Quarterly Journal
OeconomiA
copernicana

2014 Volume 5 Issue 3, September

p-ISSN 2083-1277, e-ISSN 2353-1827
www.oeconomia.pl

Jurkowska B. (2014), *The Federal States of Germany – Analysis and Measurement of Development using Taxonomic Methods,* „Oeconomia Copernicana", Volume 5, Issue 3, pp. 49-73, DOI: http://dx.doi.org/10.12775/OeC.2014.019

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The Federal States of Germany – Analysis and Measurement of Development Using Taxonomic Methods

JEL Classification: *C38*

Keywords: *taxonomic methods; federal states of Germany; regional development*

Abstract: *This article presents an analysis of the socio-economic development of the 16 federal states of Germany as compared to the whole country. The main goals of the analysis are to measure the development with the use of selected taxonomic methods, to examine the similarities and differences between the states inasmuch as that development is concerned, as well as to illustrate the distance existing between the new eastern states (Brandenburg, Mecklenburg-Vorpommern, Saxony, Saxony-Anhalt, and Thuringia) and the remaining states of Germany.*

The analysis is preceded by an illustration of the present socio-economic situation of the German states. Germany is characterized by internal diversity as regards the socio-economic development, and the policy of supporting the East German economy has failed to reach its goals. An unfavourable demographic situation is a factor that effectively inhibits the development of the new states. A falling birth rate, an increasing population beyond retirement age, as well as great numbers of

Date of submission: March 18, 2014; date of acceptance: May 26, 2014

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people emigrating to West Germany all contribute to the depopulation of the eastern states. The taxonomic analysis of the level of socio-economic development of Germany has provided information about the diversity of that development level, but it has also made it possible to determine and set the direction of development for particular states.

Introduction

The border between the German Democratic Republic (GDR) and the Federal Republic of Germany (FRG) was opened on 9 November 1989. The fall of the Berlin Wall paved the way for the process of the reunification of Germany. As a result of the disparity in the socio-economic development between the old and the new federal states, providing support for the new states became the basis for the internal economic policy of Germany. But although the financial support allowed the East German economy to increase its competitiveness, the effectiveness of the process was highly limited[1]. An unfavourable demographic situation and a lasting trend for a considerable level of emigration constitute some of the major factors inhibiting the development of the East German states.

The aim of this article is to analyse the socio-economic development of the federal states of Germany and to draw conclusions about German regional policy. This is achieved by the use of taxonomic methods and the results of collective research conducted by the Cologne Institute for Economic Research (Institut der deutschen Wirtschaft Köln). Additionally, the analysis is based on official German statistical yearbooks, as well as on other documents and materials retrieved from the websites of particular German ministries.

The location and administrative division of Germany

Germany is located between the North Sea and the Baltic Sea in the north, the Alps in the south, the Rhine in the west, and the Oder in the east. The Federal Republic of Germany comprises, following the reunification of 1990, 16 federal (constituent) states – partly sovereign countries. The "new federal states" include: Brandenburg, Mecklenburg-Vorpommern, Saxony,

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[1] The "Solidarity Pact" came into force in 1993 and was intended to aid the development of East Germany by providing the new states with 94.5 billion euro by the end of 2004. In 2005, the Pact was extended to last until 2019 as the "Solidarity Pact II." The East German states are to receive further 156 bn euro within that period of time; http://www.spiegel.de (28.11.2012).

Saxony-Anhalt, and Thuringia. They were restored by the State Restructuring Act passed on 22 July 1990. The Act transformed the previously centralist GDR into a federal state. Also East Berlin was granted authority associated with a federal state[2]. Following the reunification, East Berlin became part of the federal state of Berlin, which had enjoyed a special status in the Federal Republic of Germany.

Figure 1. Geographical and geopolitical location of Germany

Source: own elaboration based on: Statistisches Jahrbuch 2011 für die Bundesrepublik… (2011, p. 19).

[2] Verfassungsgesetz zur Bildung von Ländern in der Deutschen Demokratischen Republik - Ländereinführungsgesetz vom 22. Juli 1990, (GBl. DDR 1990 I S. 955), art. 1.

Methods

The study of the level of socio-economic development of the federal states of Germany was conducted by the use of two taxonomic methods: the Hellwig's method and the non-model method. Both methods have already been extensively described in the literature on the subject and, therefore, only their basic assumptions are presented in this paper.

The classification of socio-economic objects by the use of taxonomic methods demands that features be determined that will describe the analysed objects in detail – those are the so-called diagnostic features (variables). The selection of diagnostic features is a particularly important and responsible process for it directly influences the final results of the study. It is crucial the diagnostic variables used in the study meet the requirements of relevance, normativity, and explicitness (Narkiewicz, 1996, p. 76.)[3].

The set of output data has been assembled to form the so-called observation matrix (Wypych, 1980):

$$X = \begin{bmatrix} x_{1I} & x_{12} & \cdots & x_{1m} \\ x_{21} & x_{22} & \cdots & x_{2m} \\ \cdots & \cdots & \cdots & \cdots \\ x_{n1} & x_{n2} & \cdots & x_{nm} \end{bmatrix}, \quad (1)$$

where:
m – number of diagnostic variables (j = 1, 2, ..., m),
n – number of spatial units (i = 1, 2, ..., n),
x_{ij} – value of the *j*th diagnostic variable in the *i*th spatial unit.

[3] The requirement of relevance demands that variables representing the most significant components of the standard of living be used in the study. The requirement of normativity denotes measures having either positive or negative influence on the analysed phenomenon. The requirement of explicitness demands that the study uses variables which explicitly specify the relations between a phenomenon represented by a given measure and other phenomena.

Spatial differentiation has been determined for each initial variable, with the variation coefficient being the main criterion. The coefficient has been calculated according to the formula:

$$v_j = \frac{S_j}{\overline{x_j}}, \qquad (j = 1,2, ..., m), \qquad (2)$$

where:

$\overline{x_j}$ - arithmetic mean of variable X_j $\overline{x_j} = \frac{1}{n}\sum_{i=1}^{n} x_{ij}$,

S_j – standard deviation of variable X_j: $S_j = \left[\frac{1}{n}\sum_{i=1}^{n}\left(x_{ij} - \overline{x_j}\right)^2\right]^{1/2}$

Variables for which $v_j \leq v^*$ are deleted from the set of potential variables (v^* is the accepted critical value of the variation coefficient). Those variables do not have sufficient discriminant ability.

The potential diagnostic features may be related, in which case they are the carriers of similar information. This, in turn, entails the necessity of determining their similarity[4]. It is usually the linear correlation coefficients between the potential exogenous variables X_j, X_k, (j, k=1, 2, ..., m) that serve as the similarity measures. They are calculated according to the formula (Nowak, 2002; Kosiedowski *et al*., 1984):

[4] "The features of the initial list can be similar to one another due to a high level of correlation and can, therefore, form the so-called clusters. Clusters are the subsets of a set of features whose minimum similarity between the features is greater than or equal to r*. Clusters contain one so-called central feature and a number of so-called satellite features. A feature is called a satellite feature of a given central feature when the similarity between them is greater than or equal to r*. Features group into clusters if they comprise a central feature and at least one satellite feature – these are called the system features. The features which do not belong to clusters are called the isolated features. Both central and isolated features are regarded as diagnostic features" (Nowak, 1990, pp. 28-29).

$$r_{jk} = \frac{\sum_{i=1}^{n}\left(x_{ij} - \bar{x}_j\right)\left(x_{ik} - \bar{x}_k\right)}{\sqrt{\sum_{i=1}^{n}\left(x_{ij} - \bar{x}_j\right)^2 \sum_{i=1}^{n}\left(x_{ik} - \bar{x}_k\right)^2}}, \qquad (j, k = 1, 2, ..., m). \tag{3}$$

These coefficients form a correlation matrix:

$$R = \begin{bmatrix} 1 & r_{12} & \dots & r_{1m} \\ r_{21} & 1 & \dots & r_{2m} \\ \dots & \dots & \dots & \dots \\ r_{m1} & r_{m2} & \dots & 1 \end{bmatrix}. \tag{4}$$

Both the Hellwig's method and the non-model method were used in the analysis of the level of socio-economic development of the states. The former is regarded as a classic method of calculating the synthetic measure. It is recognized as a model method, i.e. a method where the significance of a phenomenon is related to a certain model (Piotrowska-Trybull, 2004, p. 431).

A set of diagnostic features can comprise values having different directions of influence on the analysed phenomenon. Two groups of variables can therefore be distinguished: stimulants and destimulants. All destimulants need to be transformed into stimulants so that higher values of the transformed features could indicate greater significance of the corresponding aspect of the phenomenon. In this work, destimulants have been transformed into stimulants by calculating the inverse of each value according to the formula:

$$x_{ij}' = \frac{1}{x_{ij}}. \tag{5}$$

Furthermore, as variables forming the observation matrix are not homogeneous, it is also necessary they be made comparable by normalization (Zeliaś, 2002, p. 31)[5]. This is calculated according to the formula (Zeliaś, 2002, p. 32):

$$z_{ij} = \frac{x_{ij} - \bar{x}_j}{s_j} ; \qquad (6)$$

where:
z_{ij} – normalized value of the *j* variable for the *i* object,
x_{ij} – value of the *j* variable for the *i* object,
$\bar{x}_j$ – arithmetic mean of feature X_j,
s_j – standard deviation of feature X_j.

Such normalized data are then used for the construction of the model. There are many methods of constructing the model. Following Zeliaś (2000, p. 91), this paper defines the model as an abstract object characterized by the maximum values of each normalized variable:

$$z_{01}, z_{02}, \ldots, z_{0m} ,$$

where for:

– stimulants $$z_{0j} = \max_i \{z_{ij}\}, \qquad (7)$$

– destimulants $$z_{0j} = \min_i \{z_{ij}\}. \qquad (8)$$

[5] It is crucial in taxonomic study to achieve the comparability of the final diagnostic variables. This entails, among others, the necessity to strip variables of their natural units in which the diagnostic features are expressed as well as to normalize variables to the state of comparability which, in turn, requires their range of variability to be smoothed.

Next, the distance from such defined model is calculated for each analysed object. This paper uses the following method of calculating distance from the model (Piotrowska-Trybull, 2004, p. 431):

$$c_{io} = \left[\sum_{j=1}^{m} v_j \left(z_{ij} - z_{oj} \right)^2 \right]^{1/2} , \qquad (i = 1, 2, ..., n). \qquad (9)$$

where:
c_{io} – generalized Euclidean distance of the *i* object from the model,
v_j – weight assigned to the *j* variable from the set of diagnostic variables,
z_{ij} – normalized value of the *j* variable for the *i* object,
z_{oj} – normalized value of the *j* variable for the *i* model.

Sets of diagnostic features usually comprise features having different information resources inasmuch as the aim of the study is concerned. Diagnostic features were assigned weights in order to take account of different significance of particular features (Zeliaś, 2000, pp. 45-50; Nowak, 1990, pp. 33-35). The development of the *i*th spatial unit is measured by a synthetic indicator which the literature on the subject defines as a taxonomic indicator of development (Hellwig, 1968, p. 307). These indicators are calculated according to the formula (Wypych, 1980, p. 23):

$$d_i = 1 - \frac{c_{io}}{c_o} , \qquad (10)$$

where:

$$c_o = \overline{c_o} + 2S_o ,$$

$$\overline{c_o} = \frac{1}{n} \sum_{i=1}^{n} c_{io} ,$$

$$S_0 = \left[\frac{1}{n} \sum_{i=1}^{n} \left(c_{i0} - \overline{c_0} \right)^2 \right]^{1/2} .$$

d_i – value of the synthetic indicator for the *i*th object,
c_{io} – generalized Euclidean distance of the *i*th object from the model,
c_o – normalizing factor,

$\overline{c}_0$ – arithmetic mean of the generalized Euclidean distance of the objects from the model,
s_0– standard deviation of the generalized Euclidean distance of the objects from the model.

The quantity d_i is interpreted as follows: the higher the value, the higher the level of development of a given object. The value of the synthetic indicator is within the range [0,1].

The second method used in this study is the non-model method, which uses normalized values of diagnostic features given by the formula (6). In the non-model method, the value of the synthetic indicator of development is determined by calculating the weighted mean of the normalized values of diagnostic variables:

$$d_{i*} = \sum_{j=1}^{m} v_j z_{ij} , \qquad (11)$$

where:
d_{i*} – value of synthetic indicator for the *i* object,
v_j – weight assigned to the *j* variable,
z_{ij} – normalized value of the *j* variable for the *i* object.

The indicator is interpreted as follows: the higher the value of d_i indicator, the higher the level of development of a given object.

Synthetic indicators enable a linear classification of objects. It is conducted on the basis of distinguishing groups of elements which are similar to one another with regard to the synthetic indicator of development. For the purposes of this study, classifications were carried out by the use of two methods: the standard deviation method and the three-means method (Nowak, 1990, p. 93).

The standard deviation method uses two parameters of synthetic indicators (z_i): arithmetic mean ($\overline{x}$) and standard deviation (s_z). In this study, the method of object classification divided the objects into 4 classes:

– Class 1 (high level of development): $z_i \geq \overline{x} + s_z$
– Class 2 (medium-high level of development): $\overline{x} + s_z > z_i \geq \overline{x}$;
– Class 3 (medium-low level of development): $\overline{x} > z_i \geq \overline{x} - s_z$;
– Class 4 (low level of development): $z_i < \overline{x} - s_z$.

The second of the methods used in the study, the three-means method, is based on arithmetic mean $\bar{x}$ calculated for each set of indicators $z_1, z_2, \ldots, z_N$. A given set is then divided into two subsets. The elements of the first subset (N1) fulfil the inequality $z_i \leq \bar{x}$, while the elements of the second subset (N_2) – $z_i > \bar{x}$. Arithmetic means of such calculated subsets are then denoted by $\bar{z}_1$ and $\bar{z}_2$. The final classification of objects is as follows:

– Class 1 (high level of development): $z_i > \bar{z}_2$;
– Class 2 (medium-high level of development): $\bar{x} < z_i \geq \bar{z}_2$;
– Class 3 (medium-low level of development): $\bar{z}_1 < z_i \geq \bar{x}$;
– Class 4 (low level of development): $z_i \geq \bar{z}_1$.

In order to evaluate the correctness of the classification results, it is necessary to measure the degree of similarity between units belonging to the same typological groups as well as the degree of variation of objects belonging to different subsets (Nowak, 1990, p. 190). For that purpose the following function is used to calculate the measures of intragroup similarity and intergroup similarity (Piotrowska-Trybull, 2004, p. 441):

$$J_S = \frac{\frac{1}{6}\sum_{p=1}^{3}\sum_{q=p+1}^{4} D_{pq}^{S}}{\frac{1}{4}\sum_{p=1}^{4} D_{pp}^{S}}, \tag{14}$$

where:
J_S – value of classification quality function for the *s*th division,
D_{pq}^{S} – average intergroup distance for the *p*th and *q*th groups in the *s*th division,
D_{pp}^{S} – average intragroup distance for the *p*th group in the *s*th division.

The division for which function J reaches the maximum is considered as the best.

The analysis of the level of socio-economic development of the federal states of Germany was conducted by the use of 32 initial variables (see Table 1). Their selection was dictated by the requirements of relevance, normativity, and explicitness that apply to the process of feature selection. The data used in the analysis were retrieved from the official website of the German Federal Statistical Office.

Table 1. Set of initial variables

No.	Variable	Standard deviation	Variation coefficient
1.	GDP per capita, in EUR	8627.016	0.283
2.	GDP per person employed, in EUR	9432.112	0.157
3.	GDP per hour worked	7.419	0.177
4.	Value added gross per person employed, in EUR	8423.356	0.157
5.	Gross Domestic Expenditure on R&D, in %	0.949	0.391
6.	Number of lawfully convicted adults per 1000 inhabitants	1.750	0.201
7.	Unemployment rate among the population aged 15-24 years old, in %	3.121	0.384
8.	Number of the unemployed per 1000 inhabitants	15.710	0.363
9.	Unemployment rate, in %	2.819	0.355
10.	Share of unemployed in the the population aged 15-65 years old, in %	2.826	0.343
11.	Employment rate, in %	3.098	0.042
12.	Activity rate, in %	2.773	0.039
13.	Persons employed per 1000 inhabitants	19.561	0.040
14.	Share of employed in industry and construction in total persons employed, in %	5.383	0.206
15.	Share of employed in services in the total number of the employed, in %	2.272	0.088
16.	Density of population, in people per km^2	1093.082	1.633
17.	Share of population aged 25 and under in total population, in %	2.005	0.082
18.	Share of population aged 65 and over in total population, in %	1.723	0.081
19.	Number of births per 1000 inhabitants	0.706	0.088
20.	Number of deaths per 1000 inhabitants	1.187	0.109
21.	Infant deaths per 1000 live births	1.039	0.288
22.	Population growth per 1000 inhabitants	1.783	0.632
23.	Internal migration balance per 1000 inhabitants	2.438	27.278
24.	Number of students per 10,000 inhabitants	80.674	0.304
25.	Usable floor space of a dwelling per person, in m^2	3.583	0.0847
26.	Number of dwellings per 1000 inhabitants	32.921	0.064

Table 1 Continued

No.	Variable	Standard deviation	Variation coefficient
27.	Average number of persons per a dwelling	0.123	0.063
28.	Number of newly built dwellings per 1000 inhabitants	0.746	0.373
29.	Average number of people in households	0.126	0.064
30.	Number of aborted pregnancies per 10,000 inhabitants	8.248	0.490
31.	Number of hospital beds per 100,000 inhabitants	66.511	0.105
32.	Number of doctors (including stomatologists) per 10,000 inhabitants	20.629	0.148

Source: own elaboration.

The variables whose variation coefficient did not exceed the set level of 10 per cent were deleted from the set. The features that were highly correlated with others were deleted in the next stage of the elimination process. This was achieved by the use of the already-described correlation matrix (according to the formula 3). The so-called satellite features of central features were deleted in the course of the analysis of the correlation matrix, with the critical value of the correlation coefficient being set at the level of 0.7.

Five features that carried information also through other variables were distinguished among diagnostic variables, i.e. the so-called satellite features. Isolated features are the carriers of specific information and are not significantly correlated with any other variable. The analysis distinguished five isolated features (see Table 2).

Table 2. Central, satellite, and isolated features

<table>
<tr><th colspan="2">Diagnostic features</th></tr>
<tr><th>Central features</th><th>Satellite features</th></tr>
<tr><td>GDP per hour worked</td><td>– GDP per capita, in EUR;
– GDP per person employed, in EUR;
– value added gross per person employed, in EUR.</td></tr>
<tr><td>Number of aborted pregnancies per 10,000 inhabitants</td><td>– number of lawfully convicted adults per 1000 inhabitants;
– number of students per 10,000 inhabitants.</td></tr>
</table>

Table 2 Continued

Diagnostic features	
Central features	**Satellite features**
Number of deaths per 1000 inhabitants	– population growth per 1000 inhabitants; – internal migration balance per 1000 inhabitants.
Unemployment rate, in %	– number of the unemployed per 1000 inhabitants; – unemployment rate among 15-24 year olds; – share of unemployed 15-65 year olds, in %.
Density of population in persons per km^2	– share of employed in industry and construction in the total number of the employed, in %.
Isolated features	
– infant deaths per 1000 live births; – number of newly built dwellings per 1000 inhabitants; – number of hospital beds per 100,000 inhabitants; – Number of doctors (including stomatologists) per 10,000 inhabitants; – Gross Domestic Expenditure on R&D, in per cent.	

Source: own elaboration.

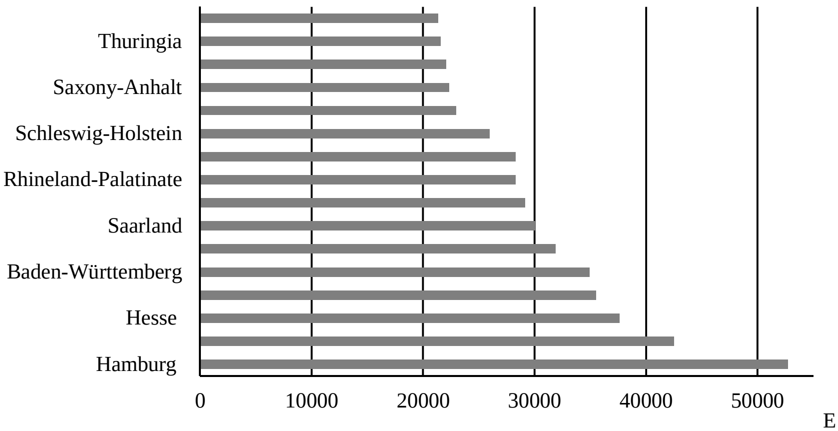
Eventually, 10 features were selected for the study:

- GDP per hour worked;
- Gross Domestic Expenditure on R&D, in per cent;
- unemployment rate, in per cent;
- density of population in people per km^2;
- number of deaths per 1000 inhabitants;
- infant deaths per 1000 live births;
- number of newly built dwellings per 1000 inhabitants;
- number of hospital beds per 100,000 inhabitants;
- number of doctors (including stomatologists) per 10,000 inhabitants;
- number of aborted pregnancies per 10,000 inhabitants.

Of the above-mentioned features, four (number of deaths per 1000 inhabitants, infant deaths per 1000 live births, unemployment rate in per cent, and number of aborted pregnancies per 10,000 inhabitants) were defined as destimulants, which were subsequently transformed into stimulants according to the formula (5).

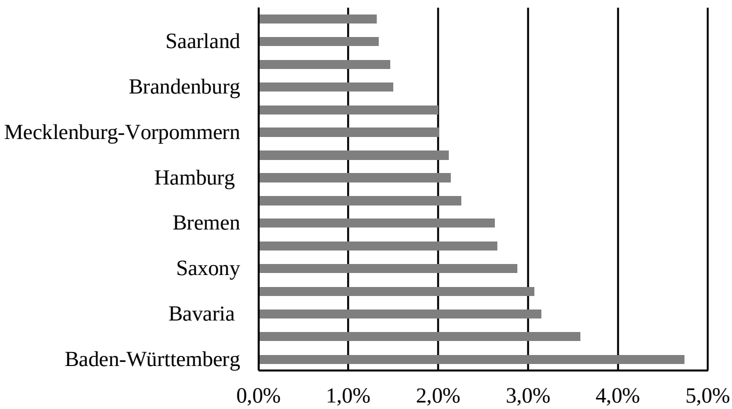
The values of all of the diagnostic features for the federal states of Germany were correct as at the end of 2011. Two selected variables and their values for particular federal states are additionally presented in Figure 1 and 2.

Figure 1. GDP per capita, in EUR, in 2011



Source: own elaboration based on: Wirtschaftsdaten Neue…(2012, p. 3).

Figure 2. Gross Domestic Expenditure on R&D, in per cent, in 2011



Source: own elaboration based on: Statistisches Bundesamt: https://www.destatis.de/DE/Startseite.html (20.10.2012).

In the following stage of the research, the diagnostic features were assigned weights which – according to the author of this study – reflected the relevance of the influence the diagnostic features exerted on the development of the federal states (see Table 3).

Table 3. Set of diagnostic variables and their weights

Variable	Variable weight
GDP per hour worked	0.15
Gross Domestic Expenditure on R&D, in %	0.15
unemployment rate, in %	0.15
number of aborted pregnancies per 10,000 inhabitants	0.1
number of deaths per 1000 inhabitants	0.075
density of population, in people per km^2	0.075
infant deaths per 1000 live births	0.075
number of newly built dwellings per 1000 inhabitants	0.075
number of hospital beds per 100,000 inhabitants	0.075
number of doctors (including stomatologists) per 10,000 inhabitants	0.075

Source: own elaboration.

Results and Interpretation

The analysis of the states' development performed by the use of the Hellwig's method used normalized variables. The coordinates of the model of development were as follows:

$$Z_{o1} = 3{,}031;\ Z_{o2} = 1{,}707;\ Z_{o3} = 1{,}761;\ Z_{o4} = 2{,}433;\ Z_{o5} = 1{,}880;$$
$$Z_{o6} = 1{,}417;\ Z_{o7}\ 2{,}117;\ Z_{o8} = 2{,}062;\ Z_{o9} = 2{,}515;\ Z_{o10} = 1{,}984.$$

The results of normalization are shown in Table 4.

Table 4. Values of normalized variables for the federal states of Germany

States	Z_{i1}	Z_{i2}	Z_{i3}	Z_{i4}	Z_{i5}	Z_{i6}	Z_{i7}	Z_{i8}	Z_{i9}	Z_{i10}
Baden-Württemberg	-0.345	1.707	0.077	1.717	0.806	1.167	-1.502	-0.091	2.515	0.686
Bavaria	-0.461	0.839	0.198	2.433	1.880	1.418	-0.402	0.110	0.784	0.553
Berlin	3.031	1.707	0.198	-1.056	-0.940	-1.410	-0.865	-0.491	1.253	-0.167
Brandenburg	-0.548	-0.383	-0.342	-0.712	0.806	-0.172	-0.315	-1.793	-1.012	-1.040
Bremen	0.853	-0.383	-1.830	-0.957	-1.208	-1.825	2.117	1.862	0.218	0.963
Lower Saxony	-0.483	-0.066	-1.240	0.259	0.403	1.331	-1.429	-0.591	0.251	0.137
Hamburg	1.608	1.254	-0.601	-0.145	0.134	-1.148	0.597	1.161	-0.315	1.984
Hesse	-0.357	0.740	-0.342	0.545	-0.134	0.016	-0.590	-0.591	0.697	1.159
Mecklenburg-Vorpommern	-0.563	-0.533	1.316	-0.930	1.074	-0.575	0.018	-0.241	-0.457	-1.326
North Rhine-Westphalia	-0.138	0.101	-0.824	-0.292	-0.134	0.429	0.713	0.360	-0.467	0.651
Rhineland-Palatinate	-0.438	-0.148	-0.037	0.898	0.403	1.331	0.033	0.410	-0.337	0.167
Saarland	-0.256	-1.084	-0.753	0.096	-0.940	-0.352	0.062	2.062	-1.186	0.129
Saxony	-0.419	-1.084	1.761	-0.638	-0.940	-0.024	0.105	-0.842	0.491	-1.253
Saxony-Anhalt	-0.522	-1.567	0.464	-0.944	-1.477	-0.441	1.060	-0.391	-1.044	-1.043
Schleswig-Holstein	-0.462	-0.148	0.198	0.096	1.208	0.653	-0.981	-0.692	-1.208	-0.198
Thuringia	-0.500	-0.953	1.761	-0.371	-0.940	-0.397	1.379	-0.241	-0.184	-1.401

Source: own elaboration.

Synthetic indicators of development d_i were calculated on the basis of the distance between each analysed state and the model (the closer the value of the indicator was to 1, the higher the level of development of a given unit). Next, each state was assigned a rank which determined its position in relation to the remaining analysed objects. The results of the calculations are shown in Table 5.

Table 5. Synthetic indicators of development of the federal states of Germany according to the Hellwig's method

States	Distance from the model	Synthetic indicator of development	Position
Bavaria	1.634	0.476	1
Baden-Württemberg	1.679	0.462	2
Hamburg	1.976	0.367	3
Hesse	1.993	0.361	4
Rhineland-Palatinate	2.041	0.345	5
North Rhine-Westphalia	2.220	0.288	6
Lower Saxony	2.337	0.251	7
Berlin	2.343	0.249	8
Bremen	2.468	0.209	9
Schleswig-Holstein	2.519	0.192	10
Saarland	2.546	0.184	11
Saxony	2.610	0.163	12
Thuringia	2.611	0.163	13
Mecklenburg-Vorpommern	2.683	0.140	14
Brandenburg	2.870	0.080	15
Saxony-Anhalt	2.930	0.061	16

Source: own elaboration.

The Table shows a markedly inferior position of the eastern states (with the exception of Berlin) in comparison with the remaining federal states of Germany. While Bavaria ranked first with regard to the level of development in 2011, Saxony-Anhalt ranked last. The highest values of synthetic indicators of development were noted in Bavaria and Baden-Württemberg, whereas the lowest – in Mecklenburg-Vorpommern, Brandenburg, and Saxony-Anhalt.

In the non-model method, the variables were normalized prior to the calculation of the synthetic indicators of development of each analysed state. The method of normalization was the same as the one used in the case of the Hellwig's method. Normalized values of variables were used to calculate the synthetic indicator of development for each analysed unit by calculating the weighted mean. Next, based on the received value d_{i*}, the position of the states was determined with respect to the level of development (the higher the value of the indicator d_{i*}, the higher the level of development of a given object). The results are shown in Table 6.

Table 6. Results of the analysis of the development of the federal states of Germany according to the non-model method

States	Synthetic indicator of development d_{i*}	Position
Baden-Württemberg	0.9033	1
Bavaria	0.8694	2
Hamburg	0.4252	3
Hesse	0.2662	4
Rhineland-Palatinate	0.2589	5
Berlin	0.0614	6
North Rhine-Westphalia	0.0324	7
Lower Saxony	-0.0254	8
Bremen	-0.0431	9
Schleswig-Holstein	-0.1967	10
Saarland	-0.2474	11
Thuringia	-0.2954	12
Saxony	-0.3187	13
Mecklenburg-Vorpommern	-0.3842	14
Brandenburg	-0.6249	15
Saxony-Anhalt	-0.6812	16

Source: own elaboration.

As a result of the analysis performed by the use of the non-model method, Baden-Württemberg ranked first, closely followed by Bavaria, while Hamburg was ranked third. The results confirm the conclusions derived from the analysis carried out using the model method: the eastern states (with the exception of Berlin) were the least developed states in the country. Saxony-Anhalt proved to be the least developed state, with a synthetic indicator of development 1.3 times lower than the corresponding indicator of Baden-Württemberg.

The results enabled the states to be classified into four distinct groups (see Table 7).

Table 7. Classification of the federal states of Germany according to the standard deviation method and three-means method

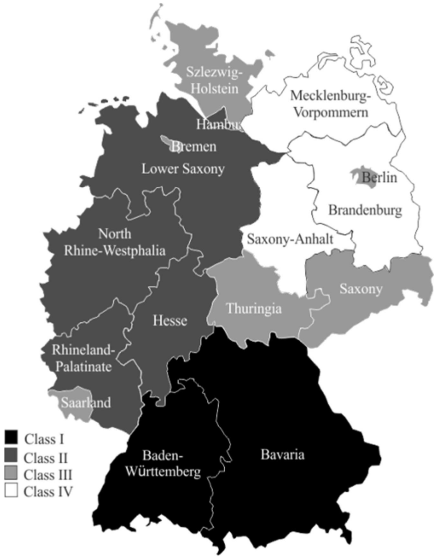
Class	Hellwig's method		Non-model method	
	standard deviation method	three-means method	standard deviation method	three-means method
I	1. Bavaria 2. Baden-Württemberg	1. Bavaria 2. Baden-Württemberg 3. Hamburg	1. Bavaria 2. Baden-Württemberg	1. Baden-Württemberg 2. Bavaria 3. Hamburg
II	3. Hamburg 4. Hesse 5. Rhineland-Palatinate 6. North Rhine-Westphalia 7. Lower Saxony	4. Hesse 5.Rhineland-Palatinate 6.North Rhine-Westphalia 7. Lower Saxony	3. Hamburg 4. Hesse 5. Rhineland-Palatinate 6. Berlin 7. North Rhine-Westphalia	4. Hesse 5. Rhineland-Palatinate 6. Berlin 7. North Rhine-Westphalia
III	8. Berlin 9. Bremen 10. Schleswig-Holstein 11. Saarland 12. Saxony 13. Thuringia	8. Berlin 9. Bremen 10. Schleswig-Holstein 11. Saarland 12. Saxony 13. Thuringia	8. Lower Saxony 9. Bremen 10. Schleswig-Holstein 11. Saarland 12. Thuringia 13. Saxony 14. Mecklenburg-Vorpommern 15. Brandenburg	8. Lower Saxony 9. Bremen 10. Schleswig-Holstein 11. Saarland 12. Thuringia
IV	14. Mecklenburg-Vorpommern 15. Brandenburg 16. Saxony-Anhalt	14. Mecklenburg-Vorpommern 15. Brandenburg 16. Saxony-Anhalt	16. Saxony-Anhalt	13. Saxony 14. Mecklenburg-Vorpommern 15. Brandenburg 16. Saxony-Anhalt

Class 1 – high level of development,
Class 2 – medium-high level of development,
Class 3 – medium-low level of development,
Class 4 – low level of development.

Source: own elaboration.

The cartographic representation of the results of the study (see Figures 2, 3, 4, 5) shows that Germany is highly diverse with respect to the level of socio-economic development of its particular states. Bavaria and Baden-Württemberg constitute the group of the most highly-ranked states in the country: both of these federal states have large and dynamically developing agglomerations.

Figure 2. Level of socio-economic development of the federal states of Germany (according to the Hellwig's method, grouped according to the standard deviation method)

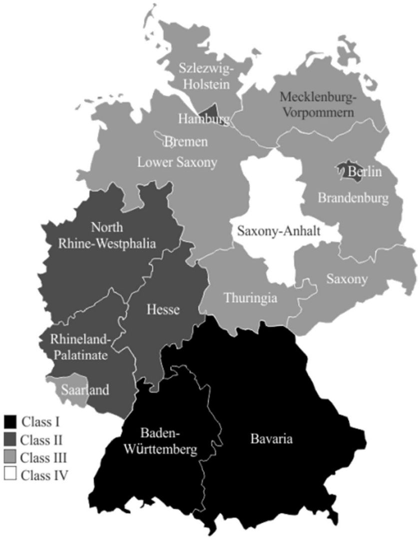


Source: own elaboration.

Figure 3. Level of socio-economic development of the federal states of Germany (according to the Hellwig's method, grouped according to the three-means method)

Source: own elaboration.

Figure 4. Level of socio-economic development of the federal states of Germany (according to the non-model method, grouped according to the standard deviation method)



Source: own elaboration.

Figure 5. Level of socio-economic development of the federal states of Germany (according to the non-model method, grouped according to the three-means method)

Source: own elaboration.

In order to make the received results comparable, the value of classification function was calculated according to the formula (14). The results were as follows:

for the Hellwig's method:

J = 1,477793 (according to the standard deviation method),

J = 1,27176 (according to the three-means method),

for the non-model method:

J = 1,4318275 (according to the standard deviation method),

J = 1,194075 (according to the three-means method).

As the classification function reached the maximum for the Hellwig's method (according to the standard deviation method), it was the results received for this method that were used for further analysis.

Discussion

A comparative analysis showed that the group representing the most socio-economically developed states comprised Bavaria and Baden-Württemberg. In relation to GDP per hour worked, the states ranked 4th (Baden-Württemberg) and 6th (Bavaria) in the country. The states were also characterized by the lowest unemployment rate and a high gross domestic expenditure on research and development. Furthermore, both states ranked low in the number of aborted pregnancies per 10,000 inhabitants and the number of deaths per 1000 inhabitants.

The second class comprised five states: Hamburg, Hesse, Rhineland-Palatinate, North Rhine-Westphalia, and Lower Saxony. The majority of relevant diagnostic variables indicated an average level of development. The states: Hesse, North Rhine-Westphalia, Rhineland-Palatinate, and Lower Saxony were characterized by a high level of labour efficiency, defined as GDP per hour worked (respectively: 2nd, 5th, 7th, and 8th), and ranked low in the number of infant deaths per 1000 live births (10th, 14th, 9th, and 15th in the country). Hamburg was the highest ranked state in the class, which was mainly owing to its high rates of labour efficiency, density of population, and the number of doctors (including stomatologists) per 10,000 inhabitants.

The most numerous class was Class III. It comprised 6 states: Berlin, Bremen, Schleswig-Holstein, Saarland, Saxony, and Thuringia. The highest ranked state in the class was Berlin, which was also ranked first in density of population. It also ranked lowest in the number of infant deaths per 1000 live births (15th in the country). Bremen ranked high in the number of hospital beds per 100,000 inhabitants (1st), as well as in gross domestic expenditure on research and development (2nd). In contrast, it was characterized by a high rate of unemployment and ranked rather low in the number of aborted pregnancies per 10,000 inhabitants. Further, Schleswig-Holstein stood out with respect to the number of newly built dwellings per 1000 inhabitants (2nd), but it was also characterized by the lowest gross domestic expenditure on research and development (16th, the last in the country). Saarland, another state representing this class, was ranked highest in the number of doctors (including stomatologists) per 10,000 inhabitants (1st), but at the same time it was ranked almost lowest in gross domestic expenditure on research and development (15th). The remaining states of Class III, Saxony and Thuringia, represented a similar level of socio-economic development. Saxony was ranked as the 6th in regard to the rate of unemployment, whereas Thuringia was ranked as the 7th in the country. Both states also ranked low in the number of infant deaths per 1000 live births.

The last class comprised three states: Mecklenburg-Vorpommern, Brandenburg, and Saxony-Anhalt. They were characterized by low rates of expenditure on research and development and labour efficiency. They also ranked the lowest in density of population: Mecklenburg-Vorpommern – 16th, Brandenburg – 15th, and Saxony-Anhalt – 14th in the country.

The assessment of the level of socio-economic development of the federal states of Germany corresponded to the assessment presented in a publication of the Cologne Institute for Economic Research, *Initiative Neue Soziale Marktwirtschaft. 10. Bundesländerranking 2012. Bundesländer im Vergleich*, which ranked Bavaria and Baden-Württemberg as the 1st and the 2nd in the country. Furthermore, the ranking indicated the East German states to be the least developed in the country: Berlin was ranked as the 16th, Saxony-Anhalt – 15th, Mecklenburg-Vorpommern – 14th, Brandenburg – 13th, Thuringia – 11th, and Saxony – 10th (Initiative Neue Soziale, 2012, p. 11).

Conclusions

The research provided information on the variation of the analysed states in respect to their socio-economic development. Of all the analysed federal states of Germany, Bavaria achieved the highest value of the synthetic indicator of development, whereas Saxony-Anhalt – the lowest. The rates of GDP and unemployment were considered as the features of the highest importance to the evaluation of the level of socio-economic development. GDP per capita is the major criterion used in the distribution of financial aid to less-developed regions within the European Union's regional policy. Of all the federal states, the one showing the highest rate of GDP was Hamburg, closely followed by Bremen. Conversely, the rate of GDP was found to be the lowest in Mecklenburg-Vorpommern, Thuringia, Brandenburg, Saxony-Anhalt, and Saxony – these states were ranked, respectively, 16th, 15th, 14th, 13th, and 12th in the country. The ranking is also similar with regard to the rate of unemployment: while the lowest rate was observed in Bavaria and Baden-Württemberg (respectively, 1st and 2nd), the highest rate was found in East Germany – in Berlin, Saxony-Anhalt, Mecklenburg-Vorpommern, Brandenburg, and in Thuringia (the states were ranked, respectively, 16th, 14th, 13th, 12th, 11th, and 10th in the country).

The difficulties encountered in the labour market are also connected with migration. It is estimated that over 4 million people migrated to the western states between 1989–2010. The volume and direction of migration are influenced by a number of factors, with the intention of taking up a new job in order to improve one's financial situation being the most frequently indicated reason for migration. The majority of the migrant population were young people, aged 21–25, a fact only further contributing to the adverse changes in the population structure and in East Germany's potential for development.

While the gap existing between the new federal states and the old ones has indeed been narrowed, the socio-economic situation of the new states is still considerably worse and will not be evened up in the near future. The disparities in the level of development between the states can only be reduced if the western part of Germany develops together with the new states, a process that cannot be achieved without introducing relevant reforms at a national level.

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