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Measures of river channel efficiency through cross-sectional form

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Abstract. The efficiency of an open river channel refers to how effectively the channel conveys water with minimal energy loss due to friction with the perimeter. Cross-sectional form, roughness characteristics and slope play pivotal roles in governing a channel's efficiency. Previous studies commonly used hydraulic radius (R_h) and width/depth ratio ($\zeta = w/\bar{d}$) to assess channel efficiency. R_h lacks a dimensionless representation for comparing rivers of different sizes, while the ζ value has no specific scale (e.g., 0 to 1) to fix a channel's efficiency level. This research specifically investigates the influence of channel cross-sectional form on efficiency, assuming a given slope and roughness, and presents novel tools for measuring channel efficiency based solely on cross-sectional form. It is widely acknowledged that a semi-circular cross-section represents the most efficient channel form due to its minimal wetted perimeter. However, little attention has been given to comparing an open channel with an ideal semi-circular form and establishing a fixed efficiency parameter. Hence, three handy tools are proposed to measure the efficiency of open-channel cross-sectional forms. The first tool (E1) compares the cross-sectional area of the observed channel with the area of a semicircle having an equivalent perimeter; we define channel efficiency, E1, as the "ratio of the cross sectional area (A) of the channel, to the area of a *theoretical semicircle* (A_c) having the same wetted perimeter P". The second tool (E2) compares the wetted perimeter of the channel with the perimeter of a semicircle of the same area as the observed cross-section. Finally, the third tool (E3) assesses the width/depth ratio of the observed channel by comparing it to the width/depth ratio of a semicircle of the same area. The effectiveness of these tools was validated using a computed dataset of size 1080 and a field-based dataset comprising 45 cross-sections obtained from the Jalangi River in India. The major highlights of the methods are as follows.

- Three new indices of channel efficiency are proposed using the cross-sectional form.
- The proposed methods are unit-free, with known scale limits for comparison.
- The proposed indices are validated using field data from a tropical river.

Key words:
hydraulic radius;
width/depth ratio;
the most efficient channel,
cross-sectional form;
Jalangi River

Introduction

The measurement of channel efficiency is a classical field of inquiry in fluvial hydraulics. Pickup (1976) viewed an efficient channel for the optimum transportation of bed-load. He suggested that bed-load channels with a given slope, roughness and sediment load tend to shape channels optimally for transporting the load. Using methods from calculus, Guo and Hughes (1984) not only suggested the most hydraulically efficient channel-cross section but also worked for the least construction cost for an artificial channel. They suggested a trapezoidal section with a freeboard. The application of duality theory in the optimisation of hydraulic efficiency for channels with a simple cross-section was introduced by Froehlich (1994), Monadjemi (1994) and Guo (2005). This theory allows for the consideration of two primary objectives: minimising the excavated channel area and maximising the delivery capacity. Huang and Nanson (2000) stated that, by introducing the ζ ratio, the self-adjusting mechanism of alluvial channels can be directly illustrated using basic flow relations of continuity, resistance and sediment transport. Natural channel flow reaches an optimum state of Maximum Flow Efficiency, and the channel form adjusts to exhibit regular hydraulic geometry relations at the dominant or bankfull stage. Their study indicates that the principle of least action provides a physical explanation for the existence of *Maximum Flow Efficiency, Maximum Sediment Transporting Capacity and Minimum Stream Power.* Nanson (2010) suggested the variation in the width/depth ratio determining flow fields in the channel. The concept of probabilistic design of open drainage channels was introduced by Easa (1992). Easa (2009) also suggested the cross-section consisting of two parabolic segments smoothly connected as the least-cost design. Therefore, the evolutionary progress in the concept of channel efficiency is vital to comprehend the nature of previous measurements. Constructional ease and economic advantages of parabolic sections have been recommended by Mironenko et al. (1984), Loganathan (1991) and Laycock (1999). Anwar and de Vries (2003) proposed increasingly efficient channel sections in terms of ease of construction, economic and hydraulic efficiency in the context of power-law cross-section. Das (2007) and Han et al. (2017) introduced the optimal design of cross-sections with horizontal bottom and parabolic sides. A more precise compromise between different most hydraulically efficient and the least construction & maintenance cost sections has been proposed by Dehghan and Shojaeefard (2022), Zheng et al. (2022) and Barkdoll (2022).

Therefore, it appears from the above review that the least-cost section differs from the most hydraulically efficient section, and the former depends on the channel path and land acquisition cost (Blackler and Guo 2009) while the latter depends on the channel's cross-sectional form, channel slope (S), bed roughness and nature of bedload exerting shear stress (τ_0) resisting flow. Tabata and Hickin (2003) found higher flow resistance and lower efficiency in the anastomosing channel. Swamee et al. (2001) suggested that the least earthwork cost is possible for an optimal circular section. They also suggested that, from the viewpoint of earthwork cost, the trapezoidal section is more economical than triangular and rectangular sections.

The maximum hydraulic radius (R_h) or the least wetted perimeter with a given cross-sectional area (A) of a channel was described as the most efficient channel by Rubey (1952), Charlton (2008) or the "best hydraulic section" Chow (1959). Rubey (1952) stated, "The stream makes for itself the channel of the maximum hydraulic radius or maximum efficiency of flow that it can maintain under its conditions of the character of bed, amount of load and other controlling variables." According to Chow (1959), "The hydraulic radius (A/P) is an important indicator of the efficiency of the channel cross section. A more efficient section has a higher hydraulic radius, which leads to higher flow velocities for the same slope and roughness." The efficiency of a cross-section of a river channel is therefore defined as the rate of observed flow (Q_o) through the given cross-sectional area (A) of a channel to the estimated flow (Q_e) through the same cross-sectional area (A) of a theoretical channel, keeping all other conditions (slope, bed and grain roughness) constant (Eq. 1).

$$E = \frac{Q_o}{Q_e} \tag{1}$$

According to Huang and Nanson (2000), the maximum flow efficiency of the channel section is the maximum sediment transporting capacity per unit of available stream power. In this context "Minimum stream power" was proposed by Chang (1979a, b, 1988). Hoover Mackin (1948) suggested that, under given conditions, a river channel tends to form its cross-section with a width (w)/ depth ($\bar{d}$) ratio that provides "the maximum silt-carrying capacity". Abdulrahman (2007) acknowledged the semi-circle as the most efficient channel cross-section.

Acknowledging a semi-circle as the most efficient channel cross-section, Abdulrahman (2007) recommended a composite cross-section- a trapezoidal section at the bottom and a rectangular section at the top or a half-octagonal shape. Jain (2000) mentioned that half-square and half-hexagon are the best hydraulic sections of rectangular and trapezoidal shapes, respectively. Neal et al. (2015) developed a model for wetted perimeter (P) and incorporated the uncertainty of channel cross-section geometry into the flood inundation model.

According to Huang and Nanson (2000), the ζ of alluvial channels can be illustrated with the flow continuity, resistance and sediment transport. In another study by Huang et al. (2004), the Bélanger-Böss theorem of critical flow was presented as the principle of minimum energy (Yang et al. 1981), maximum efficiency and minimum friction of a channel section. Han and Easa (2017) calculated that a three-and-one-third parabolic cross-section is the superior channel cross-section in terms of flow efficiency. They calculated w/d = 2.1273 as the most efficient channel.

Therefore, the hydraulic efficiency of a river is largely influenced by its cross-sectional form, and comparing natural channel forms with the most efficient semicircular form can provide valuable insights into the efficiency of natural channels. However, such comparative attempts in fluvial hydraulics are rare in the literature. Therefore, it would be a novel attempt to fill this crucial gap by developing these comparative tools and consequently advancing the understanding of hydraulic efficiency in river systems. To this end, the present work systematically addresses a few objectives: (1) to assess the geometrical properties of different cross-sectional forms like triangular, rectangular, trapezoidal, half-octagonal and semicircular, (2) to trace out the limitations of existing measures (hydraulic radius & width/depth ratio) for estimating the hydraulic efficiency of a channel, (3) to propose a novel and handy tool in estimating channel efficiency based on the channel form, and (4) to demonstrate the applicability of the proposed method with computed and field-based datasets. The results of this study are expected to have significant implications for further research in relevant fields and will provide valuable handy tools for measuring the hydraulic efficiency of different channel cross-sectional forms, facilitating informed decision-making in river engineering and management practices.

Method details

Different cross-sectional forms and their width, depth & wetted perimeter

From the studies mentioned above, it appears that channel cross-sectional forms have significant control over flow resistance and, in turn, the channel's flow efficiency. Different scholars have suggested different cross-sections from different perspectives (flow efficiency, least construction cost, ease of construction, durability, etc.) that have been synthesised by Abdulrahman (2007) (Table 1).

Table 1. Synthesis of properties of optimum open channel cross-sections by Abdulrahman (2007) based on previous literature

Shape	Average depth ($\bar{d}$)	Cross-sectional area (A)	Wetted perimeter (P)
Trapezoidal	$0.968\left(\frac{nQ}{\sqrt{S_0}}\right)^{3/8}$	$1.622\left(\frac{nQ}{\sqrt{S_0}}\right)^{3/4}$	$3.353\left(\frac{nQ}{\sqrt{S_0}}\right)^{3/8}$
Rectangular	$0.917\left(\frac{nQ}{\sqrt{S_0}}\right)^{3/8}$	$1.682\left(\frac{nQ}{\sqrt{S_0}}\right)^{3/4}$	$3.668\left(\frac{nQ}{\sqrt{S_0}}\right)^{3/8}$
Triangular	$1.297\left(\frac{nQ}{\sqrt{S_0}}\right)^{3/8}$	$1.622\left(\frac{nQ}{\sqrt{S_0}}\right)^{3/4}$	$3.353\left(\frac{nQ}{\sqrt{S_0}}\right)^{3/8}$
Composite (Half-octagon)	$0.984\left(\frac{nQ}{\sqrt{S_0}}\right)^{3/8}$	$1.604\left(\frac{nQ}{\sqrt{S_0}}\right)^{3/4}$	$3.261\left(\frac{nQ}{\sqrt{S_0}}\right)^{3/8}$
Semicircular	$1.004\left(\frac{nQ}{\sqrt{S_0}}\right)^{3/8}$	$1.583\left(\frac{nQ}{\sqrt{S_0}}\right)^{3/4}$	$3.154\left(\frac{nQ}{\sqrt{S_0}}\right)^{3/8}$

Width/depth ratio ($w/\bar{d}$) and hydraulic radius (R_h) are two classical measures of channel efficiency based on channel cross-sectional parameters. In a flume study, Mohammad Nezhad et al. (2022) found that a triangular channel with a 45° side wall is less resistant to flowing water than a channel with a 30° side wall. Amongst triangular cross-sections, the isosceles triangular (diagonally half of a square) channel is the most efficient (Das 2015b). For an isosceles triangular cross-section, area (A) is formulated using Eq. 2 (Fig. 1).

$$A = \frac{a^2}{2} \tag{2}$$

Where a is the length of a side of a square or regular hexagon, or regular octagon.
In terms of area (A), the width (w) of an isosceles triangular channel is analogous to Eq. 3.

$$w = \sqrt{2A} \tag{3}$$

The wetted perimeter (P) of the triangular channel is twice the side (a) formulated in Eq. 4.

$$P = 2\sqrt{A} \tag{4}$$

Amongst the rectangular group of channels, a cross-section having a width/depth ratio of 2:1 has been calculated to be the most efficient (Jain 2000; Swamee et al. 2001; Das 2015a). Of the above-mentioned rectangular section, the area is formulated in Eq. 5.

$$A = w \times \bar{d} = \frac{w^2}{2} \tag{5}$$

Where $\bar{d}$ is the average depth of the channel and is equal to half of w.
In terms of area (A), width (w) is given in Eq. 6.

$$w = \sqrt{2A} \tag{6}$$

Wetted perimeter is formulated using Eq. 7.

$$P = 2 \times \sqrt{2A} \tag{7}$$

The cross-sectional area of the most commonly used trapezoidal or half-hexagonal channel is given in Eq. 8.

$$A = \frac{3\sqrt{3}\,a^2}{4} \tag{8}$$

Where a is the side of a regular hexagon.
The width (w) of a regular trapezoidal cross-section is expressed in Eq. 9.

$$w = 2a = 2\sqrt{\frac{2A}{3\sqrt{3}}} \tag{9}$$

And the wetted perimeter of the trapezoidal channel is derived through Eq. 10.

$$P = 3a = 3\sqrt{\frac{2A}{3\sqrt{3}}} \tag{10}$$

A half-octagonal channel section was suggested as the best channel shape (Abdulrahman 2007). The area of the regular half-octagonal channel cross-section is mentioned using Eq. 11.

$$A = \left(1 + \sqrt{2}\right) \times a^2 \tag{11}$$

Where a is the side of a regular octagon.
The formulas for the width (w) and perimeter (P) in terms of area (A) are given in Eqs. 12 & 13.

$$w = \frac{2\sqrt{A}}{\sqrt{1+\sqrt{2}}} \times \frac{1}{2\sqrt{2}-2} \tag{12}$$

$$P = 4\left(\frac{\sqrt{A}}{\sqrt{1+\sqrt{2}}}\right) \tag{13}$$

Amongst all types of open channel cross-sectional forms, the semi-circular form covers the maximum cross-sectional area (A) by the given wetted perimeter (P), creating the least resistance to flow and thereby producing the most hydraulically efficient form (Das 2015a). However, the bed shear stresses are not uniformly distributed over the wetted perimeter; it is higher on the bed than on banks. Yet, in comparison to all other forms, semicircular forms represent the least resistant one. The area of a semi-circular channel is mentioned in Eq. 14.

$$A = \frac{\pi r^2}{2} \tag{14}$$

The formulas for the width (w) and perimeter (P) in terms of area (A) are given in Eqs. 15 & 16.

$$w = 2 \times \sqrt{\frac{2A}{\pi}} \tag{15}$$

$$P = \pi \times \sqrt{\frac{2A}{\pi}} \tag{16}$$

Based on cross-sectional geometry, hydraulic radius (R_h) and width/depth ($w/\bar{d}$) are two widely used measures for channel efficiency. Abdulrahman (2007) compared $\bar{d}$, A and P in terms of Manning's roughness coefficient (n),

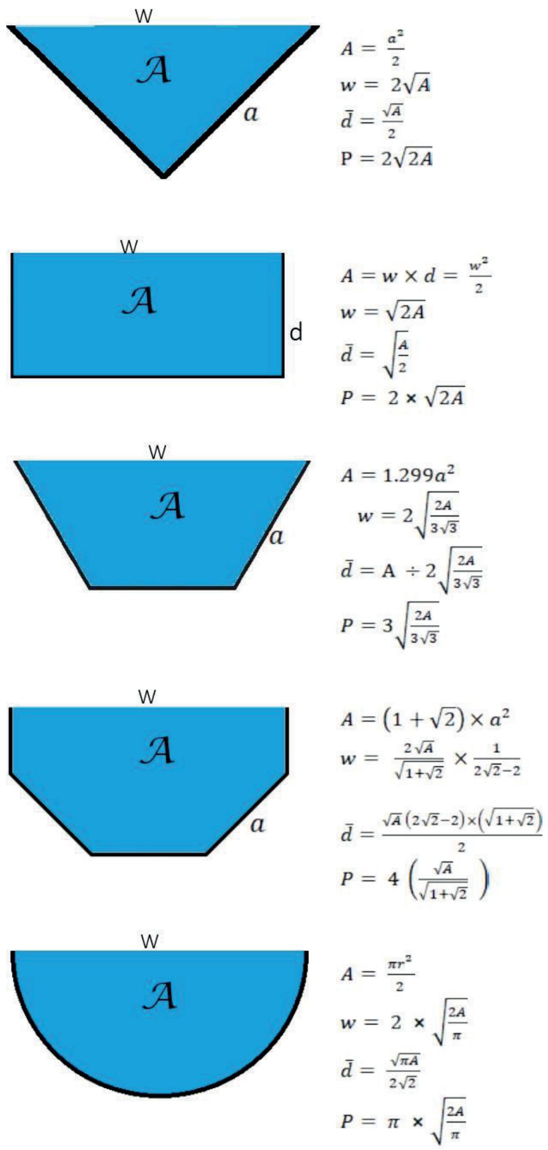

Fig. 1. Different channel cross-sections of equal area (A) and equations of their area, width & wetted perimeter needed for finding the most-efficient one

discharge (Q) and bed slope (S_0) of trapezoidal, rectangular, triangular, half-octagonal and semicircular channels for determining efficient shape (Table 2). We also compared and tabulated w, $\bar{d}$, A and P in terms of cross-sectional area (A) of triangular, rectangular, trapezoidal, half-octagonal and semicircular channel cross-sections are necessary for calculating channel efficiency (Table 2).

Limitations of using R_h and w/$\bar{d}$ for estimating the efficiency of the channel cross-sectional form

R_h is widely used as a measure of the channel's flow efficiency. Rubey (1952) suggested that "the stream makes for itself the channel of the maximum hydraulic radius or maximum efficiency of flow that it can maintain under its conditions of the character of bed, amount of load and other controlling variables". Regarding the most efficient channel, Chang (1979a, b, 1988) contended that the conditions of maximum hydraulic radius and Minimum Stream Power should intersect at a common point. Therefore, the channel form that has the highest R_h is considered the most efficient channel, and the most hydraulically efficient channel is semi-circular with the highest R_h and the least flow resistance. Huang and Nanson (2000) proposed that, under general circumstances, the maximum hydraulic radius is yielded when the ζ is equal to 2.5. In the case of a semicircular cross-sectional form, ζ is equal to 2.55. However, no study has yet proposed any tools for estimating the efficiency level of a cross-sectional form of a natural channel. Although the $\mathrm{w}/\bar{\mathrm{d}}$ ratio and R_h are widely used as measures of channel efficiency, they lack some of the basic properties of an index.

However, R_h is scale sensitive, and a river of a higher magnitude of cross-sectional parameters (A) would have a higher R_h in comparison to a smaller one. For example, two rectangular channel cross-sections A and B with $\mathrm{w}/\bar{\mathrm{d}}$ ratio 20:1, having widths of 20 m and 10 m, respectively, will produce R_h values of 0.91 m and 0.46 m, respectively, although their cross-sectional shape is similar and what varies is the size. Therefore, exclusively from the viewpoint of the channel's cross-sectional form, efficiency ought to be equal. However, R_h produces a higher value for larger rivers and a smaller value for smaller streams.

Secondly, R_h is not unit-free, which is a property of an index (Knighton 1981). As in the case of the example mentioned above, the unit of length (m) is there with the R_h value. Thirdly, R_h has no known limits for pragmatic comparison of the efficiency of different channel cross-sections of varying magnitude from the same river or different rivers (Das and Islam 2023). However, to avoid this scale sensitivity and the presence of dimension, R_h can

Table 2. Properties of trigonometric shaped channels with given cross-sectional area 'A'

Shape (1)	Area (A) (2)	Width (w) (3)	Average Depth ($\bar{d} = A/w$) (4)	Wetted Perimeter (P) (5)
Isosceles Triangle $\left(\frac{1}{2}\ square\right)$	$A = \frac{a^2}{2}$	$2\sqrt{A}$	$\frac{\sqrt{A}}{2}$	$2.828427\sqrt{A}$
Rectangle (2:1) $\left(\frac{1}{2}\ square\right)$	$A = \frac{w^2}{2}$	$1.414214\sqrt{A}$	$\frac{\sqrt{A}}{1.414214}$	$2.828428\sqrt{A}$
Trapezoidal $\left(\frac{1}{2}\ hexagon\right)$	$A = \frac{3\sqrt{3}\,a^2}{4}$	$1.754765\sqrt{A}$	$\frac{\sqrt{A}}{1.754765}$	$2.632148\sqrt{A}$
1/2 Octagon	$A = \left(1+\sqrt{2}\right) \times a^2$	$1.553774\sqrt{A}$	$\frac{\sqrt{A}}{1.553774}$	$2.574377\sqrt{A}$
Semicircle	$A = \frac{\pi r^2}{2}$	$1.595769\sqrt{A}$	$\frac{\sqrt{A}}{1.595769}$	$2.506628\sqrt{A}$

be non-dimensionalised as R_h^2/A. In that case, due to the removal of scale sensitivity, rectangular and arc-shaped channels produce the highest values of 0.125 and 0.159, respectively, which can be used for comparison with other channel shapes.

The $w/\bar{d}$ ratio is a unit-free measure and has been employed to determine channel efficiency. Although the ratio can be used, like the hydraulic radius (R_h), to compare the efficiency between different cross-sectional forms of a river or of different rivers, it fails to indicate the specific efficiency level of an individual cross-section.

2.3 Derivation of channel-efficiency methods proposed in the study

Lacey's (1958) flow equation was rewritten by Huang and Nanson (2000), incorporating Lacey's silt factor related to sediment size D_{50} (mm), channel's mean depth $\bar{d}$, hydraulic radius R_h and channel slope S as (Eq. 17).

$$v = \frac{\bar{d}^{1/4}\sqrt{R_h S}}{N_a} \tag{17}$$

where N_a=0.0225$f^{0.25}$, in which f is Lacey's silt factor related to sediment size and f= 1.6 $D_{50}^{0.5}$.

It has been conjectured that Maximum Sediment Transporting Capacity (MSTC) could serve as a suitable indicator of flow efficiency for river channels characterised by sediment-transporting active beds and erodible banks (Griffith 1927; Kirkby 1977). MSTC can be used to indicate channel efficiency because it reflects the channel's ability to move the greatest amount of sediment with the available stream power, signalling an optimal balance between energy use and sediment transport. This condition represents a state of dynamic equilibrium, where the channel form is naturally adjusted for maximum hydraulic and geomorphic efficiency. The sediment transport equation of DuBoys (1879) incorporates factors of flow shear stress (τ_0), critical shear stress (τ_c), channel width (w), and coefficient C_d and was formulated by Huang and Nanson (2000) as (Eq. 18):

$$Q_s = C_d \tau_0 (\tau_0 - \tau_c)\, w \tag{18}$$

In terms of cross-sectional form, the sediment transport equation was shaped (Huang and Nanson 2000) as (Eq. 19):

$$Q_s = K_1 \frac{\zeta^{10/11}}{(\zeta+2)^{7/11}} \left[K_2 \frac{\zeta^{5/11}}{(\zeta+2)^{9/11}} - \tau_c\right] \tag{19}$$

where, $\zeta = w/\bar{d}$

Replacing $K_1=\rho C_d N_a^{8/11} S^{8/11} Q^{7/11}$, $K_2=\rho N_a^{4/11} S^{9/11} Q^{4/11}$, and τ_c with their respective values, the formula can be rewritten only in terms of cross-sectional shape, i.e. ζ, as (Eq. 20).

$$Q_s = 18.88 \frac{\zeta^{10/11}}{(\zeta+2)^{7/11}} \left[4.74 \frac{\zeta^{5/11}}{(\zeta+2)^{9/11}} - 0.0889\right]$$

Therefore, with the given C_d,τ_0 ,τ_c and S channel efficiency (flow and sediment transport) can be expressed solely in terms of cross-sectional form.

Both theoretically and practically, the semi-circular cross-section has been proven as the most efficient channel form (Pickup 1976; Swamee et al. 2001; Das 2015a). That is why, based on cross-sectional geometry, in this present study, to determine the level of efficiency of a channel cross-section, the properties of the semi-circular channel have been considered as a standard. Based on properties of semicircular cross-section, we proposed three equations to estimate the level of hydraulic efficiency of the cross-sectional form of a channel.

First, we define channel efficiency, E1, as the "ratio of the cross sectional area (A), to the area of a *theoretical semicircle* (A_c) having the same wetted perimeter P" Eq. 21 & 22.

$$\mathrm{E}_1 = \frac{A}{A_c} \times 100 \tag{21}$$

$$\mathrm{E}_1 = \frac{2\pi A}{P^2} \times 100 \tag{22}$$

Instead of area, to estimate channel efficiency, one can also compare the wetted perimeter (P) of a channel cross-section with the perimeter of a semicircle having the same area as the channel. Hence, we define the channel's ***geometric efficiency***, E2, as the "ratio of the ***minimum possible wetted perimeter (Pc)***—that of a semicircle having the same cross-sectional area—to the channel's ***actual wetted perimeter (P)",*** expressed as (Eq. 23 & 24).

$$\mathrm{E}_2 = \frac{P_c}{P} \times 100 \tag{23}$$

$$\mathrm{E}_2 = \frac{\sqrt{2\pi A}}{P} \times 100 \tag{24}$$

Combining equations of width (w) and average depth ($\bar{d}$) of a semicircular cross-section from Table 2, the ideal width-to-depth ratio can be written as (Eq. 25 & 26).

$$\frac{w}{\bar{d}} = 2\sqrt{\frac{2A}{\pi}} \div \frac{\sqrt{\pi A}}{2\sqrt{2}} \tag{25}$$

$$\frac{w}{\bar{d}} = \frac{8}{\pi} = 2.546 \tag{26}$$

We define channel's geometric efficiency as the "ratio of the width-to-depth ratio (2.546) of the ideal semicircular section to that of the measured width to depth ratio" (Eq. 27). This formulation ensures a dimensionless efficiency bounded between 0 and 1, with unity corresponding to the most hydraulically efficient (semicircular) shape while deviation from unity (>1 or <1) indicate less efficient channels.

$$\mathrm{E}_3 = \frac{2.546}{\zeta} \times 100 \tag{27}$$

All other geometric shapes other than a circle cover a lesser area with a given perimeter; therefore, A_c is always greater than A; and all other geometric shapes other than a circle have larger perimeters to encompass a given area; therefore $Pc < P$. That is why the values of indices (Eq. 22 & 24) range from 0% to 100% or zero to unity (if not expressed in %). Zero indicates the absence of efficiency, while the higher values indicate the higher hydraulic efficiency of the channel cross-sectional form. It is important to note that, in cases where $\zeta < 2.55$, the equation (e.g., Eq. 27) yields an absurd and theoretically impractical solution. To address this, the first part of the equation [2.54/ζ] should be replaced by its reciprocal [ζ/2.54].

Method application

To apply the tools (Eqs. 20 to 22) suggested in this study for estimating the hydraulic efficiency of channel cross-sections, two steps were followed: firstly, we applied the tools on a computed database, and secondly, the application was made on a field-based dataset.

Applicability on the computed database

Utilising Microsoft Office Excel software, we computed various hydraulic parameters like w (width), $\bar{d}$ (depth), ζ (width-to-depth ratio), P (wetted perimeter), R_h (hydraulic radius), E1 (hydraulic efficiency factor I) and E2 (hydraulic efficiency factor II), for 1080 cross-sectional forms. The statistical summary shows (Table 3) the variation in the width (w) of a rectangle while keeping its depth (d) constant at 2.0, resulting in a wide range of w/dw/d ratios (0.5 to 538.5). The perimeter of a semicircle (P_c) corresponding to a given area (A) varies from 3.5 to 116.3, while the area (A_c) for a given perimeter (Pr) ranges

Table 3. Descriptive statistics of 1080 computed channel cross-sections having fixed depth and varying channel width

Statistic	Varying width of rectangle (w)	Fixed depth of rectangle (d)	w/d	Perimeter (Pc) of a semicircle having area ‚A'	Area of a semicircle (Ac) having perimeter (Pr)	E1=A/Ac	E2=Pc/Pr	E3=2.55/ζ	R_h
Max	1077.0	2.0	538.5	116.3	185982.3	0.8	0.9	0.85	2.0
Min	1.0	2.0	0.5	3.5	4.0	0.0	0.1	0.00	0.4
Average	540.5	2.0	270.2	77.7	62654.2	0.1	0.2	0.03	2.0
Median	540.5	2.0	270.3	82.4	47186.3	0.0	0.2	0.01	2.0
Q1	270.8	2.0	135.4	58.3	12014.2	0.0	0.1	0.02	2.0
Q3	810.3	2.0	405.1	100.9	105520.2	0.0	0.2	0.00	2.0

widely as well (4.0 to 185,982.3). Efficiency E1 (A/Ac) and E2 (Pc/Pr) remain low overall, with E1 peaking at 0.8 and E2 at 0.9, suggesting limited geometric efficiency. E3 fluctuates up to 0.85, while R_h remains fixed at 2.0 in most cases, except for a minimum value of 0.4. The median and average values indicate a generally symmetric distribution centred around mid-range values for most metrics.

Our findings reveal a logarithmic correlation between R_h and ζ (R2 = 0.9202), with R_h increasing as the cross-sectional area expands. However, the rate of R_h increase is gradual until $\zeta = 1$, after which it experiences rapid growth until $\zeta = 5$. Beyond $\zeta = 5$, the rate of increase in R_h diminishes substantially, approaching negligible change per unit change in ζ.

We conducted a comparison using 1080 observations of E1 and E2, contrasting them with ζ values associated with rectangular channel forms of a fixed depth (2 m) and varying width. Our analysis revealed that the highest efficiency for the channel form (E1 = 0.79 & E2 = 0.89) was attained at $\zeta = 2$, and $P/w = 1.57$ (the value of a semicircular cross-section), thereby affirming the suitability of tools E1 and E2 for estimating the efficiency of the cross-sectional form of a river channel (Fig. 2). Additionally, the similar appearance of the E1 and E2 plots and the correlation coefficient between them (E2 = 0.2666 + 0.8506 × E1, $R^2 = 0.9834$) indicates a high level of consistency between the two measures.

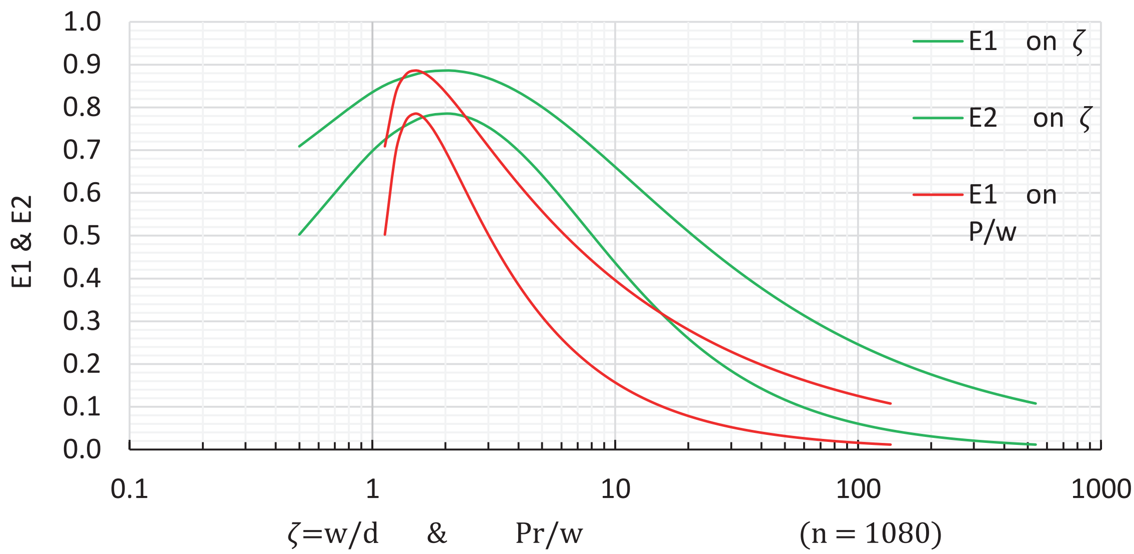

Fig. 2. The highest efficiency of rectangular channel form (E1 = 0.79 & E2 = 0.89) is attained at $\zeta = 2$ and at P/w = 1.57

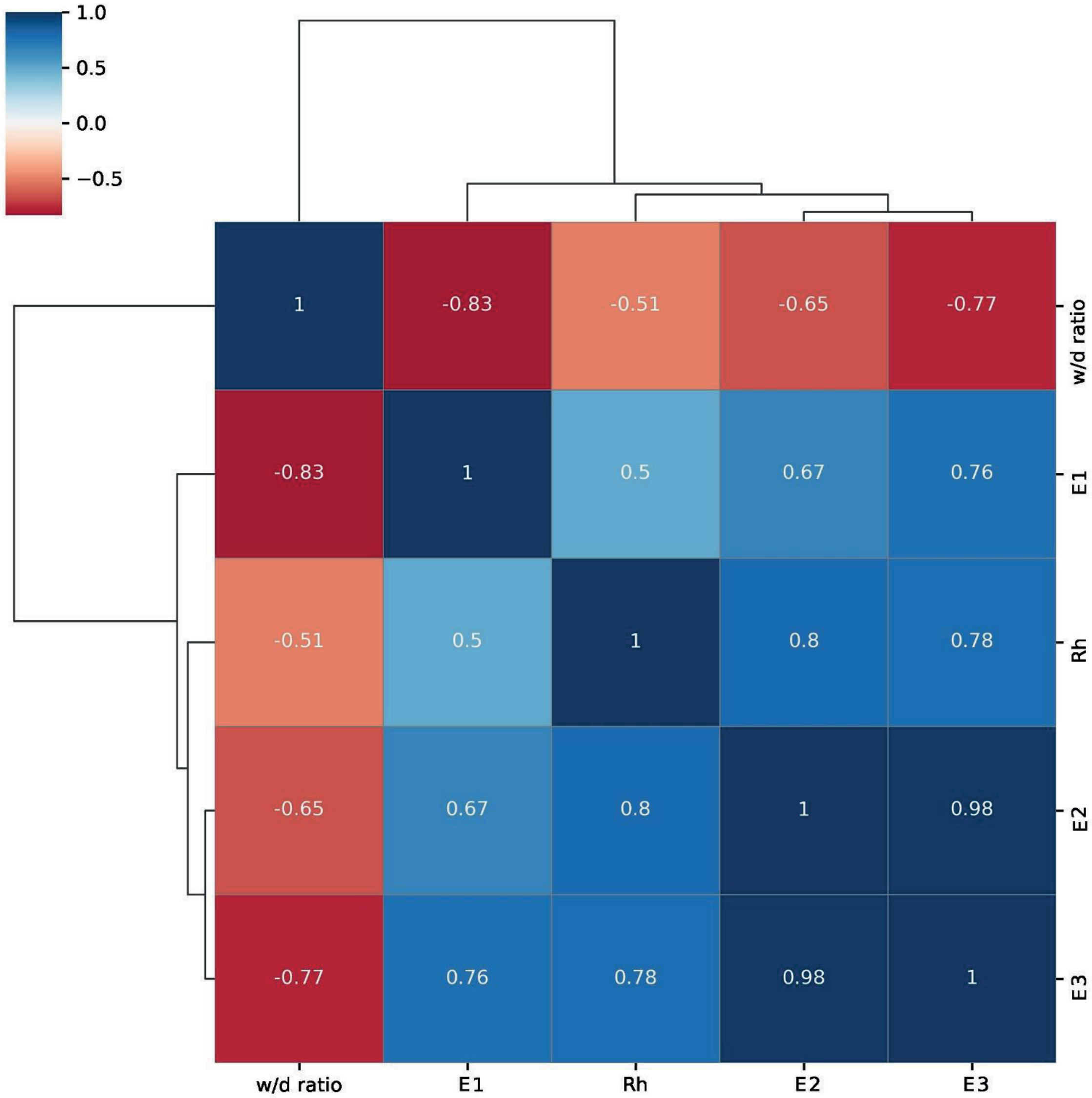

Fig. 3. Correlation between the proposed indices (E1, E2, & E3) and previous method (R_h and w/d̄ ratio).

Applicability to the field-based dataset

In this section, we again followed two steps: firstly, we calculated the Pearson's correlation coefficient between indices formulated (Eq. 22, 24 & 27) in this paper and previous measures (R_h & ζ) to estimate the degree of association among indices (Fig. 3). For this purpose, we used data from 45 cross-sections of the Jalangi River channel in West Bengal, India (Fig. 4). .

Correlation coefficient (R) between R_h and E1 is 0.5, whereas E2 and E3 portray a higher magnitude of correlation (~0.8) (Fig. 3). However, there is a negative relation between ζ and E1, E2, E3 and R_h. The correlations are statistically significant, as indicated by the $p<0.05$.

Secondly, to validate the methods proposed in this study (Eqs. 16 to 18), we took 45 cross-sections across the river Jalangi (Fig. 4), West Bengal, India and applied the proposed tools of the present study to estimate the level of hydraulic efficiency of different sections of the channel.

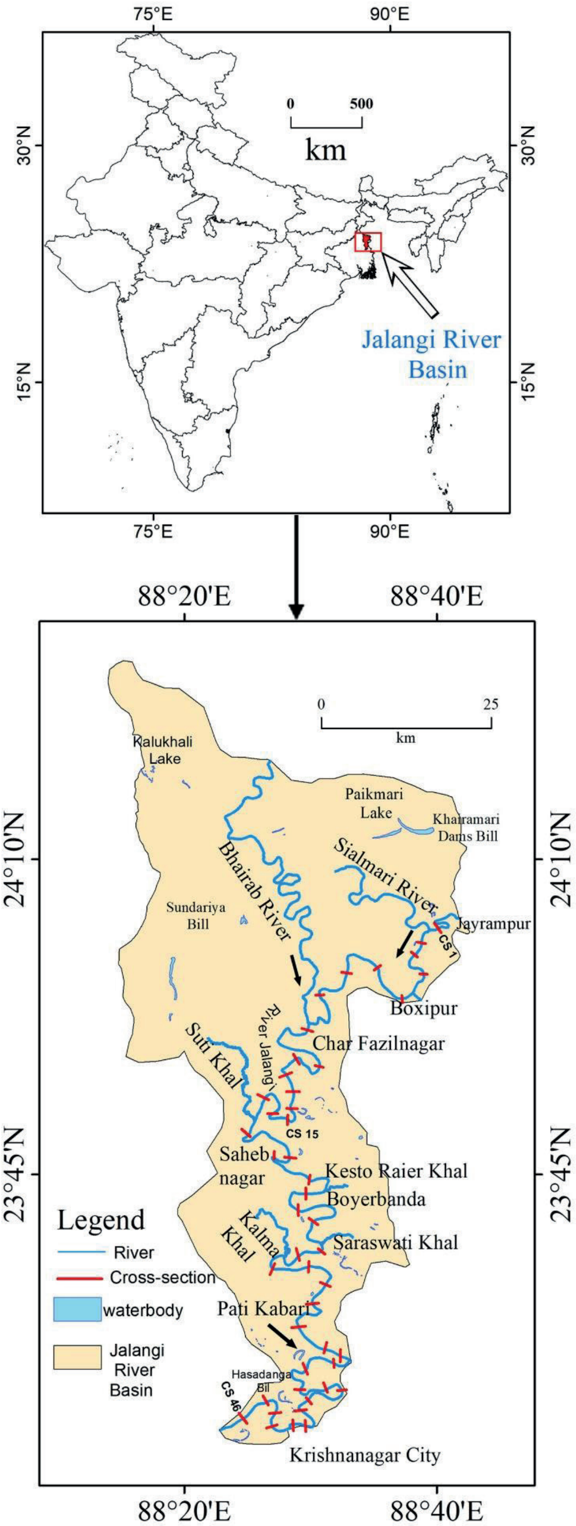

Fig. 4. Location of the cross sections (CS) on the River Jalangi, India

The Jalangi River

Jalangi is a distributary of the sacred Ganga River. Originating at 24°05′26″ N and 88°4′53″ E in the extreme north of Nadia district, West Bengal, India, the river follows a sinuous path covering a distance of 233 km (Das and Bhattacharya 2020) before its confluence with the river Bhagirathi at Swarupganj at 23°29′23″ N and 88°28′57″ E. The course of the Jalangi River includes a stretch of 96 km along the border of the Nadia and Murshidabad districts, with the remaining 137 km traversing through the Nadia district. Notably, a 51-km-long section from the off-take to the Jalangi-Bhairab confluence at Char Moktarpur village in Nadia district constitutes a "dead channel", meaning it carries little or no flow during certain periods (Das 2013).

The Jalangi River's 4,300 km² catchment receives water from tributaries like the Sialmari, Bhairab, Suti Nala and Kalma Khal, which drain backswamps. Spill channels such as Bhairab Khal, Kesto-Raier Khal, Saraswati Khal and Anjana relieve excess flow during peak periods, reflecting a dynamic hydrological system (Das et al. 2025 a,b,c). The basin spans the moribund tract of Murshidabad and Nadia districts in deltaic West Bengal, a gently south-sloping plain dotted with swamps, paleo-channels, oxbow lakes and meander scars – evidence of the river's geomorphological past (Dasgupta 1997). Geologically, it comprises Palaeocene to lower Palaeocene alluvium of the Jalangi formation, overlain by recent alluvium (Majumder 1978). Fertile silt deposits atop these layers enhance agriculture and attract settlements. Soil types – from sand to clay – affect erosion and channel changes (Guchhait et al. 2016). The basin has a tropical monsoon climate, with 1,473 mm annual rainfall mainly from June to September (Majumder 1978), supporting agriculture, which employs 47.04% of the population (DCH, Nadia 2011). The river has shaped deltas, marshes, levees and meander scars, creating habitats for flora, fauna and inland fish, all vital to regional socio-economics (Das 2025a,c).

Channel efficiency of the Jalangi River

Figure 5 shows the spatial variation of four efficiency measures – R_h, E1, E2 and E3 – along a 233-km stretch from offtake to the confluence point. R_h and E3 remain relatively low and stable throughout the distance, indicating minimal spatial variability. In contrast, E1 and especially E2 exhibit higher values and significant fluctuations, with noticeable peaks around 40–100 km and a sharp increase after 180 km, suggesting possible changes in cross-sectional form near the confluence. Overall, E2 shows the greatest sensitivity to spatial changes, while R_h and E3 appear more stable.

It is apparent from Figure 6A that, in deltaic Bengal, channel efficiency (E1 %) is inversely related (power function) to ζ (having $R^2 > 0.79$).

Figure 6B also depicts that the % efficiency of all the cross-sections of the channel is within the limit of 0% to 100%. The cross-sectional area and wetted perimeter are the parameters for R_h, which indicates higher R^2 values (>0.6) with E2 and E3. However, and R_h do not portray a strong association between them.

Conclusions

The efficiency of river channels holds significant importance in understanding the complex interplay between stream energy and the sediment load nexus. The cross-sectional geometry of river channels is a crucial factor in determining their overall efficiency. Traditionally, quantifying and comparing channel efficiency for rivers of varying sizes posed challenges, especially concerning using R_h and ζ methods. The tools proposed in this research offer simple yet effective solutions for detecting even subtle changes in channel efficiency resulting from alterations in channel geometry. As the research lays the groundwork for measuring hydraulic efficiency exclusively from the viewpoint of channel geometry, future endeavours should focus on employing these methods in various climatic and morpho-tectonic settings to obtain comparable results and expand the knowledge base on channel efficiency. The findings of this study are expected to pave the way for more effective river management strategies and sustainable development of river systems worldwide.

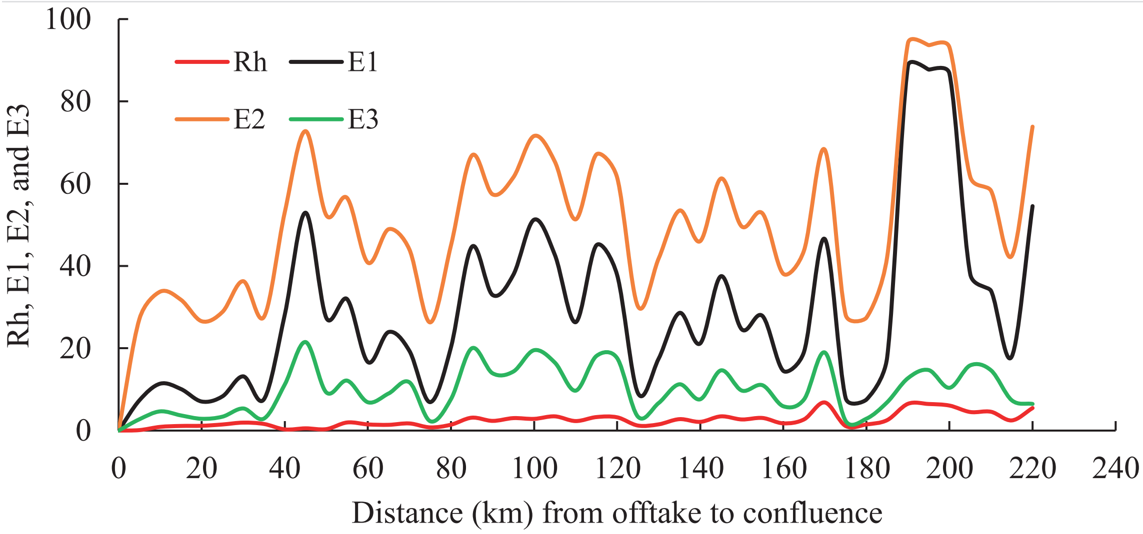

Fig. 5. Downstream variation in Rh, E1, E2, and E3 of the Jalangi river

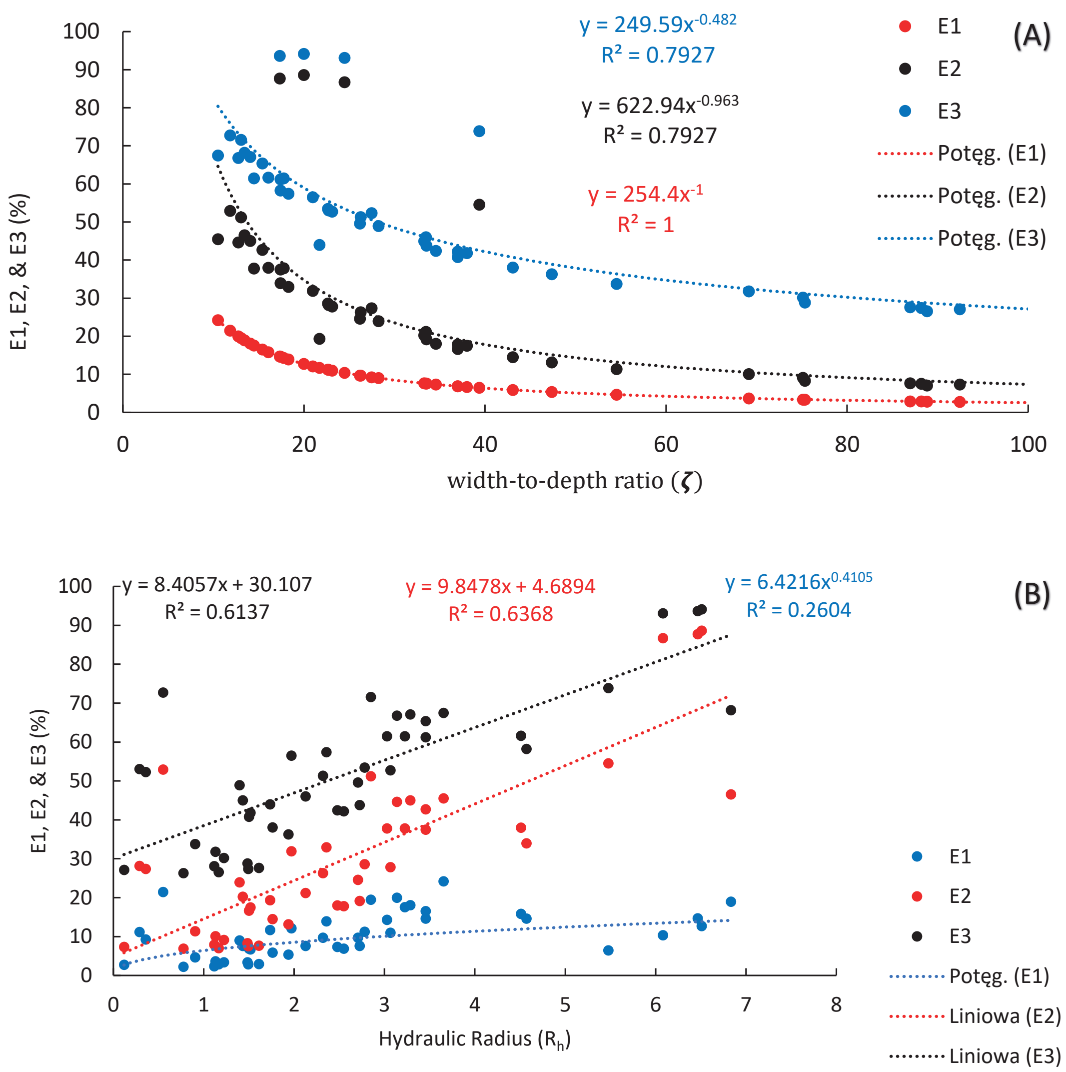

Fig. 6. Scatter plots of E_2, E_2 and E_3 showing their relations with (A) ζ and (B) R_h

Disclosure statement

No potential conflict of interest was reported by the authors.

Author contributions

Study design: BCD; data collection: BCD, AI; statistical analysis: BCD, AI, UD; manuscript preparation: BCD, UD; literature review: BCD, AI.

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