Topological Methods in Nonlinear Analysis Volume 50, No. 2, 2017, 489–512 DOI: 10.12775/TMNA.2017.015

© 2017 Juliusz Schauder Centre for Nonlinear Studies Nicolaus Copernicus University

ON SOME APPLICATIONS OF CONVOLUTION TO LINEAR DIFFERENTIAL EQUATIONS WITH LEVITAN ALMOST PERIODIC COEFFICIENTS

Adam Nawrocki

ABSTRACT. We investigate some properties of Levitan almost periodic functions with particular emphasis on their behavior under convolution. These considerations allow us to establish the main result concerning Levitan almost periodic solutions to linear differential equations of the first order. In particular, we state a condition, which guarantees that a special linear equation possesses a Levitan almost periodic solution. We also compare the class of Levitan almost periodic functions and the class of almost periodic functions with respect to the Lebesgue measure, and simultaneously, give an answer to the open question posed by Basit and Günzler in the paper [2].

1. Introduction

In the years 1924–1926, Bohr introduced the class of uniformly almost periodic functions (see [4]–[6]). The basic notion of almost periodicity was extended in many directions. This led to creation of many classes of almost periodic functions. Among many classes of almost periodic functions an important role play the classes of Stepanov, Weyl and Besicovitch almost periodic functions

²⁰¹⁰ Mathematics Subject Classification. Primary: 42A75, 42A85; Secondary: 34A30. Key words and phrases. Almost periodic function in view of the Lebesgue measure; convolution; linear differential equation; Levitan almost periodic function; N-almost periodic function; Levitan almost periodic coefficients.

The author thanks Professor Dariusz Bugajewski for his valuable remarks and comments. The author thanks the anonymous referee for his/her suggestion to consider the complex case of equation (5.1).