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DYNAMICS ON SENSITIVE AND EQUICONTINUOUS FUNCTIONS

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ABSTRACT. The notions of sensitive and equicontinuous functions under semigroup action are introduced and intensively studied. We show that a transitive system is sensitive if and only if it has a sensitive pair if and only if it has a sensitive function. While there exists a minimal non-weakly mixing system such that every non-constant continuous function is sensitive, and a topological dynamical system is weakly mixing if and only if it is sensitive consistently with respect to (at least) any two non-constant continuous functions. We also get a dichotomy result for minimal systems — every continuous function is either sensitive or equicontinuous.

1. Introduction

By a topological dynamical system (t.d.s. for short) we mean a pair (X,T) where X is a compact metric space with metric d and $T: X \to X$ is a continuous map. The collection of all continuous real-valued functions on a given t.d.s. (X,T) is denoted by $C(X,\mathbb{R})$.

In [7], Glasner and Weiss first discovered the link between ℓ_1 -structure via coordinate density for elements of $C(X,\mathbb{R})$ and the topological entropy of (X,T). Later Kerr and Li [8], [9] completely characterized this connection. In particular,

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