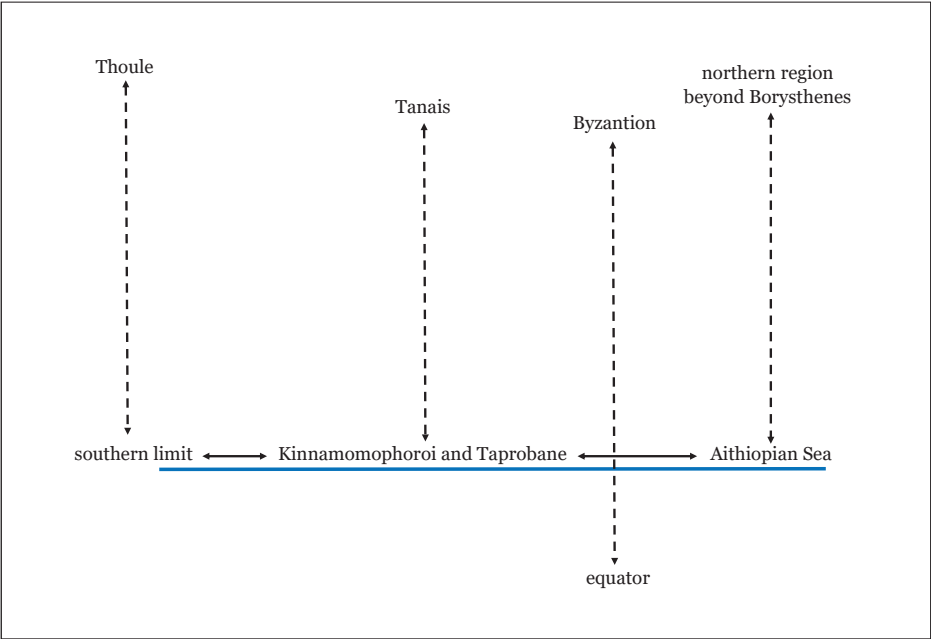
<sup>37</sup> Eratosth. F60 Str. 2.5.38–41.

<sup>38</sup> Eratosth. F38 ‘Measurement of the Entire Inhabited Earth 1’ (Mullerus 1885: 424).

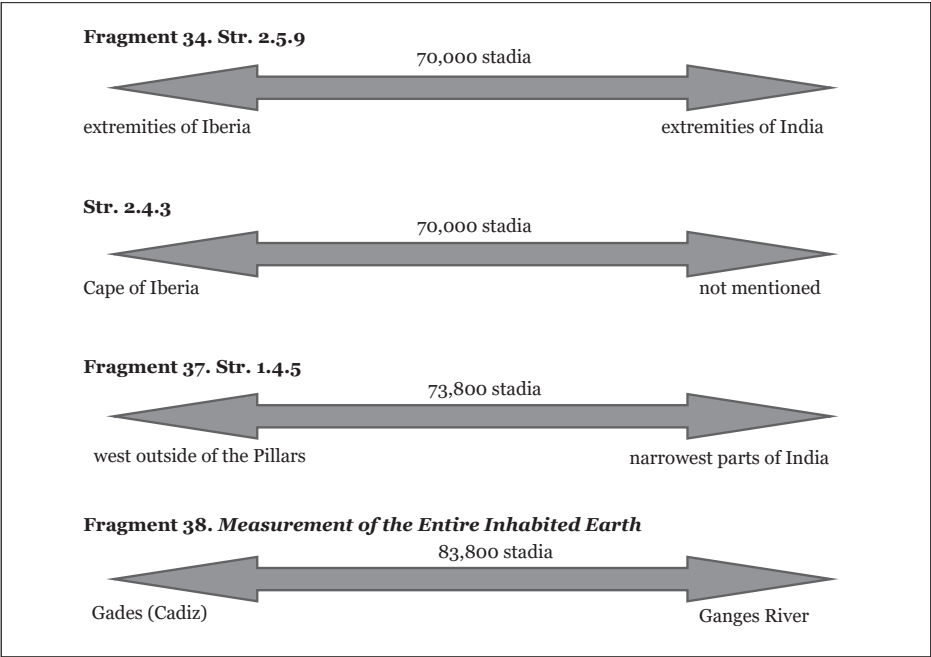
<sup>39</sup> Eratosth. F35 Str. 1.4.2.

<sup>40</sup> The Arabian Sea.

<sup>41</sup> Eratosth. F30 Str. 2.5.5–6, F34 Str. 2.5.7–9, F54 Str. 2.1.35, 2.2.7, 2.1.12, 2.1.13.



3. The north-south variances found in the fragments of Eratosthenes, illustrating their different start and end points (Elaborated: S. Sink).



4. The east-west variances found in the fragments of Eratosthenes (Elaborated: S. Sink).

It is difficult to discern from the four fragments the exact coordinates of the Aithiopian Sea, the equator, the Kinnamomophoroi, and the southern limit of the inhabited world, and where Eratosthenes precisely positioned these in and around the Near East and North Africa; however, Eratosthenes' descriptive geographical narrative in these passages provides an opportunity for deductive reasoning. The equator is the most southerly point recorded in what survives of Eratosthenes' *Geographika*. The equator is not to be confused with the description and placement in Fragment 34 as the 'southern limit of the inhabited world', which in fact refers to the latitude in which the Kinnamomophoroi reside. Fragment 35 confirms that the Kinnamomophoroi also shared the same latitude as Taprobane.<sup>42</sup> The Kinnamomophoroi are in Aristotle, Herodotus, Eratosthenes, Hipparchos, Strabo, and Ptolemy.<sup>43</sup> Geographically, they are inhabitants near the southern part of the Red Sea, positioned off the Somali coast in the Horn of Africa.<sup>44</sup>

Their name (Kinnamomophoroi) associates them with their craft as traders of cinnamon, not producers of it.<sup>45</sup> Cinnamon is not native to East Africa, but archaeobotanical studies have proven that cinnamon was grown in Sri Lanka (and India) and was transported via maritime trade routes from there to the Near East and North Africa.<sup>46</sup> They were the 'middlemen' – the traders between the ancient Mediterranean world and that of Taprobane.<sup>47</sup> Many scholars agree that these 'cinnamon bearers' most probably lived in the region of Punt.<sup>48</sup> Through archaeological finds and botanical analysis, the region of Punt was a known location in Egyptian sources as a trading hub for exotic goods like incense and myrrh.<sup>49</sup> The material culture indicates evidence of long-distance trade between the region of Punt and Sri Lanka.

The exact location of the ancient region of Punt remains a subject of scholarly debate; however, the combination of historical and archaeological sources allows for a general range of modern coordinates, placing the Kinnamomophoroi somewhere between 10–15° N and 40 to 50° E. Sites such as Ras Hafun (near ancient Opone), and Cape Guardafui lie within the presumed location of the region of Punt.<sup>50</sup> In the Hellenistic period, the two main locales in the Horn of Africa were Ras Hafun located at 10.4° N, 51.3° E and Cape

<sup>42</sup> According to modern measurements, these two places do not lie on the same latitude.

<sup>43</sup> Arist. *BNJ* 3c646 T; Jacoby *FGrHist* 646; Herod. 3.111 mentions them without name; Eratosth. F53 Str. 2.5.14, F57 2.5.35, F58 2.2.2, F98 17.1.1–2, Hipp. F44 in Dicks 1960: 93; Ptol., *Geog.* 4.7.34.

<sup>44</sup> Eratosth. F34 Str. 2.5.7–9; Dalby 2003: 87–88; Casson 1959: 122–124; Roller 2010: 24, 152, 197, 236.

<sup>45</sup> Str. 2.1.14, 15.1.22, 16.4.19; Roller 2010: 152, 169, 197, 236; Schneider 2016: 185–202.

<sup>46</sup> Str. 2.1.14, 15.1.22, 16.4.19; Casson 1959: 179; Sidebotham 2011: 34, 38; *Strabo* 2024: 88, 794, 877, 883, 887; Raaflaub, Talbert (Eds) 2010: 83.

<sup>47</sup> Roller 2010: 152, 236; Sidebotham 2011: 38.

<sup>48</sup> Casson 1959: 9–11, 13. The earliest clear mentions of the land of Punt date as early as the Fifth Dynasty (Sahure) and possibly Fourth Dynasty Khufu (c. 2500–2400 BC). Raaflaub, Talbert (Eds) 2010: 5, 173; *Urk.* II: 320, 15; 324, 11; 342, 15; 345, 1; Sidebotham 2011: 34; Kaplan 2018: 196.

<sup>49</sup> Str. 2.1.14, 15.1.22, 16.4.19; Casson 1959: 179; Sidebotham 2011: 34, 38; *Strabo* 2024: 88, 794, 877, 883, 887; Bopearachchi 2004: 543; Raaflaub, Talbert (Eds) 2010: 83. Excavations include Damelein (Somalia), Hafun (ancient Opone), Zeyla.

<sup>50</sup> Smith, Wright 1988: 115; Casson 1989: 16, 29, 45, 59, 101, 115, 123, 130, 132, 134, 136, 251, 280; Ptol., *Geog.* 1.17.



Guardafui located at 11.8° N, 51.2° E, placing them both latitudinally in line with the possible region of Punt, inhabited by the Kinnamomophoroi.<sup>51</sup>

Some scholars consider Ras Hafun, near the ancient site of Opone, as Eratosthenes' 'southern limit of the inhabited world' location, since it was an ancient key trading area and entry port for the Red Sea;<sup>52</sup> Ras Hafun fits both the geographic and economic profile implied in Fragment 34. Pottery fragments of Hellenistic origin have been identified here.<sup>53</sup> At least since the third millennium BC eastern coastline of Africa have been part of a maritime trade network.<sup>54</sup> Ras Hafun most likely was in use until the third century AD.<sup>55</sup> It is a potential toponym for Fragment 34, which is supported by ceramic evidence. Furthermore, ceramics from Ras Hafun were determined to be imported, which additionally emphasises Ras Hafun's role and connection in long-distance trade.<sup>56</sup>

From the combined material, the 'southern limit of the inhabited world' is not the equator; rather, it lies in line with the coast of Somalia. This line of latitude is situated between the equator and Meroë, ranging from 10° N to 15° N. Drawing from both the literary sources and material culture, I propose that the 'southern limit of the inhabited world' most likely corresponds with the region of the Punt.<sup>57</sup> The 'southern limit of the inhabited world' is also in line with the ancient island of Taprobane, which spans from 6° N to 9° N. Coordinates, 06° 57' 10" N, 79° 50' 41" E to 08° 59' 22" N, 79° 59' 53" E, represent known ports in antiquity (Colombo and Mantai, respectively) and thus, either port could very well be associated with the latitude that was drawn from the Kinnamomophoroi (off the Somali coast) to ancient Taprobane.<sup>58</sup>

While Eratosthenes states that the Kinnamomophoroi represent the southern limit of the inhabited world, he continues onward and claims that from the boundary of the inhabited world to the equator is an additional 8,800 stadia (see **Fig. 1**).<sup>59</sup> Although Eratosthenes placed the southern limit as the Kinnamomophoroi, it was also already known that the Nile River went further than this – all the way to Ethiopia.<sup>60</sup> Eratosthenes clearly states that places, lands, and waterways existed outside of the known inhabited world. It is not only known but also rationalised that it is beyond the boundary of their world as they knew it. Fragment 33 (not addressed in this paper) directly specifies this connection – that

<sup>51</sup> Casson 1989: 132.

<sup>52</sup> See above, footnote 50.

<sup>53</sup> Smith, Wright 1988: 120, 124.

<sup>54</sup> Smith, Wright 1988: 115.

<sup>55</sup> Smith, Wright 1988: 122, 125, 138, 140.

<sup>56</sup> Smith, Wright 1988: 138, 140; Casson 1989: 132; Phillips 1997: 449.

<sup>57</sup> See above footnote 48.

<sup>58</sup> The latter latitudinal coordinates given are for the Port of Colombo (Meza 2025).

<sup>59</sup> Eratosth. F34 Str. 2.5.7–9.

<sup>60</sup> Herod. 2.28–29; Str. 1.2.27–28, 17.1.2, 17.1.53; Diod. Sic. 1.32; Arist., *Mete.* 1.14, 2.5. While the ancient Greeks knew that the Nile went down to Aithiopia, in modernity, Burundi is the most southern country that the Nile flows to (Thompson 2003: 106; Shoup 2017: 3). Even Eratosthenes himself knows that the Nile River ends, and the land connects again in the south; Eratosth. F30 Str. 2.5.5–6, F33 Str. 1.4.6–8. Before Eratosthenes, several scholars, such as Herodotus, Hekataios, Ctesias of Cnidus, Ephorus, and Aristotle, refer to places and tribes further south than Meroë. These southern areas are typically referred to as Aithiopia.

Eratosthenes chose the last known people group or last known toponym – as the furthest boundaries when defining the inhabited world; yet in theory, and in knowledge, and through maths, he knew of lands that lay beyond it (**Fig. 5**).<sup>61</sup>

This brief analysis demonstrates a mathematical borderscape of Eratosthenes. The north-south mathematical borderscape required comprehending the fragment, understanding toponymal placement of ‘the southern limit of the inhabited world’, knowledge of GIS, mathematical modelling, and the conceptualisation of this entire border. As ‘the southern limit of the inhabited world’, it acts as a border in name only; however, it is not the southern limit at all, since the equator lies beyond that. While my claim cannot be fully expanded upon within the limitations of this paper, there are several instances where Eratosthenes’ geography reveals a motif of borderscapes beyond the known world.<sup>62</sup> Even in Fragment 34, this is described in relation to the northern boundary beyond the Borys-thenes River. Moreover, in Fragment 60, the theoretical existence of the north and south poles was known, and Eratosthenes knew Kerch (the mouth of Lake Maeotis) was halfway between the equator and the north pole.<sup>63</sup> In Eratosthenes’ geography, examples like these confirm a pattern that boundaries were selected by the location of the most extreme people group or by the most extreme known location. Although these locations were drawn as final boundaries, simultaneously, it was known that places beyond these boundaries existed and thus, measurements were still created for the furthest edges based on theoretical practices. In the case of Fragment 34, the equator lies beyond the southern limit, and furthermore, he acknowledges that land lies beyond the equator. This reiterates the modern concept that borderscapes include intangible practices, are intentionally broad, and change dynamically over time and across space.<sup>64</sup> Eratosthenes’ borderscapes include both theoretical and empirical perspectives.

## CONCLUSIONS

This paper highlighted the furthest north-south boundary measurements of the *oikoumene*, measurement differences and inconsistencies, and the interpretation of geographical and visual representation of Fragments 38, 35, and 34. In summary, Eratosthenes’ north-south measurement variances, ranging from 30,000 to 38,000 stadia, suggest the inhabited world extended further than was originally calculated. These fragments reveal the depth and breadth of Eratosthenes’ calculated and known measurements from Thoule to the equator. Not all places were consistently defined in Eratosthenes’ fragments, leaving some of the north-south distances with imprecise starting or end points. Identifying interpretations of geographical locations, such as ‘the southern limit of the inhabited world’ – perhaps the region of the Punt

<sup>61</sup> Eratosth. F33 Str. 1.4.6–8, F30 Str. 2.5.5–6.

<sup>62</sup> Eratosth. F34 Str. 2.5.7–9, F35 Str. 1.4.2, F37 Str. 1.4.5.

<sup>63</sup> Eratosth. F60 Str. 2.5.38–41, F30 Str. 2.5.5–6.

<sup>64</sup> Rajaram, Grundy-Warr (Eds) 2007: xxvii, 165, 183, 198; Brambilla *et. al.* (Eds) 2015: 1–2, 7, 18, 43, 124, 217–218, 222, 231, 237; Kricher 2019: 2, 5, 6, 14.



5. The *oikoumenic* boundaries by Eratosthenes as described in *Geographika* (Elaborated: S. Sink).

(for Kinnamomophoroi) and either Mantai or Colombo (for Taprobane) – were necessary to understand this particular mathematical borderscape.

Although the precise locations of many of the sites mentioned by Eratosthenes remain uncertain, this paper attempts the conversions of latitudinal degrees and stadia as analytical tools to engage with his construction of the *oikoumene* as a ‘mathematical borderscape’ – a space delineated not only by empirical observation but defined by geometric reasoning, conceptual boundaries, and material and theoretical practices. These spatial reconstructions were intended as an investigative and intellectual curiosity that allows us to explore how Eratosthenes used mathematics to structure and divide the inhabited world. Eratosthenes’ work was interdisciplinary. By converting ancient measurements, it becomes possible to identify patterns, inconsistencies, and systematic (and logical) practices in Eratosthenes’ geographical framework – especially where natural features (mountains, rivers, coastlines, islands) intersect with calculated lines (meridians and parallels). His work demonstrates the interconnectedness of the entire *oikoumene*, of mathematical, practical, and theoretical methods, and of his diverse source material, although the latter was not discussed in this paper.

**Figs 1–5** and **Tables 1–7** were used to illustrate the denseness and difficulty in understanding the technical, geographical work of Eratosthenes and his many methods. The analyses of the passages revealed complexities and inconsistencies in defining the boundaries of the inhabited world. The borderscapes of Fragments 38, 35 and 34 demonstrated that they are mathematically and geographically accessible and comprehensible today.

These mathematical borderscapes showcased not only Eratosthenes' innovation, but served a greater purpose, allowing the modern public to understand just how far the world spread and just how far these places were from one another.

This quantitative analysis of the ancient fragments provided a critical, necessary, and initial entry into understanding the spatial thinking of Hellenistic science more broadly, and the ways in which applied mathematics was used as an innovative standardisation to depict and construct the world as a knowable, measurable entity. For Eratosthenes, whether boundaries were mathematically established or chosen based on natural landscape features, they were conceptually made to encompass only inhabited places, but theoretically other places existed without bounds, beyond the created boundaries.

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