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## $L_2$ -THEORY FOR TWO INCOMPRESSIBLE FLUIDS SEPARATED BY A FREE INTERFACE

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Dedicated to the memory of Professor Marek Burnat

ABSTRACT. The paper is devoted to the problem of non-stationary motion of two viscous incompressible fluids separated by a free surface and contained in a bounded vessel. It is assumed that the fluids are subject to mass forces and capillary forces at the interface. We prove the stability of a rest state under the assumption that initial velocities are small, a free interface is close to a sphere at an initial instant of time, and mass forces decay as  $t \to \infty$ .

## 1. Introduction

The paper deals with unsteady motion of a two-phase fluid in a container. Both phases are assumed to be viscous and incompressible; they are immiscible and separated by an unknown closed interface on which the surface tension is taken into account. The motion of a drop in a liquid medium is governed by the Navier–Stokes system including mass forces, initial and boundary conditions and, in addition, by the initial configuration of the drop.

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