

REALIZATION OF A GRAPH AS THE REEB GRAPH OF A MORSE FUNCTION ON A MANIFOLD

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ABSTRACT. We investigate the problem of the realization of a given graph as the Reeb graph $\mathcal{R}(f)$ of a smooth function $f: M \rightarrow \mathbb{R}$ with finitely many critical points, where M is a closed manifold. We show that for any $n \geq 2$ and any graph Γ admitting the so-called good orientation there exist an n -manifold M and a Morse function $f: M \rightarrow \mathbb{R}$ such that its Reeb graph $\mathcal{R}(f)$ is isomorphic to Γ , extending previous results of Sharko and Masumoto–Saeki. We prove that Reeb graphs of simple Morse functions maximize the number of cycles. Furthermore, we provide a complete characterization of graphs which can arise as Reeb graphs of surfaces.

1. Introduction

The Reeb graph, denoted by $\mathcal{R}(f)$, has been introduced by Reeb [11]. It is defined for a closed manifold M and a smooth function $f: M \rightarrow \mathbb{R}$ with finitely many critical points by contracting the connected components of level sets of f . Reeb graphs are widely applied in computer graphics and shape analysis (see [1]).

Sharko [12] and Masumoto–Saeki [8] proved that every graph which admits a good orientation (see Definition 2.2) can be realized as the Reeb graph of a function with finitely many critical points on a surface. This leads to a natural problem:

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