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## NONAUTONOMOUS CONLEY INDEX THEORY. THE HOMOLOGY INDEX AND ATTRACTOR-REPELLER DECOMPOSITIONS

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ABSTRACT. In a previous work, the author established a nonautonomous Conley index based on the interplay between a nonautonomous evolution operator and its skew-product formulation. This index is refined to obtain a Conley index for families of nonautonomous evolution operators. Different variants such as a categorial index, a homotopy index and a homology index are obtained. Furthermore, attractor-repeller decompositions and conecting homomorphisms are introduced for the nonautonomous setting.

In [4], the author defined a nonautonomous Conley index relying on the interplay between an evolution operator (1) and a skew-product formulation. Isolatted invariant sets obtained in the skew-product setting give rise to an index for a related nonautonomous evolution operator.

An important technical detail of defining the index is the class of index pairs under consideration. In [4], index pairs are always obtained in the skew-product formulation. In this paper, it will be proved that, roughly speaking, the same index can be defined using a broader class of index pairs based on the evolution operator instead of the skew-product formulation.

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<sup>(1)</sup> or process