ON DIRECTIONAL DERIVATIVES
FOR CONE-CONVEX FUNCTIONS

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ABSTRACT. We investigate the relationship between the existence of directional derivatives for cone-convex functions with values in a Banach space Y and isomorphisms between Y and c_0.

1. Introduction

Starting from the seminal results of Asplund, many efforts are devoted to characterisations of Banach spaces in terms of differentiability properties of some classes of functions. In the present paper we investigate the relationships between directional differentiability of cone-convex functions and the properties of their image spaces. We prove sufficient conditions ensuring that a Banach space Y does not contain an isomorphic subspace of c_0 (does not contain a copy of c_0). These sufficient conditions are expressed in terms of directional differentiability of cone-convex functions taking values in Y.

Our aim is to relate the existence of directional derivatives for K-convex functions $F\colon X \to Y$, where cone K is normal, to the existence of isomorphisms between image space Y and c_0.

Isomorphisms of a Banach space Y and the space c_0 have been investigated e.g. in [4], [10], [17], [18]. For example in [18] such spaces were investigated in