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LOWER AND UPPER BOUNDS FOR THE WAISTS OF DIFFERENT SPACES

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Abstract. In this paper we prove several new results around Gromov's waist theorem. We give a simple proof of Vaaler's theorem on sections of the unit cube using the Borsuk–Ulam–Crofton technique, consider waists of real and complex projective spaces, flat tori, convex bodies in Euclidean space; and establish waist-type results in terms of the Hausdorff measure.

1. Introduction

Using his version of Morse theory, Almgren showed that for any smooth map $f\colon \mathbb{S}^n \to \mathbb{R}^k$, under certain general position assumptions, there exists a $y \in \mathbb{R}^k$ such that the Riemannian volume of the fiber $f^{-1}(y)$ is at least the Riemannian volume of the sphere $\mathbb{S}^{n-k}$ (here $\mathbb{S}^i := \{x \in \mathbb{R}^{i+1} : |x| = 1\}$), and the bound on

$$\inf_f \sup_{y\in Y} \operatorname{vol}_{n-k}(f^{-1}(y))$$

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