

EXISTENCE OF SOLUTIONS FOR A CLASS OF DEGENERATE ELLIPTIC EQUATIONS IN $P(X)$ -SOBOLEV SPACES

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ABSTRACT. We study the Dirichlet problem for degenerate elliptic equations of the form

$$-\operatorname{div} a(x, u, \nabla u) + H(x, u, \nabla u) = f \quad \text{in } \Omega,$$

where $a(x, u, \nabla u)$ is allowed to degenerate with respect to the unknown u , and $H(x, u, \nabla u)$ is a nonlinear term without sign condition. Under suitable conditions on a and H , we prove the existence of bounded and unbounded solution for a datum $f \in L^m$, with $1 \leq m \leq \infty$.

1. Introduction

Let Ω be a bounded subset of \mathbb{R}^N , $N \geq 2$. In [10], the authors have studied the quasi-linear elliptic problem

$$A(u) + H(x, u, \nabla u) = f \quad \text{in } \Omega,$$

where $Au = -\operatorname{div}((a(x, u)\nabla u))$ is a Leray Lions operator from $H_0^1(\Omega)$, the Carathéodory function H satisfies the growth conditions and no sign condition is posed (i.e. $H(x, s, \xi)s \geq 0$), the data f belongs to $L^m(\Omega)$. They showed the existence of weak solutions if $m > N/2$, and existence of entropy solutions if

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