

## EXISTENCE OF SOLUTIONS FOR NONLINEAR $p$ -LAPLACIAN DIFFERENCE EQUATIONS

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**ABSTRACT.** The aim of this paper is the study of existence of solutions for nonlinear  $2n^{\text{th}}$ -order difference equations involving  $p$ -Laplacian. In the first part, the existence of a nontrivial homoclinic solution for a discrete  $p$ -Laplacian problem is proved. The proof is based on the mountain-pass theorem of Brezis and Nirenberg. Then, we study the existence of multiple solutions for a discrete  $p$ -Laplacian boundary value problem. In this case the proof is based on the three critical points theorem of Averna and Bonanno.

### 1. Introduction

Consider the fourth-order  $p$ -Laplacian difference equation

$$(1.1) \quad \Delta^2(\varphi_{p_2}(\Delta^2 u(k-2))) - a\Delta(\varphi_{p_1}(\Delta u(k-1))) \\ + V(k) \varphi_q(u(k)) = \lambda f(k, u(k))$$

where  $p > 1$ ,  $\varphi_p(t) = |t|^{p-2}t$ ,  $V: \mathbb{Z} \rightarrow \mathbb{R}$  is a  $T$ -periodic positive function for  $T$  a fixed integer and  $f: \mathbb{Z} \times \mathbb{R} \rightarrow \mathbb{R}$  is a given function with growth conditions.

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The above equation is a discretization of a fourth order  $p$ -Laplacian equation studied by authors in [18], where the existence of solution for a periodic problem involving  $p$ -Laplacian differential equation is considered. The partial cases where  $p_1 = p_2 = 2$  are known as stationary extended Fisher–Kolmogorov equation (see Peletier and Troy [17], [19] and references therein).

The theory of nonlinear difference equations is widely used in the study of discrete models in different fields of science. Recently, the problems for difference equations are treated by topological and variational methods. Topological methods for higher order difference equations using Green’s functions and fixed point theorems are used in [2], [3]. The variational methods coupled with critical point theory have been extensively applied to the solvability of problems for difference equations during the last decade. We refer the reader to [1], [12], [20] and references therein. A survey on applications of critical point theory to existence results for difference equations is given in [7]. Periodic and homoclinic orbits for  $2n^{\text{th}}$  order difference equations are studied in [8] using linking theorem and in [9] by mountain-pass and symmetric mountain-pass theorems.

This paper is divided in two parts. The first part is based on the mountain-pass theorem of Brezis and Nirenberg [5]. Following the steps of [6], we obtain the existence of a nontrivial homoclinic solution of equation (1.1), i.e. a nonzero solution  $u$ , such that  $\lim_{|k| \rightarrow +\infty} |u(k)| = 0$ .

In the second part, we obtain the existence of at least three solutions for the difference equation with  $p_1 = p_2 = q$  and the Dirichlet boundary conditions, by generalizing a result given in [10] to the problem

$$\begin{aligned} -\Delta(\varphi_p(\Delta u(k-1))) &= \lambda f(k, u(k)), \quad k \in [1, T], \\ u(0) &= u(T+1) = 0. \end{aligned}$$

Such a result is obtained by applying [10, Theorem 2.1], which is a modification of the theorem of Averna and Bonanno (see [4]), to our boundary value problem.

In both cases, we show how our result should be modified for higher order problems.

The study of  $p$ -Laplacian difference equations has been developed in the literature. In addition to the previously mentioned [6], [10], we refer to [13], where the following problem is studied:

$$\begin{aligned} \Delta(\varphi_p(\Delta u(k-1))) + a(t) f(k, u(k)) &= 0, \quad k \in [1, T+1], \\ \Delta u(0) &= u(T+2) = 0, \end{aligned}$$

where  $a(t)$  is a positive function. Moreover, in [21], the existence of three positive solutions of this problem is studied.