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FIRM VALUATION WITH DATA ENVELOPMENT ANALYSIS: BOSRA İSTANBUL EXAMPLE

Keywords: valuation, Data Envelopment Analysis, market-based valuation approach, Borsa İstanbul.

J E L Classification: G17, G32.

Abstract: Objective of this study is to present an example of firm valuation with Data Envelopment Analysis (DEA), which is an alternative and new firm valuation method, and to contribute this limited field of research area with new findings. With this objective in mind, after giving information about the theory of valuation, market-based valuation, DEA and methodology of firm valuation with DEA, BIST 100 indexed firms in Borsa İstanbul are valued with DEA in this study. Results shows that 69% of the estimated value ranges and 73% of the maximum values estimated cover the market values of the relevant firms. It can be said that this method, which is understood to have more successful estimation results in case the number of firms in peer groups

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increases, is a very useful method for testing the results obtained from other firm valuation methods.

■■■ INTRODUCTION

Widely used firm valuation approaches in the literature are “accounting-based”, “market-based” and “cash flow-based” approaches. Firm valuation with DEA method also emerges as a tool that can be used to find out what the value of a firm should be. Valuation with DEA tool can be evaluated within the scope of the market-based valuation approach, as it searches for companies that are comparable to the firm whose value is being estimated, and thus estimates the firm's value by comparison.

Although it is seen that there are limited number of studies in the literature in which the firm valuation with DEA tool is applied, it has been understood that very satisfactory results have been obtained from these studies. Reviewing the literature, it is found that there is a doctoral thesis, a master's thesis and a scientific article written on this subject. Their information will be given in the literature review part of this study.

Our main motive in doing this study is to test this method with new empirical data and expand this limited research area. Thus, this method's awareness will increase too.

It should also be noted that in the literature, there are examples of valuation of companies operating in similar sectors (for example, the manufacturing sector) with DEA, but only one study was found in which the success of the results were tested by performing a valuation analysis of companies operating in different sectors with DEA. This single study is also a relatively old study (made in 2013) and the results of this study have not been tested with other studies. And also to the best of our knowledge, this study is the first valuation study in Turkey using DEA tool for firms operating in different sectors altogether.

And also, in literature, there is no correlation analysis made for the number of firms in the peer groups and the number of firms whose market value is covered by the value found with DEA in those groups. In this study, this correlation analysis was conducted too.

THE RESEARCH METHODOLOGY AND THE COURSE OF THE RESEARCH PROCESS

This study was carried out firstly by presenting the theoretical information on firm valuation, DEA and valuation with DEA. Then, with the light of theoretical knowledge an application is implemented. In the application phase, biggest 100 companies' (BIST 100 indexed) data in İstanbul Stock Exchange is collected. Within the scope of the study, analysis were made with the data for the end of 2019. All data have been obtained from www.isyatirim.com.tr that is a web site of the one of the biggest investment service provider in Turkey.

After collecting the data, data envelopment analysis was conducted by a software called Efficiency Masurement System. Nevertheless, correlation analysis for finding the correlation coefficient between the number of firms in the peer groups and the "number of companies within the market value estimation range" variables was conducted with the help of SPSS software.

FIRM VALUATION AND MARKET BASED VALUATION APPROACH

Valuation is the process of estimating the value of an asset or liability according to the International Valuation Standards Council (IVSC, 2019) and is a very important concept as they form the basis of decisions involving significant amounts of money or wealth transferred from one party to another (Rawley & Gup, 2010).

Firm valuation can also be defined as the activities of determining the value of a firm based on the above definition, and it is done in order to determine the appropriate and reasonable market value of the firm subject to the valuation (Chamber, 2011).

Although many valuation methods are used, starting from very simple methods to more complex methods, some of these methods are classified together because they have some common features. With the classification of these valuation methods, which have a common feature, the broader concept of "valuation approach" has emerged (Sipahi et al., 2016).

Firm valuation with DEA, which will be used in the implementation phase of the study, is mainly seen as an extension of a market-based valuation approach. In this approach, the value of an asset is valued based on how similar assets are priced in the market. For example, a prospective home buyer decides how much

to pay for a home by looking at the prices paid for similar homes in the neighborhood (Damodaran, 2006).

With market-based methods, when searching for whether a publicly traded and therefore market-priced firm is overvalued or undervalued, first of all, comparable firms are determined and then the average multipliers of these firms are found. Finding a multiplier is essentially a standardization (normalization) operation to make an absolute value comparable.

Firm valuation with DEA follows a similar principle. Here too, first of all, the relative efficiency of Decision Making Units (DMUs) that produce similar outputs by using similar inputs in the value creation process is measured with DEA. As a result of this measurement, the peer DMUs that the inefficient DMUs take as examples for themselves are determined. Here in valuation with DEA by using "taking as an example" feature, the most similar DMUs are determined.

DATA ENVELOPMENT ANALYSIS

The assessment of institutions efficiency is focused on the ability of adequate current and future evaluation of the supervised entities operations, based on the available information and making proper (correct) decisions on this basis (Slimen, Belhaj, Hadriche & Ghroubi, 2022). At this point, DEA comes across as a very useful tool as it determines a frontier and analyzes the efficiency of each unit in the comparison set relative to its distance from this frontier. DEA determines the relative efficiencies of each unit by calculating the ratio of weighted outputs to weighted inputs, using observed inputs and outputs. This method can provide the integrity that traditional methods cannot provide for the evaluation of multiple inputs and multiple outputs with the logic of total factor productivity. In the DEA technique, the weights of inputs and outputs are used for DMUs to maximize their efficiency ratios. Weights are allocated for the inputs and outputs of each decision-making unit separately (Kutlar & Babacan, 2008).

DEA is essentially a method developed for measuring the comparative efficiency of homogeneous DMUs using the same type of inputs and obtaining the same type of output (Benli & Bozdan, 2019).

It should be noted here that classic DEA models have two type of orientation, defined as input and output orientated models. DEA model that while input-oriented models ask, "How much can inputs be proportionally reduced by

keeping output constant?", output-oriented approaches ask "by how much can output quantities be increased proportionally by keeping the input amount constant?" (Torun, Atan & Ayanoğlu, 2020).

Below is a mathematical representation of the output oriented CCR (initials of Charnes, Cooper, and Rhodes) multiplier model which is the first DEA model put forward by Charnes, Cooper, and Rhodes in 1978 (Zhou, Ang & Poh, 2008). CCR model is the basic DEA model and there are also models developed apart from this.

If there are M inputs and N outputs for each of the K DMUs in DEA:

$$\max \sum_{m=1}^M u_m Y_{mo}$$

s. t.

$$\sum_{m=1}^M u_m Y_{mk} - \sum_{n=1}^N v_n X_{nk} \leq 0 \quad (1)$$

$$k = 1, 2, \dots, K.$$

$$\sum_{n=1}^N v_n X_{xo} = 1$$

$$u_m, v_n \geq 0, \quad m = 1, 2, \dots, M;$$

$$n = 1, 2, \dots, N.$$

u_m : weight given to the mth output by the K decision unit,

v_n : weight given to the nth input by the K decision making unit,

Y_{mk} : mth output produced by K decision making units,

X_{nk} : nth input used by K decision making units.

Accordingly, K models are created for K DMUs in the above model, and K maximization models are solved so that the relative efficiency of each DMU can be measured.

Below is the dual of the above multiplier form (input-oriented CCR envelopment model):

$$\begin{aligned}
 & \text{Min} \theta_k \\
 & \sum_{n=1}^N \lambda_k X_{nk} \leq \theta_k X_{no}, \quad n = 1, 2, \dots, N, \\
 & \sum_{m=1}^M \lambda_k Y_{mk} \geq Y_{mo}, \quad m = 1, 2, \dots, M, \\
 & \lambda_k \geq 0, \quad k = 1, 2, \dots, K.
 \end{aligned} \tag{2}$$

The envelope form is preferred more frequently in applications because it contains fewer constraints than the multiplier model. Again, in envelope form, instead of weights related to inputs and outputs, the weights on the DMUs, that is, the density vector λ , are obtained. The density vector λ indicates how much an inefficient DMU should resemble the efficient DMUs (Yücel İşbilen, 2017).

The basic DEA model given above takes into account the assumption of constant returns to scale. This type of modeling is used when a proportional relationship between inputs and outputs can be assumed. For problems where there is no proportional relationship between inputs and outputs, DEA models with variable returns to scale were derived by Banker, Charnes, and Cooper in 1984. The BCC (initials of Banker, Charnes, and Cooper) model can be set by adding a convexity constraint to CCR model, which makes the sum of the λ variables equal to 1.

FIRM VALUATION WITH DEA

Literature on Firm Valuation with DEA

The purpose of the first study (Simak, 2000), which deals with the firm valuation with DEA, is to find the group of publicly traded companies that are most similar to the firm to be valued. In this study, with the input-oriented BCC model, efficiency analysis was carried out by using the input and output data of the year 1997 of 51 publicly traded companies operating in the production sector and then by the output of this analysis valuation is conducted (Simak, 2000).

In the second study on this subject, the first study conducted by Simak was expanded and it was revealed that DEA could be used not only for the firm value range, but also for maximum value calculations for firms found inefficient and minimum value for firms found efficient (Anadol, 2000).

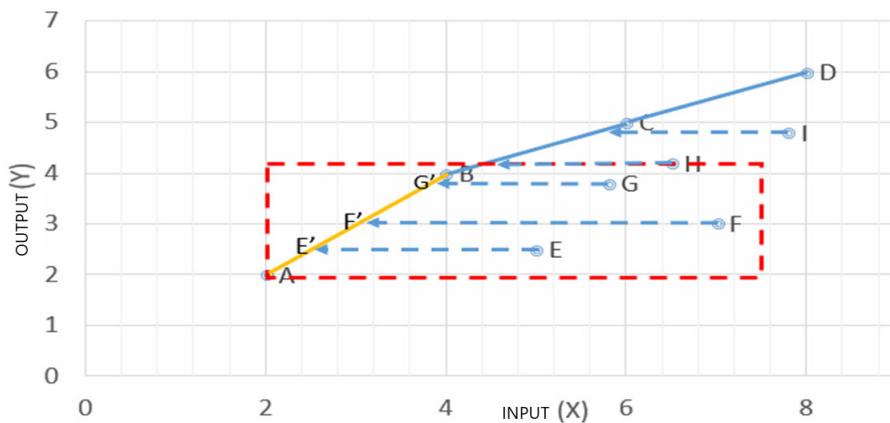
In the third study on firm valuation with DEA, variable returns to scale (VRS) assumption was used, as in the first two studies. Main difference in this study is that the slack based model (SBM) is used instead of the BCC model. It has been stated that the SBM model is used because both inputs and outputs are important in deciding the efficiency state of a DMU and it is difficult to say which is more critical. In this study, 2013 data of 500 firms randomly selected from more than 6.000 United States firms operating in different sectors were analyzed (Anadol, Paradi, Simak & Yang, 2014).

Firm Valuation Methodology with DEA

The DEA valuation approach is essentially a market-based valuation method, and its key component is finding the cluster of companies most similar to the company to be valued. Similarity should exist in all relevant dimensions of the company's operating characteristics. DEA allows for such a multidimensional comparison between firms.

The simple two-dimensional example below illustrates the DEA process. First, the efficient frontier is determined, which consists of the best practice units (A, B, C, D). Second, for each DMU, efficiency measures reflecting its distance from the border is calculated. For every inefficient firm, there is a peer group, a set of efficient firms that are "closest" to the firm being evaluated.

Figure 1. Two-Dimensional DEA



Source : developed by the authors.

For example, the peers of firms E, F, G are companies A and B, and firms B and C are peer of H and I. When considering the input-oriented projection, companies belonging to the same peer group have similar levels of output. Using this feature of peer groups, DEA can be used to identify similar firms multidimensionally. Firms E, F, and G have the same set of peers, and their lambda values show how close they are to each other. In this example, the three relevant lambda vectors for firms E, F, and G are [0.75, 0.25, 0, 0], [0.5, 0.5, 0, 0] ve [0.1, 0.9, 0, 0] (the corresponding efficient firms are A, B, C and D). Lambda values also show that company E is closer to company F than company G (in terms of output). This represents a two-dimensional example, but the same theory applies in multiple dimensions. The difference indicator calculation has been developed to move this two-dimensional example to multiple dimensions. Variable δ_{ij} for each company i, difference indicator between companies i and j as follows (Anadol, 2000):

$$\delta_{ij} = \sum (\lambda_{ik} - \lambda_{jk})^2 \quad (3)$$

Here k is the coefficient of all efficient companies in the analysis. For companies belonging to the same peer group, the presence of a low difference indicator value is an indicator of the similarity in the output dimensions of the DEA model.

The λ value in the same formula shows the contribution level of efficient firms to the virtual DMU created by reflecting an inefficient DMU to the efficiency frontier. These units, which contribute to the creation of the imaginary unit, will have non-zero dual weights, namely lambda, and will form the reference set for the DMU (the DMU in question). The reference set for the DMU therefore consists only of efficient units and serves as a basis for calculating the DMU's efficiency score.

Another indicator to look for similarity is the efficiency score. The efficiency score is an input radial measure of efficiency and can be used to determine similarity across input dimensions. In the example above, since firm E has an efficiency score of 0.5 and firm F has an efficiency score of 0.43, it can be seen that they are somewhat similar in the input-oriented dimension.

In order to understand the valuation method with DEA, the example given by Simak will be used. In Simak's study, data from 51 publicly traded companies operating in the manufacturing sector for 1997 were used and 13 firms were

found to be efficient and the rest were found to be inefficient. As a result of the DEA, for example, the peer companies of the company no. 2 were determined as firms no. 19, 22 and 45. Other firms that take the same firms as a peer are firms no. 14, 32, 33, 34, 42 and 43. Firms no. 19, 22, and 45, which are the peer companies of the firm no. 2, as well as firms no. 14, 32, 33, 34, 42 and 43, which take no 19, 22 and 45 firms as peer just like the firm no. 2 were examined altogether in terms of similarity for the firm no. 2.

The δ_{2j} values, which are the indicator of the difference between these companies and companies no. 2, were calculated. Difference indicators calculated together with lambda values are given in the table below. For example, the difference indicator between firm no. 2 and firm no. 42 is found by solving the following formula:

$$\delta_{2,42} = \sum(\lambda_{2,k} - \lambda_{42,k})^2 = (0.57 - 0.65)^2 + (0.19 - 0.09)^2 + (0.24 - 0.26)^2 = 0.018$$

Table 1. Firm No. 2 Example

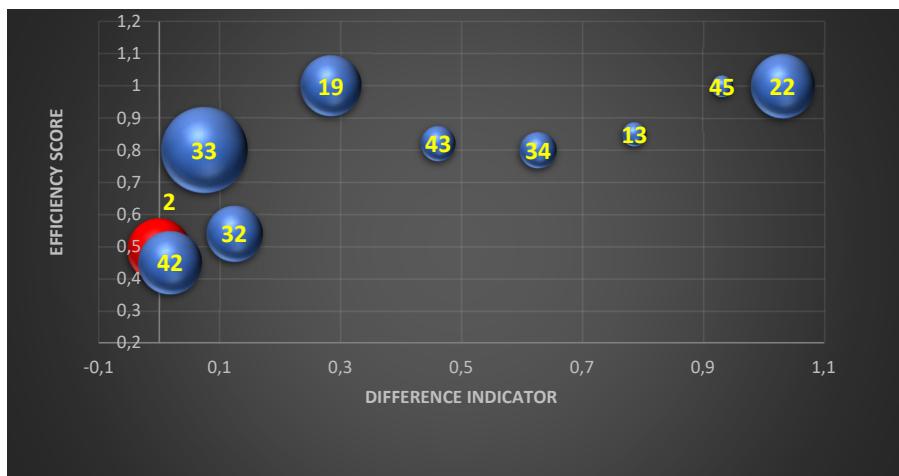
Firm Number	Lambda Values of Efficient Firms			Difference Indicator δ_{2j}	Efficiency Score θ	Market Value Million \$
	λ_{19}	λ_{22}	λ_{45}			
2	0.566	0.192	0.242	0	0.49	372
42	0.65	0.09	0.26	0.018	0.45	375
33	0.387	0.396	0.217	0.074	0.8	690
32	0.456	0.04	0.54	0.124	0.54	295
19	1	0	0	0.284	1	341
43	0.1	0.164	0.736	0.462	0.82	109
34	0.054	0.106	0.84	0.627	0.8	122
13	0.042	0.021	0.937	0.787	0.85	54
45	0	0	1	0.932	1	43
22	0	1	0	1.032	1	380

Source: Simak, 2000, p. 109.

Firms with a low difference indicator and an efficiency score close to 0.49 will be similar to firm 2. The most similar firm in the data set is firm 42 with the lowest difference indicator and with its 0.45 efficiency score is which is very

close to firm 2. Firms 32 and 33 have a relatively low difference indicator, but efficiency scores of 0.8 and 0.54 may be too far from 0.49 to establish a sufficiently high degree of similarity. If we were to use the market value of firm 42 (\$375M) to estimate the market value of firm 2, we would be very close to the real market value of \$371M. The next two similar firms define the \$295 million to \$690 million range, but are less usable for market cap estimation as noted there is a larger difference in efficiency scores to that of company 2. Representation of this analysis for the firm no 2 is given in below figure in which the size of the bubbles is proportional to the market values of these firms.

Figure 2. Balloon Chart for Firm No.2



Source : Simak, 2000.

Accordingly, the closest balloon to the balloon (shown in red) representing firm 2 (firm numbers are shown with yellow numbers) is the balloon belonging to firm 42 and they are almost exactly the same size. Firm 32 is the second closest firm, and the size of the bubble is slightly smaller than the bubble of firm 2. The bubble of firm 33 is farther away from firm 2 (compared to firm 32) and noticeably larger. When these results are evaluated together, it is concluded that the values of companies no. 42 and 32 can be used to determine the value range of firm no. 2.

With DEA, not only value range estimation can be made, but also maximum value estimation can be made for firms that are found to be inefficient as a result of DEA. For this, the lambda values of the efficient firms, of which the firms that are found to be inefficient as a result of DEA, are taken as a peer, are multiplied by market values of efficient firms and then summed together. Value found in the total is the maximum value for the inefficient firm analyzed.

If this analysis is to be done for firm 13 in the above example, the lambda values of firms no. 19, 22 and 45, which are the efficient firms that firm no. 13 takes as a peer are multiplied with the market values of these firms and after that those multiplications are summed together for finding the maximum value of firm 13. This operation will be as follows:

$$\text{Max Value}_{\text{Firm 13}} = (0.042 * 341 \text{ M\$}) + (0.021 * 380 \text{ M\$}) + (0.937 * 43 \text{ M\$}) = 62.5 \text{ M\$}$$

The market value of firm 13 is \$54 Million which is less than the maximum value found as a result of the analysis. Based on this example, analysis of maximum firm (no 13) value with DEA gives a reasonable result.

APPLICATION

In studies using DEA, the selection of decision-making units, DEA model and input-output data set is extremely important. In the following titles, the reasons and results regarding their selection are discussed and finally the results of the application are presented.

Selection of DMUs

In DEA, the efficiencies of units that convert to similar outputs using similar inputs are usually compared. This approach is generally equivalent to making comparisons of units operating in the same sector. However, when the data obtained from the financial statements are used as input and output, it is seen that the efficiency comparison of the companies that are not in the same sector is made on the assumption that each firm produces similar outputs (for example, net profit) using similar inputs (eg assets). In this case, it is assumed that each business just like a factory produces some of the data in the financial statements as output, by using the data that is included in the financial state-

ments as input. The basic assumption here is that the inputs and outputs determined from the financial statement data bring businesses to a common denominator. In line with the same assumption within the scope of this study, it will be assumed that BIST-100 indexed firms are in the same sector. This assumption is nothing but a different interpretation of the criterion that DMUs to be included in the analysis must operate in the same sector when conducting DEA, and it is consistent with the basic logic of DEA.

While determining the DMUs, it is also very important to determine the number of DMUs. Dyson, Allen, Camanho, Podinovski, Sarrico and Shale (2001) argued that the number of DMUs should be determined at least twice as much as the product of their number of inputs and number of outputs. Cooper, Li, Seiford, Tone, Thrall and Zhu (2001) stated that the number of DMUs should be more than three times the sum of the number of inputs and the number of outputs. Norman and Stoker (1991) emphasize that the number of DMUs included in the analysis should be at least twenty. There are also opinions stating that if the number of inputs selected is m and the number of outputs is s , then at least $m + s + 1$ DMU is a necessary constraint for the reliability of the research (Okursoy & Tezsürücü, 2014).

BIST-100 indexed firms are included in this study. The BIST 100 index is the index that includes the biggest firms quoted in Borsa İstanbul, Turkey's only stock exchange. As can be seen under the next heading, the analysis was carried out with two inputs and four output. Considering that the number of firms included in the analysis is 100 and the sum of the number of inputs and outputs is six, the number of DMUs included in this analysis (100) satisfies all the above-mentioned constraints.

Within the scope of the study, analysis were made with the data for the end of 2019. All data have been obtained from www.isyatirim.com.tr that is a web site of the one of the biggest investment service provider in Turkey.

Selection of Inputs and Outputs Included in DEA

The second step in the application of DEA is the selection of the input and output variables to be used in the analysis. Since DEA is a data-based efficiency measurement technique, the accuracy of the measurement results depend selecting significant inputs and outputs (Okursoy & Tezsürücü, 2014).

With DEA, service efficiency of companies and their ability to transform input into output can be analyzed. Considering this logic, inputs and outputs that reflect the firm's value creation process and measure its ability to transform assets into profits must be included in valuation with DEA (Anadol, 2000).

The variables included in this study were selected based on the input and output variables used in previous DEA and valuation studies in the literature. In this direction, within the scope of the study, total assets and total liabilities (total of short and long-term liabilities) were used as inputs, and total revenue, net profit, equity and cash flow from operations were used as output.

Model Selection

In the model selection in DEA, input-oriented models are selected if the relevant DMU has control over the inputs, and output-oriented models are selected if it has control over the output. While input-oriented models are used in the analysis of operations and management, output-oriented models are preferred in planning and strategy analysis (Acer, 2021).

In the application phase of this study, input-oriented DEA model was preferred, because it was evaluated that the control over the input set was more than the control over the output set.

Another issue in model selection is the scale relationship between inputs and outputs. If there is a fixed return on scale of DMUs, the CCR model should be preferred, and if there is a variable return to scale, the BCC model should be preferred (Yücel İşbilen, 2017). Since the rate of changes in inputs and outputs are thought to be unequal in the model established within the scope of this study, variable return to scale approach was chosen.

DEA Analysis and Valuation Findings

In order to carry out DEA, Efficiency Measurement System, a software for solving DEA problems, is used. As a result of the analysis carried out with the help of this program, productivity scores, peer firms and lambda values for inefficient firms were found.

As a result of the analysis, 20 of the 100 companies were found to be efficient. In the valuation phase, firms that take the same efficient firms as a peer

for themselves and the efficient firms that these companies take as a peer are considered together as a peer group.

If an inefficient firm other than itself does not take a peer for the same group, no peer group definition has been made for these groups, since there are not enough comparable firms for this group.

In the value range analysis, the difference indicators of "inefficient firms" within peer groups were used. Together with the low difference indicator, the closeness to the efficiency score of the company whose value range is sought is also taken into account in the value range estimation. For example, while the efficiency score of the inefficient firm whose value range is sought is 10%, a 100% efficient firm, even if the difference indicator is very low (even if it is close), is excluded from the value range estimation and instead, firm with second lowest difference indicator in the peer group is selected if this second best firm has a proximate efficiency score to the firm whose value is sought.

Summary of Analysis

The summary results of the DEA and valuation analysis for BIST 100 companies are given in the table below. As can be seen, there are 16 peer groups in total. The number of firms in peer groups was between 4 and 17.

Table 2. Valuation Results with DEA

No	Number of Firms in the Peer Group	Value Range		Maximum Value	
		Number of Firms Whose Market Value in Estimation Range	Number of Firms Whose Market Value Out of Estimation Range	Number of Firms with Market Value Below the Estimated Maximum Value	Number of Firms with Market Value Above the Estimated Maximum Value
1	7	2	1	2	1
2	9	4	0	4	0
3	6	1	1	0	2
4	6	1	1	1	1
5	5	1	1	2	0
6	5	1	1	2	0
7	6	1	1	2	0

Table 2. Valuation...

No	Number of Firms in the Peer Group	Value Range		Maximum Value	
		Number of Firms Whose Market Value in Estimation Range	Number of Firms Whose Market Value Out of Estimation Range	Number of Firms with Market Value Below the Estimated Maximum Value	Number of Firms with Market Value Above the Estimated Maximum Value
8	5	1	1	1	1
9	6	2	1	2	1
10	9	3	2	4	1
11	5	1	1	2	0
12	8	4	1	5	0
13	17	8	5	8	5
14	9	6	0	1	5
15	9	5	2	7	0
16	4	2	0	2	0
Total		43	19	45	17
Percentage		69%	31%	73%	27%

Source : authors' computation, 2022.

As can be seen from the table above, 69% of the value range estimates made with DEA include the market values of these firms. Again, 73% of the estimated maximum value is above the market value of these firms.

Furthermore, if the correlation analysis is made with the help of the data in the table above (Table 2), the correlation coefficient for the number of firms in the peer group and the "number of companies within the market value estimation range" variables is found 90%, and the correlation coefficient for the number of firms in the peer group and "the number of firms below the estimated maximum value" variables is found 77%. This results show that if the number of firms in the peer group increases, the probability of the estimation interval and the estimated maximum value to include the market value of the relevant firm increases and there is a high and positive relationship between these variables.

■■■ CONCLUSION AND RECOMMENDATIONS

Firm valuation method with DEA is a tool that can be used to find out what the value of a firm should be. It has been observed that there are a limited number of studies in the literature regarding the use of this tool, whose main area of use is efficiency comparison, in firm valuation.

With this study, which have not been conducted before for Stock Market Istanbul firms to the best of our knowledge, it has been tested whether the value ranges and maximum values, found by using DEA results for BIST 100 indexed firms operating in "different sectors", cover these firms' market values. Firms in the BIST-100 index are assumed as if they were factories producing the same outputs (total revenue, net profit, equity and cash flow from operations) with the same inputs (total assets and liabilities).

As a result of the application, 69% of the value range estimates made with DEA include the market values of the firms whose value is estimated. Again, 73% of the estimated maximum value is above the market value of these firms. This results show that DEA is a very useful tool for firm value estimation. These results also overlap with the results of other studies on firm valuation with DEA in the literature. It has also been determined that if the number of firms in the peer group increases, the probability of the estimation interval and the estimated maximum value to cover the market value of the relevant firm increases. This finding has also been discovered in the literature on the firm valuation practice with DEA, but no correlation has been given regarding this. In this study, this finding was supported by this analysis.

As a result, this method, which is understood to be able to find more accurate value ranges as a result of increasing number of peer companies, is considered to be very useful for testing the results obtained from other company valuation methods.

While valuation with multiples is done by taking into account a single parameter that creates value in the valuation process, the inclusion of more than one input and output in the value creation process in the modeling of the valuation with DEA can be seen as an advantage of valuation with DEA compared to valuation with multipliers.

The data used in this study are data for 2019, and it is thought that this test can be expanded further with data for different years in future studies. Nevertheless, by comparing the valuation results obtained from different input and

output sets that can be included in the DEA with future studies, it is thought that evaluations can be made about what the most appropriate input and output set might be. In this way, input and output titles that can better predict value ranges will be presented to valuation experts.

■■■ REFERENCES

Acer, A., (2021). Determining the Efficiency of Entrepots in Logistics Activities by Data Envelopment Analysis. *Journal of Business Research-Turk*, 13(4), 2976–2989. <https://doi.org/10.20491/isarder.2021.1302>.

Anadol, B., (2000). Valuing Private Companies: A DEA Approach, Master Thesis, University of Toronto, Toronto, Canada. <https://tspace.library.utoronto.ca/bitstream/1807/15165/1/MQ54101.pdf> (accessed: 07.07.2021).

Anadol, B., Paradi, J., Simak, P., & Yang, X. (2014). Valuing Private Companies: A DEA Approach. *International Journal of Business and Management*, 9(12), 16-27. <https://doi.org/10.5539/ijbm.v9n12p16> (accessed: 7.10.2021).

Chambers, N. (2011). Firm Valuation (Beta Releases, İstanbul).

Cooper, W.W., Li, S., Seiford, L.M., Tone, K., Thrall, R.M., & Zhu, J. (2001). Sensitivity and Stability Analysis in DEA: Some Recent Developments. *Journal of Productivity Analysis*, 15, 217-246. <https://doi.org/10.1023/A:1011128409257>.

Damodaran, A., (2006). Valuation Approaches and Metrics: A Survey of the Theory and Evidence, Working paper (Stern School of Business). Available at: <http://people.stern.nyu.edu/adamodar/pdffiles/papers/valuesurvey.pdf> (accessed: 15.01.2022).

Dyson, R.G., Allen, R., Camanho, A.S., Podinovski, V.V., Sarrico, C.S., Shale, E.A. (2001). Pitfalls And Protocols in DEA. *European Journal of Operational Research*, 132(2). 245-259. [https://doi.org/10.1016/S0377-2217\(00\)00149-1](https://doi.org/10.1016/S0377-2217(00)00149-1).

International Valuation Standards Council (2019). International Valuation Standards (IVS), <https://www.rics.org/globalassets/rics-website/media/upholding-professional-standards/sector-standards/valuation/international-valuation-standards-rics2.pdf> (accessed: 07.09.2021).

Keskin Benli, Y., & Bozdan, D.N. (2019). Measuring the Efficiency of Deposit Banks in Turkey with Data Envelopment Analysis (İksad Publishing House).

Kutlar, A., & Babacan, A. (2008). CCR Efficiency-Scale Efficiency Analysis in Public Universities in Turkey: Application of DEA Technique. *Kocaeli University Journal of Social Sciences*, 15/1, 148-172.

Norman, M., & Stoker, B. (1992), Data Envelopment Analysis The Assessment of Performance, John Wiley&Sons, England.

Okursoy, A., & Tezsürücü, D. (2014). Comparison of the Relative Efficiencies by the Data Envelopment Analysis: An Application in Turkey for the Provinces of Cultural Indicators. *Journal of Management & Economics*, 21(2), 1-18. <https://doi.org/10.18657/yecbu.92031>.

Rawley, T., & Gup, B. E. (2010). *The Valuation Handbook : Valuation Techniques From Today's Top Practitioners* (John Wiley & Sons, Inc., Hoboken, New Jersey). <https://onlinelibrary.wiley.com/doi/10.1002/9781118268179> (accessed: 29.12.2021).

Simak, P.C. (2000). Inverse and Negative DEA And Their Application To Credit Risk Evaluation, University of Toronto, Doctorate Thesis, Toronto, Canada. <https://tspace.library.utoronto.ca/bitstream/1807/13948/1/NQ49942.pdf> (accessed: 06.06.2021).

Sipahi, B., Yanık, S., & Aytürk, Y. (2016). *Company Valuation Approaches Principles – Methods – Practices* (Seçkin Publishing, Ankara).

Slimen, R.B., Belhaj, F., Hadriche, M., & Ghroubi, M. (2022). Banking Efficiency: A Comparative Study Between Islamic and Conventional Banks in GCC Countries. *Copernican Journal of Finance & Accounting*, 11(1), 89–106. <http://dx.doi.org/10.12775/CJFA.2022.005>.

Torun, N., Atan, M., & Ayanoglu, Y. (2020). An Analysis Of Financial Efficiency Of Public Hospitals According To Statistical Region Classification. *World of Accounting Science*, 22(3), 476–505. <https://doi.org/10.31460/mbdd.672704>.

Yücel İşbilen, L. (2017). Data Envelopment Analysis, Example of Portfolio Activity Measurement with Frontier Analysts and WinDeap, and Various Applications (İstanbul:DER Publications).

Zhou, P., Ang, B.W., & Poh, K.L. (2008). A Survey Of Data Envelopment Analysis in Energy And Environmental Studies. *European Journal of Operational Research*, 189 (1), 1-18. <https://doi.org/10.1016/j.ejor.2007.04.042>.