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## CONVENIENT MAPS FROM ONE-RELATOR MODEL TWO-COMPLEXES INTO THE REAL PROJECTIVE PLANE

MARCIO COLOMBO FENILLE

ABSTRACT. Let f be a map from a one-relator model two-complex  $K_{\mathcal{P}}$  into the real projective plane. The composition  $\varrho \circ f_{\#}$  of the homomorphism  $f_{\#}$  induced by f on fundamental groups with the action  $\varrho$  of  $\pi_1(\mathbb{R}P^2)$  over  $\pi_2(\mathbb{R}P^2)$  provides a local integer coefficient system  $f_{\#}^{\varrho}$  over  $K_{\mathcal{P}}$ . We prove that if the twisted integer cohomology group  $H^2(K_{\mathcal{P}};_{f_{\#}^{\varrho}}\mathbb{Z}) = 0$ , then f is homotopic to a non-surjective map. As an intermediary step for the proof, we show that if  $H^2(K_{\mathcal{P}};_{\beta}\mathbb{Z}) = 0$  for some local integer coefficient system  $\beta$  over  $K_{\mathcal{P}}$ , then  $K_{\mathcal{P}}$  is aspherical.

## 1. Introduction

The existence of strong surjections from a finite and connected n-dimensional CW complex K (a n-complex, to shorten) into a closed n-manifold Y has been investigated for at least a decade, specially from the viewpoint of the topological root theory.

For a strong surjection from K into Y we mean a (continuous) map  $f: K \to Y$  whose free homotopy class  $[f] \in [K; Y]$  has just surjective maps. In this case, we say also that f is strongly surjective. In the context of topological root theory, a map  $f: K \to Y$  which is not strongly surjective is said to be root free.

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