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EPISTEMIC EROTETIC SEARCH SCENARIOS

Abstract. The aim of this paper is to introduce erotetic search scenarios known from Inferential Erotetic Logic by using the framework of epistemic erotetic logic. The key notions used in this system are those of askability and epistemic erotetic implication. Scenarios are supposed to represent all rational strategies of an agent solving the problem posed by the initial question where the interaction with an external information source is seen as a series of updates of the agent's knowledge.

Keywords: questions; Inferential Erotetic Logic (IEL); Dynamic Epistemic Logic (DEL); e-scenarios, agenda

Introduction

We take ideas from A. Wiśniewski's Inferential Erotetic Logic (IEL) [18, 22] as a point of departure in this paper; however, we rely on their epistemic interpretation as proposed and discussed in detail in [13, 12]. Such an approach allows us to discuss the problem-solving and questioning agenda in the context of the agents' interaction and takes the agents' knowledge into account.

The pragmatic intuition underlying the concept of erotetic search scenarios (e-scenarios for short) is that an e-scenario

> [...] provides information about possible ways of solving the problem expressed by its principal question: it shows what additional data should be collected if needed and when they should be collected. What is important, an e-scenario provides the appropriate instruction for every possible and just-sufficient, i.e. direct, answer to a query: there are no "dead ends". [21, p. 110]

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E-scenarios have proven to be powerful logical tools for modelling cognitive goal-directed processes (cf. [20, 19, 23, 15, 16]). Erotetic search scenarios were also employed in the field of problem-solving (see [5] and Section 1 of this paper). What is more, the pragmatic account of the scenarios was used in the Turing test adequacy debate (see [6, 7, 10]). See also [14] for the connection between e-scenarios and proof theory. E-scenarios were also used as a basis for the procedure of generating cooperative responses useful for the interfaces of information systems — see [9] and [8]. The procedure presented in [9] was implemented in Prolog and is available for download.[1]

This paper is structured as follows: Similarities and differences between our approach and the already existing frameworks for agents using questioning agendas are discussed in Section 1. In Section 2 a motivational example explaining our main intuitions and types of epistemic situations that we are addressing in our framework is analysed. Section 3 introduces elementary concepts of epistemic logic with questions such as the askability of a question, answerhood conditions, and epistemic erotetic implication. Section 4 covers epistemic erotetic search scenarios. We introduce basic epistemic scenarios and present how they are enveloped into questioning agendas — epistemic erotetic search scenarios. We also discuss how the knowledge structure of an agent influences generation of epistemic erotetic search scenarios. The last section provides a summary of our ideas and possible directions of their further development, especially an extension of the proposed approach to multi-agent settings, where a group of agents is cooperatively solving a complex problem.

1. Related work

The framework presented here stems directly from the IEL. Our idea of an epistemic erotetic scenario (e-e-scenario for short) is adapted from the concept of the erotetic search scenario presented for the first time by Wiśniewski in [20]. The key difference is that our scenarios are based on the notion of the epistemic erotetic implication in contrast to the erotetic implication of Wiśniewski. As discussed in the introduction, this allows

[1] The program *COOP Q-RESPONSES* is available at the page of IntQuestPro project: `https://intquestpro.wordpress.com/software-data/`. At the same page the Prolog program *CPC e-scenarios*, that generates atomic e-scenarios for yes-no questions is also available.

us to consider the questioning agenda issue in the epistemic context. The core intuitions about the structure of search scenarios and their purpose remain the same.

A more straightforward inspiration from the IEL is [15] which shows how we may use e-scenarios for a formal modelling of the agents' hidden agendas for questioning. From this paper we take the idea that there are certain cases when an agent does not want to (or simply cannot) reveal her initial question. Moreover, examples analysed in [15] show that e-scenarios used in such contexts offer the possibility of differentiating questions posed by an interrogator to herself from the ones that an interrogated person should actually be asked. We address this issue in our framework. We also aim at clear differentiation of the factual data gathered by an agent and the inferences which she takes on this data. At this point [5] should also be mentioned where a three-valued logic is employed in order to express the lack of information involved in the problem-solving process. We exploit this idea in the approach presented in this paper. A situation when an agent encounters a lack of information after having asked the initial question is a crucial point for our basic epistemic scenario — this is a point from which an agent starts developing her questioning agenda.

Also another interesting approach to problem-solving *via* questioning, which stems from Hintikka's Interrogative Model of Inquiry framework, is worth mentioning. In [1] Genot introduces a framework for analysing questions in interaction based on Interrogative Games and game theory. The main objective is to apply these in the field of Belief Revision Theory as presented in [11]. The idea is to investigate the agent's questioning agenda and its modifications during the information-seeking process. Genot [1] presents this process in a form of a game between two agents where two types of moves are allowed: deductive ones (when a questioning agent states something) and interrogative ones (when the agent asks whether something is true). We use a similar idea in our definition of an epistemic erotetic search scenario. A full e-e-scenario is developed from the basic epistemic scenario (which is a form of the initial epistemic situation for our questioning agent) *via* expansion steps (deductive and erotetic ones). Moreover, the framework proposed in our paper allows us to model agents' agendas for questioning and grasps the idea of dependence between the knowledge of an agent and the agenda structure. What is more, thanks to the well-defined concepts of epistemic logic with questions we can extend our framework into a multi-agent setting in order to model the interaction of many agents collaborating on solving a given problem.

2. Motivation

Our main focus in this paper is to address a process of solving a problem with the use of questioning. It is often the case that we ask a question and we do not receive an answer to this question. Such a situation results in the developement of a questioning agenda based on the initial question. Let us illustrate this by the following example.

Example 1. Carol has just come to a party. She obviously wants to know who else will be there. So she asks, "Are Andrew and Barbara also coming?" She does not get an answer. She thinks that people around her either do not know or took her too literally, and if they do not know a full answer they prefer to keep silent than give at least partial information. Or it might be too difficult to answer this question at this stage of the party. Carol does not give up and asks, "Is Andrew at the party?" "No," somebody replies. It is now clear to Carol that if the answer had been "Yes", it would make sense to ask about Barbara, but not now, because her question has already been answered. She also notices that the one who replied "No" could already have answered the first question, if he had bothered to think a bit.

The situation might also be different. Coming to a party the following week she knows very well that Andrew will be there (because she had a secret date with him the night before), but she would like to know if Barbara is there as well. However, asking about Barbara directly might lead to some unwanted question like "Why is she interested only in Barbara? How does she know that Andrew is here?" Carol therefore decides to solve the problem by asking the same question as the week before — thus hiding the information she has and getting the information she needs. Obviously, if it came to the second question concerning Andrew again (in the case of no direct answer) and this question was answered positively, Carol would get a full answer and asking about Barbara would not give her any new information.

Carol's strategy in the example was rather straightforward:

1. Do not ask questions about things you know.
2. Ask directly for maximum information.
3. If you don't get a reply, ask a simpler, easier to answer, question(s), leading to the answer to your original question.

4. Try to infer the answer to the initial question on the basis of the obtained replies.

The example presents an agent-questioner who wants to obtain an answer to her (initial/main) question. She asks that question and expects an answer from other agents. What follows after the question has been asked influences our agent-questioner's epistemic state. The agent either gets a simple straightforward answer or no answer at all. Both these situations will influence her next step in the questioning strategy.

Now we will introduce the epistemic logic with questions in order to express the intuitions underlying the interplay of questions and knowledge in a formal way.

3. Epistemic logic with questions

Our basic framework will be the standard S5 epistemic logic supplemented with erotetic operators. The language consists of a countably infinite set of atomic formulas $\mathcal{P} = \{p, q, \dots\}$ and formulas defined by BNF as follows:

$$\psi ::= p \mid \neg\psi \mid \psi \to \psi \mid K\psi$$

We interpret the modality K as "the agent knows that". We will also use the modality $\hat{K}$, i.e., "the agent considers it possible that", which is defined as a dual to K:

$$\hat{K}\psi := \neg K \neg\psi$$

In this paper we use the standard S5 semantics based on Kripke frames. A (Kripke) frame is an ordered pair $\langle W, R\rangle$ where W is a non-empty set of possible worlds and the relation $R \subseteq W^2$ is an equivalence relation. A (Kripke) model $M = \langle W, R, V\rangle$ is a (Kripke) frame with a valuation function V, defined in the usual manner.

The satisfaction relation $\models$ is defined in the standard way:

- $(M, w) \models p$ iff $w \in V(p)$;
- $(M, w) \models \neg\psi$ iff $(M, w) \not\models \psi$;
- $(M, w) \models \psi_1 \to \psi_2$ iff $(M, w) \models \psi_1$ implies $(M, w) \models \psi_2$;
- $(M, w) \models K\psi$ iff $(M, v) \models \psi$, for each v such that wRv.

Other propositional connectives ($\wedge, \vee, \leftrightarrow$) are introduced in the standard way. The notions of satisfaction in a model or in a frame and validity are defined as usual.

3.1. Questions

Formally, a question is an expression of the form $?\{\alpha_1, \alpha_2, \ldots, \alpha_n\}$. An intended reading of a question of this form is: "Is it the case that α_1 or is it the case that α_2 … or is it the case that α_n?" We call α_1, α_2, …, α_n *direct answers to the question*. The set of direct answers is finite, the formulas α_1, …, α_n are syntactically distinct, and we require that there are at least two direct answers ($n \geqslant 2$). The language for $\mathsf{S5Q}$ is an extension of epistemic $\mathsf{S5}$:

$$\psi ::= p \mid \neg\psi \mid \psi \to \psi \mid K\psi \mid ?\{\psi_1, \ldots, \psi_n\}$$

In what follows, we use a metavariable Q (possibly with subscripts) for questions, and we write dQ for the corresponding set of direct answers. We will use the following abbreviations: $?\alpha$ denotes a simple yes-no question $?\{\alpha, \neg\alpha\}$, and $?|\alpha, \beta|$ denotes a binary conjunctive question of the form: $?\{\alpha \wedge \beta, \alpha \wedge \neg\beta, \neg\alpha \wedge \beta, \neg\alpha \wedge \neg\beta\}$.[2]

Askability. Askability is the central notion for our framework. It specifies conditions under which an agent is justified in asking a question. The intuitions behind the askability are as follows. If an agent (honestly) asks, for example, a question $?\{\alpha, \beta\}$ the addressee obtains the following information:

1. The agent does not know whether α or β.
2. The agent considers both α and β (epistemically) possible.
3. The agent expects that it is the case that α or it is the case that β (it is not epistemically possible that neither of them holds).[3]

We illustrate this idea with a simple example (see [12, p. 62]). There is a group of three friends: Anne, Bill, and Catherine. Each of them has one card and nobody can see the cards of the others. One of the cards is the Joker and everybody knows this fact. At this point Catherine, who does not have it, asks, *Who has the Joker: Anne, or Bill?* This question has a two-element set of direct answers, namely {Anne has the Joker, Bill has the Joker}. Asking her question, Catherine expresses that: (1.) she does not know the answer to her question; (2.) she considers the answers to be possible; and (3.) she presupposes that either Anne or Bill has the Joker.

These intuitions behind askability are formalized in the following definition.

[2] For a generalised definition of conjunctive questions see [14, p. 76].

[3] We allow that both α and β hold.

Definition 1 (Askability). We say that a question $Q = ?\{\alpha_1, \alpha_2, \ldots, \alpha_n\}$ is *askable* in a state (M, w) (in symbols $(M, w) \models Q$) iff $\alpha_1, \alpha_2, \ldots, \alpha_n$ are mutually non-equivalent (i.e., it is not the case that $\alpha_i \models \alpha_j$ for some $i \neq j$) and:

(n-t) $(M, w) \not\models K\alpha_i$, for each $\alpha_i \in dQ$ (non-triviality)
(ad) $(M, w) \models \hat{K}\alpha_i$, for each $\alpha_i \in dQ$ (admissibility)
(con) $(M, w) \models K(\alpha_1 \vee \alpha_2 \vee \cdots \vee \alpha_n)$ (context)

Semantically the askability conditions mean that the set of states accessible from w by relation R, contains for each $\alpha \in dQ$ at least one state in which α holds (ad), but, at the same time, it contains at least one state in which α does not hold (n-t). Moreover, at least one direct answer α should hold in each state (con).

As a result, the askability of $Q = ?\{\alpha_1, \ldots, \alpha_n\}$ in (M, w) is equivalent to

$$(M, w) \models (\neg K\alpha_1 \wedge \cdots \wedge \neg K\alpha_n) \wedge (\hat{K}\alpha_1 \wedge \cdots \wedge \hat{K}\alpha_n) \wedge K(\alpha_1 \vee \cdots \vee \alpha_n). \quad (1)$$

It can be simplified to a conjunction of the context condition plus "extended admissibility" expressing that each direct answer is considered to be possible by the agent as well as its negation:

$$(M, w) \models [(\hat{K}\alpha_1 \wedge \hat{K}\neg\alpha_1) \wedge \cdots \wedge (\hat{K}\alpha_n \wedge \hat{K}\neg\alpha_n)] \wedge K(\alpha_1 \vee \cdots \vee \alpha_n).$$

Observe that for the safe questions (most notably for yes-no questions), i.e., when $(\alpha_1 \vee \cdots \vee \alpha_n)$ is a tautology, askability coincides with extended admissibility. The formulas above give us a reduction of (askability of) a question to a standard S5 formula.[4]

Non-askable questions. Obviously there are questions which are not askable. The simplest example is a question with tautologies/contradictions among its direct answers. The language of S5Q allows to ask questions about knowledge and questions about questions, but in a single agent set-up these kinds of questions are not askable either. The reason for this is evident: Since our background system is epistemic S5, our agent is fully introspective and hence fully aware of her knowledge and ignorance. In particular, the question $?\{K\varphi, \neg K\varphi\}$ is not askable for any φ.

[4] Originally, the definition of askability was designed for a broader set of formal systems; in particluar, for systems in which there might be a difference between non-satisfaction ($\not\models$) of a formula and satisfaction of its negation. The general study of expressive power of epistemic languages can be found in [17, Chapter 8].

Proposition 1. $(M,w) \not\models ?\{K\varphi, \neg K\varphi\}$, *for any* (M,w).

Proof. If an agent knows φ (locally) in a state, i.e., $(M,w) \models K\varphi$, then, because K is an S5-modality, she knows that she knows φ: $(M,w) \models K(K\varphi)$ (positive introspection $K\varphi \to KK\varphi$). But then, according to the non-triviality condition, $K\varphi$ cannot be among the direct answers.

It is similar for the case when an agent does not know φ. Then she does know that she does not know φ (because of the negative introspection $\neg K\varphi \to K\neg K\varphi$) hence $\neg K\varphi$ cannot be among the direct answers either. ⊣

We can also consider introspective agents with respect to questions. If a question is askable by an agent, then she knows this fact. The formula $(Q \leftrightarrow KQ)$ is valid in S5Q. (Remember that Q is a abbreviation for $?\{\alpha_1, \dots, \alpha_n\}$.)

Proposition 2. *The formula* $(Q \leftrightarrow KQ)$ *is valid in* S5Q.

Proof. Let $dQ = \{\alpha_1, \dots, \alpha_n\}$, M be an arbitrary model and $w \in W$ be an arbitrary state. Then $(M,w) \models Q$ is equivalent to (cf. (1))

$$(M,w) \models (\neg K\alpha_1 \wedge \dots \wedge \neg K\alpha_n) \wedge (\hat{K}\alpha_1 \wedge \dots \wedge \hat{K}\alpha_n) \wedge K(\alpha_1 \vee \dots \vee \alpha_n)$$

$\Updownarrow$ from the duality of $\hat{K}$ and K

$$(M,w) \models (\neg K\alpha_1 \wedge \dots \wedge \neg K\alpha_n) \wedge (\neg K\neg\alpha_1 \wedge \dots \wedge \neg K\neg\alpha_n) \wedge K(\alpha_1 \vee \dots \vee \alpha_n)$$

$\Updownarrow$ apply positive as well as negative introspection

$$(M,w) \models (K\neg K\alpha_1 \wedge \dots \wedge K\neg K\alpha_n) \wedge (K\neg K\neg\alpha_1 \wedge \dots \wedge K\neg K\neg\alpha_n) \wedge KK(\alpha_1 \vee \dots \vee \alpha_n)$$

$\Updownarrow$ because of the monotonicity axiom $K(\varphi \wedge \psi) \leftrightarrow K\varphi \wedge K\psi$

$$(M,w) \models K((\neg K\alpha_1 \wedge \dots \wedge \neg K\alpha_n) \wedge (\neg K\neg\alpha_1 \wedge \dots \wedge \neg K\neg\alpha_n) \wedge K(\alpha_1 \vee \dots \vee \alpha_n))$$

is equivalent to $(M,w) \models KQ$. ⊣

Corollary 1. *The formula* $(Q \leftrightarrow \hat{K}Q)$ *is valid in* S5Q.

As a consequence of the previous results we obtain that a question about question is not askable for any agent.

Proposition 3. $(M,w) \not\models ?(Q)$, *for any* (M,w).

Proof. Assume $?(Q)$ is askable, then it has to satisfy the admissibility condition from Definition 1. $?(Q)$ has two direct answers: Q and $\neg Q$. Admissibility requires that $(M, w) \models \hat{K}Q$ and $(M, w) \models \hat{K}\neg Q$. We know from Corollary 1 that $(M, w) \models \hat{K}Q$ implies $(M, w) \models Q$ which according to Proposition 2 entails that $(M, w) \models KQ$ and consequently $(M, w) \not\models \hat{K}\neg Q$. Hence the admissibility is violated and $?Q$ is not askable. $\dashv$

Let us note that questions about questions and questions about knowledge of other agents are possible and useful in a multi-agent set-up, see, e.g., [12]. For the purposes of this paper (single-agent set-up) we shall be considering only declarative formulas without modal operators among the direct answers to questions.

Answerhood conditions. We assume that after an agent has asked a question, she expects an answer. Below we specify the *answerhood conditions*, i.e., when we say that a question is answered in a state (for an agent):

1. A question Q is *(completely) answered* in a state (M, w) iff there exists $\alpha \in dQ$ such that $(M, w) \models K\alpha$.
   (The agent knows at least one of the direct answers.)
2. A question Q is *partially answered* in a state (M, w) iff there exists $\alpha \in dQ$ such that $(M, w) \models K\neg\alpha$.
   (The agent can exclude one of the direct answers.)

The answerhood conditions are related to the first two conditions in the definition of askability (Definition 1). This in particular means that questions completely or partially answered in a state are not askable in this state.

3.2. Epistemic erotetic implication and erotetic entailment relation

In what follows we introduce the notion of *epistemic erotetic implication* (e-e-implication henceforth) which describes how one can move from one question to another preserving askability. This leads us to the concept of a search scenario employing the e-e-implication.

The notion of an *epistemic erotetic implication* is a counterpart to the *erotetic implication* (e-implication) from Inferential Erotetic Logic [18, 22]. In IEL, erotetic implication is a semantic relation between a question, Q, a (possibly empty) set of declarative well-formed formulas, Γ, and a question, Q_1 (where Q is called an *interrogative premise* or simply *initial question*, the elements of Γ are *declarative premises* and the question Q_1 is the *conclusion* or the *implied question* — see [22, pp. 51–52]). The core intuition behind e-implication may be expressed as follows. Let us imagine an agent who is trying to solve a certain (possibly) complex problem. The problem is expressed by her initial question. We assume that the agent does not have resources to answer the initial question on her own so the initial question has to be processed.[5] This processing is aimed at replacing the initial question with the one which is — possibly — easier to answer and such that each answer to it is useful for answering the initial question. The auxiliary question obtained as a result of the processing should have certain characteristics. First of all, it should keep to the main topic. In other words, no random questions should appear here. However, the main characteristics that we are aiming at here is that the answer provided to the auxiliary question should be at least a partial answer to the initial question (i.e., it should narrow down the set of direct answers to the initial question, see [22, p. 43]). It should bring our agent closer to solving the initial problem. IEL provides the following conditions of validity for such an erotetic inference (see [22, pp. 52–53]) making the described intuitions more precise:

1. If the initial question has at least one true direct answer (with respect to the underlying semantics) and all the declarative premises are true, then the question which is the conclusion must have at least one true direct answer.
2. For each direct answer B to the question which is the conclusion there exists a such non-empty proper subset Y of the set of direct answers to the initial question that the following condition holds:

   if B is true and all the declarative premises are true, then at least one direct answer $A \in Y$ to the initial question must be true.

Epistemic erotetic implication addresses the same intuitions. However, it takes askability as a central notion. What is ensured by e-e-implication is a 'transfer of askability'. As such, it allows us to tackle the idea of justification for certain erotetic moves. Whether an agent can ask a question depends on the agent's knowledge. Such a solution suits our needs for epistemic erotetic search scenarios. One of the drawbacks of using askability in our framework is that it does not allow us to tackle the goal-directness of the original notion of e-implication as expressed

[5] See also more linguistically oriented approaches to such question processing in, e.g., [3], [4] or [2].

by the second condition above. This has certain consequences for e-e-scenario construction where such a feature has to be imposed on the level of rules of construction (while in Wiśniewski's framework it results from underpinning search scenarios with erotetic implication).

We start with the notion of a pure epistemic erotetic implication where we do not assume anything about the agent's background knowledge. In the epistemic approach to questions, knowledge of an agent is given implicitly — by a particular state of an epistemic (Kripke) model. However, in the proposed framework of epistemic erotetic scenarios it is much more natural to work with the agent's knowledge explicitly.

Definition 2 (Pure e-e-implication). A question Q_1 *(purely) e-e-implies* a question Q_2 in a state (M, w) iff whenever Q_1 is askable in (M, w), Q_2 is askable as well. Formally:

$$(M, w) \models (Q_1 \rightarrow Q_2) \text{ iff } (M, w) \models Q_1 \text{ implies } (M, w) \models Q_2.$$

We say that Q_1 *(purely) entails* Q_2 iff $(M, w) \models (Q_1 \rightarrow Q_2)$ for each state w and and each model M (the formula $(Q_1 \rightarrow Q_2)$ is valid).

Let us stress that the implication used in Definition 2 is a standard implication of standard epistemic logic.[6]

Epistemic erotetic implication has an expected property:

Corollary 2 (Transitivity of pure implication). *If* $(M, w) \models Q_1 \rightarrow Q_2$ *and* $(M, w) \models Q_2 \rightarrow Q_3$*, then* $(M, w) \models Q_1 \rightarrow Q_3$*, for any* (M, w)*.*

Let us now introduce a few examples of valid pure e-e-implications which are used later in this paper (more examples can be found in [12, Chapter 3.3]).

- Questions $?\alpha$ and $?\neg\alpha$ have the same askability conditions which is reflected in the following valid formulas:

$$?\alpha \rightarrow ?\neg\alpha \quad \text{and} \quad ?\neg\alpha \rightarrow ?\alpha \tag{2}$$

- A question $?\{\alpha_1, \ldots, \alpha_n\}$ e-e-implies each yes-no question based on its direct answers:

$$?\{\alpha_1, \ldots, \alpha_n\} \rightarrow ?\alpha_j \text{ , for each } j \in \{1, \ldots, n\} \tag{3}$$

- A conjunctive question e-e-implies simple yes-no questions based on one of its conjuncts:

[6] For broader discussion on the issue of implication between erotetic formulas see [12] and [22].

$$?|\alpha_1, \ldots, \alpha_n| \to ?\alpha_j \text{ , for each } j \in \{1, \ldots, n\} \tag{4}$$

As we have pointed out, the context of the epistemic erotetic search scenarios requires an epistemic erotetic implication with an explicit representation of the part of the agent's knowledge 'relevant' to a particular question. This leads us to the definition of a generalized e-e-implication, i.e., an implication which holds with respect to an auxiliary set of formulas.

First, we define what it means to say that a question is askable (for an agent) with respect to some auxiliary set of formulas.

Definition 3. We say that a question Q is *askable in* (M, w) *wrt* Γ iff $(M, w) \models \{K\gamma \mid \gamma \in \Gamma\}$ and $(M, w) \models Q$.

Let us stress that Γ need not contain all of the agent's knowledge and, generally, need not be deductively closed. The definition only requires it to be consistent (explicitly) and not to provide either a complete or a partial answer Q (implicitly). The intended reading of Γ is that it explicitly represents a part of the agent's knowledge (in a given state) which is related to the question Q.

Definition 4 (Epistemic erotetic implication). A question Q_2 is *e-e-implied by* Q_1 *wrt* Γ in a state (M, w), we write $(M, w) \models (Q_1 \underset{\Gamma}{\rightarrow} Q_2)$, iff either Q_1 is not askable in (M, w) wrt Γ or Q_2 is askable in (M, w). Moreover, we say that Q_1 *implies* Q_2 *wrt* Γ and write $(Q_1 \underset{\Gamma}{\rightarrow} Q_2)$ iff $(M, w) \models (Q_1 \underset{\Gamma}{\rightarrow} Q_2)$, for every w and M.

Strictly speaking $(Q_1 \underset{\Gamma}{\rightarrow} Q_2)$ is not a formula of $\mathsf{S5Q}$; it is a notational abbreviation in the same sense as $?Q$ is.

Let us, for example, consider Q_1 to be $?\alpha$, and Q_2 to be $?(\beta \wedge \gamma)$; then Q_1 implies Q_2 on the basis of $\Gamma = \{\alpha \leftrightarrow (\beta \wedge \gamma)\}$:

$$?\alpha \xrightarrow[\{\alpha \leftrightarrow (\beta \wedge \gamma)\}]{} ?(\beta \wedge \gamma).$$

The role of Γ is to represent explicitly not only knowledge but also a lack of knowledge. The latter is important in the context of a nontriviality condition for askability.

For example, in general it is not valid that $?(\alpha \wedge \beta) \to ?\alpha$, the question $?\alpha$ is askable in a state (M, w), only if both α and $\neg\alpha$ are epistemically possible $(\neg K \neg\alpha)$. Possibility of α (which is equivalent to $\neg K \neg\alpha$) is guaranteed by askability of $?(\alpha \wedge \beta)$ (as knowledge of α would make it non-askable) but possibility of $\neg\alpha$ needs to be assumed explicitly:

$$?(\alpha \wedge \beta) \xrightarrow[\{\neg K\alpha\}]{} ?\alpha.$$

Generally speaking, whenever an agent considers both α and $\neg\alpha$ possible, then

$$?(\alpha \circ \beta) \xrightarrow[\{\neg K\alpha, \neg K\neg\alpha\}]{} ?\alpha, \tag{5}$$

where $\circ$ is any of the connectives: $\wedge$, $\vee$, $\rightarrow$, $\leftrightarrow$. We will make use of this fact later.

Our examples of e-e-implications (2)–(5) can be interpreted as patterns of how to move from 'more complex' questions to 'less complex' questions.[7] This seems to be very useful in many cases. In Section 2, we saw some informal applications of such a move. Another example is when we ask information sources with restricted languages (e.g., query languages for databases) where we need to decompose complex questions in order to make them 'understandable' for the addressee. Sometimes we want to conceal the complex question because asking it publicly could reveal too much about our knowledge and ignorance, e.g., in the case of a police investigation (see, e.g., [15]). This viewpoint brings us again to the idea of erotetic search scenarios in epistemic logic. A general idea is to control the agent's effective communication starting with a general question $?\{\alpha_1, \ldots, \alpha_n\}$ which is fully decomposed into atomic yes-no questions if necessary.

Nonetheless, before we introduce the idea of epistemic erotetic search scenarios, let us list some important properties of e-e-implication. It is especially useful at this point to emphasise that sets of formulas used in e-e-implication is our way to display (explicitly) the knowledge relevant to the askability of the corresponding question.

Erotetic epistemic implication is 'locally' as well as 'globally' monotonic with respect to the corresponding set of declaratives:

PROPOSITION 4 (Monotonicity). *Let Γ and Δ be sets of declarative formulas. Then*:

1. *If* $(M, w) \models (Q_1 \xrightarrow[\Gamma]{} Q_2)$, *then* $(M, w) \models (Q_1 \xrightarrow[\Gamma \cup \Delta]{} Q_2)$.
2. *If* Q_1 *implies* Q_2 *wrt* Γ, *then* Q_1 *implies* Q_2 *wrt* $(\Gamma \cup \Delta)$.

[7] Compare the motivation for the term *reducibility* in [18] and (especially in this context) in [12].

Proof. *Ad* 1. Let us denote $K\Psi = \{K\psi \mid \psi \in \Psi\}$, for a set of declaratives Ψ. The claim follows from the definition of epistemic erotetic implication: if $(M, w) \models K(\Gamma \cup \Delta)$ and $(M, w) \models Q_1$ then $(M, w) \models Q_2$. Now either $(M, w) \models K(\Gamma \cup \Delta)$ is true then $(M, w) \models K\Gamma$ is true and $(M, w) \models Q_2$ holds according to the assumption or $(M, w) \models K(\Gamma \cup \Delta)$ is false then $(M, w) \models Q_2$ holds trivially.

Ad 2. Follows from 1. ⊣

Monotonicity can be used for the proof of a form of transitivity of the e-e-implication with auxiliary sets of formulas:

Proposition 5 (Transitivity). *If* $(M, w) \models (Q_1 \underset{\Gamma}{\rightarrow} Q_2)$ *and* $(M, w) \models (Q_2 \underset{\Delta}{\rightarrow} Q_3)$, *then* $(M, w) \models (Q_1 \xrightarrow[\Gamma \cup \Delta]{} Q_3)$.

Proof. If $(M, w) \models \{K\varphi \mid \varphi \in (\Gamma \cup \Delta)\}$ and $(M, w) \models Q_1$, then, from Proposition 4, $(M, w) \models (Q_1 \xrightarrow[\Gamma \cup \Delta]{} Q_2)$ as well as $(M, w) \models (Q_2 \xrightarrow[\Gamma \cup \Delta]{} Q_3)$. Thus, $(M, w) \models (Q_1 \xrightarrow[\Gamma \cup \Delta]{} Q_3)$. ⊣

4. Epistemic erotetic search scenarios

Let us return to Example 1 presented in Section 2. The tree in Figure 1 represents all the situations which could have happened. It covers:

- possible relevant questions Carol could have asked (indicated by the question mark symbol);
- possible replies she could have got (represented within square brackets) including also lack of answers (represented as [ ]).

The story described in the Example 1 corresponds to one of the branches in the tree in Figure 1, but things might have been different. Carol could have obtained an immediate positive answer — the process would have been quick and fully informative (see the leftmost branch in Figure 1). In the case of an immediate negative answer (the second branch on the left) she knows what she wanted to know but does not have full information. She does not know who is missing — whether Andrew, Barbara, or both of them. Nonetheless, the initial question is not askable any more. Our tree represents not only the ways to obtain a particular direct answer to the initial question, but also the information the agent gained in the process of asking and answering. This is remarkable in the case of a failure — the agent might not have got an answer, but

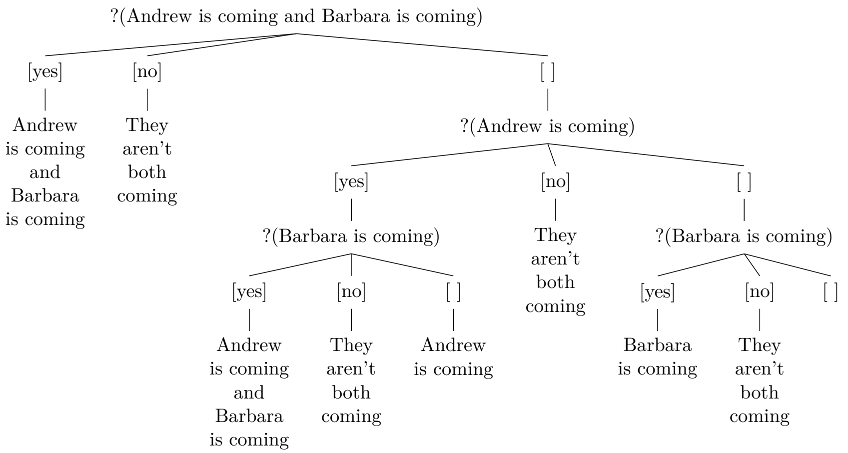


Figure 1. Representation of the possible courses of events for Example 1

still Carol knows more than before and she can continue asking other questions following a similar strategy.

Let's assume that an agent has some background knowledge Γ. Then there are two options: either the question is answered immediately (the agent obtains a direct answer) or the agent does not obtain any direct answer. An epistemic erotetic search scenario provides a pattern of questioning the agent should follow whenever she wants to solve a problem expressed by the question Q. The scenario simulates a 'dialogue' between an agent and an external information source (other agent(s)). The information coming from 'outside' is indicated by brackets [,].

Formally, questions and premises involved in such a problem-solving process are represented as a labelled tree or a scenario for a question relative to the set of premises. A question Q and an auxiliary set of epistemic $\mathsf{S5Q}$ formulas Γ is represented as a labelled tree $\Sigma = (V, E)$ where V is a finite set of nodes and $E \subseteq V^2$ is a set of edges.[8] The initial node r is labelled by Q and every other non-terminal node is labelled either by a formula in brackets or without brackets or by [ ]. Obviously the scenarios correspond to a certain kind of labelled tree and our task is now to specify these trees.

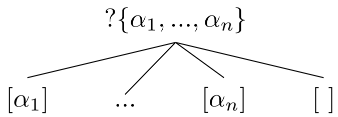
As we can see in the analysed example, the first step in the process of building a scenario is rather straightforward. The intuition behind this

[8] Nodes will be denoted by r, x, y, z, etc.; r is for root-node.

step is grasped by the notion of a *basic epistemic erotetic search scenario* (basic scenario, for short).

Definition 5 (Basic scenario). Let $Q = ?\{\alpha_1, ..., \alpha_n\}$ be a question. Basic epistemic erotetic search scenario for the question Q is a finite labelled tree Σ where:

1. the root is labelled by Q;
2. there are n+1 successors of the root, n of which are labelled by $[\alpha_i], i = 1, ..., n$, respectively, and the n+1th is labelled by [ ] indicating that the agent failed to get an answer.



Let us point out here that we are considering only questions with finite sets of direct answers, thus the resulting tree is finite.

When an agent is asking a question Q and does not receive any immediate answer, she has to prepare a strategy leading to an answer to the initial question or at least to gain the maximum information the agent can obtain. To address this we introduce *expansions*, with which we will be able to develop basic scenarios into (proper) e-e-scenarios. An e-e-scenario is an expansion of the tree corresponding to a basic scenario using either a *deductive move* or an *erotetic move*. Both moves crucially depend on knowledge the agent has collected up to the current state of the procedure. The following definition allows us to keep track of information available to the agent in the current state of the procedure.

Definition 6. Let us take a tree $\Sigma = (V, E)$ for a question Q and an auxiliary set Γ labelled in the way described above. For $x \in V$ we define a set of auxiliary formulas as follows.

1. $\Gamma(x) = \Gamma$ if x is the root of Σ.
2. Otherwise there is a unique path (sequence of nodes) $\pi(x)$ in Σ from the root x_0 to the node x: $\pi(x) = \langle x_0, \ldots, x_n, x\rangle$. Then $\Gamma(x) = \Gamma(x_n) \cup \{\varphi\}$ if x is labelled by $[\varphi]$, else $\Gamma(x) = \Gamma(x_n)$.

$\Gamma(x)$ represents, in fact, the agent's initial knowledge (Γ) and the knowledge in the form of replies to auxiliary questions collected on the path from the root to x (indicated by nodes with square brackets). On the basis of this information the agent might be able to infer a reply to some question asked earlier.

DEFINITION 7 (Deductive expansion move). A terminal node x of a labelled tree Σ for Q and Γ is *deductively expandable* iff there is a question Q' on the path from the root to x such that $\Gamma(x) \models \alpha$ for some $\alpha \in dQ'$.

The idea behind an *erotetic move* is the introduction of an auxiliary question which helps the questioner to find an answer to the initial question. Technically, the agent checks at a node x whether there is a question that is e-e-implied (with respect to $\Gamma(x)$) by a certain question appearing earlier in the e-e-scenario. We should be careful, however. When we say that Q_1 implies Q_2 with respect to an auxiliary set $\Gamma(x)$, we can read $\Gamma(x)$ in two ways. In the strict reading $\Gamma(x)$ (together with askability conditions for Q_1) should *explicitly allow* for asking the implied question, i.e., $\Gamma(x)$ and the askability of Q_1 should entail the askability conditions of Q_2. This reading is captured by the notion of a (standard) e-e-implication. We can also admit a loose reading, requiring just that $\Gamma(x)$ (together with the askability of Q_1) *does not prevent us* from asking Q_2. This reading will be captured by the notion of a *default* e-e-implication. In a nutshell, the strict reading means ‘what is not explicitly allowed is forbidden’, while the loose one means ‘what is not explicitly forbidden is allowed’.

DEFINITION 8 (Default epistemic erotetic implication). A question Q_1 *e-e-implies by default* $Q_2 = ?\{\alpha_1, \ldots, \alpha_n\}$ *wrt a set of declaratives* Γ *in a state* (M, w) iff the following conditions hold:
1. Q_1 is askable in (M, w) with respect to $\Delta = \Gamma \cup \{(\neg K\alpha_1 \wedge \cdots \wedge \neg K\alpha_n) \wedge (\hat{K}\alpha_1 \wedge \cdots \wedge \hat{K}\alpha_n) \wedge K(\alpha_1 \vee \cdots \vee \alpha_n)\}$, and
2. $(M, w) \models (Q_1 \underset{\Delta}{\longrightarrow} Q_2)$.

Then we write $(M, w) \models (Q_1 \underset{|\Gamma|}{\longrightarrow} Q_2)$.

In Definition 8 we require more than just e-e-implication. We say that the implying question is askable in a state (M, w), and this epistemic state is compatible with the agent’s knowledge collected in Γ together with the askability conditions of Q_2: see Definition 3. Using our notion of default epistemic erotetic implication we can define erotetic expansion as a move introducing a new question Q^* satisfying three conditions:
1. Q^* is implied by default by some previous question,
2. Q^* is not a repetition or reformulation of any previous question,
3. Q^* is ‘relevant’, i.e., it helps to solve the initial question.

These conditions are formally captured by the following definition.

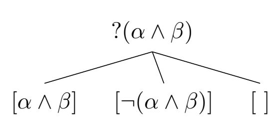
Definition 9 (Erotetic expansion move). A terminal node x of a labelled tree Σ for Q and Γ is *erotetically expandable* by a question Q^* iff the following three conditions hold:

1. there is a question Q' which is a label of a vertex on the path $\pi(x)$ from the root of Σ to x such that $(Q' \xrightarrow[|\Gamma(x)|]{} Q^*)$,
2. there is no Q'' on the path $\pi(x)$ such that Q^* purely implies Q'' and vice versa,
3. there is at least one $\alpha \in dQ^*$ such that α together with $\Gamma(x)$ entails at least one answer (direct or partial) to some preceding question Q'' (this means $\Gamma(x) \cup \{\alpha\} \models \beta$ or $\Gamma(x) \cup \{\alpha\} \models \neg\beta$ for some $\beta \in dQ''$).

Erotetic expansion moves are based on the notion of default erotetic implication (condition 1). However, the initial question might generally imply questions which are completely useless from the point of view of the corresponding scenario. From the point of view of erotetic scenarios default erotetic implication is too loose and needs to be restricted — that is the role of conditions 2 and 3.

The second condition prevents an agent asking "the same questions" and making the scenario circular or arbitrarily long. It excludes repeated asking of questions which have already been asked, but also of the questions which are (erotetically) equivalent to previously-asked questions. For example, the question $?(\varphi \wedge \varphi)$ is implied by and not identical to the question $?\varphi$, but we certainly do not want to allow it as an expansion of $?\varphi$. The same holds for $\varphi \wedge (\psi \vee \neg\psi)$. The third condition blocks questions that are implied by a previous question, are not repetitions but are still irrelevant with respect to the scenario. Consider atomic formulas p, q, r and an initial question $Q = ?(p \wedge q)$. It might happen that $(M, w) \models Q$ as well as $(M, w) \models \hat{K}\neg r$ and $(M, w) \models \hat{K} r$. Now the question $?r$ is also askable, but it does not help us to answer the initial question.

Before we define an e-e-scenario, we show what is considered an effective strategy. We use the tree presented in Figure 1 which is an (informal) e-e-scenario for the yes-no question *?(Andrew is coming and Barbara is coming)*. The process of reconstructing it in our formal language starts at the basic epistemic scenario. In the schema, α stands for *Andrew is coming* while β for *Barbara is coming*. There are no other premisses.



Now the expansion moves are employed. Let us start at the leftmost node labelled by $[\alpha \wedge \beta]$.

$$\Gamma([\alpha \wedge \beta]) = \{(\alpha \wedge \beta)\}.$$

It is deductively expandable.

$$\Gamma([\alpha \wedge \beta]) \models (\alpha \wedge \beta),$$

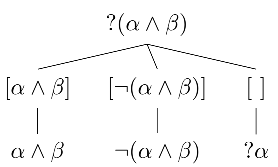
so the answer $\alpha \wedge \beta$ is placed in a child node.

Similarly for the $[\neg(\alpha \wedge \beta)]$-node. It is also deductively expandable, so the answer $\neg(\alpha \wedge \beta)$ is placed in a child node.

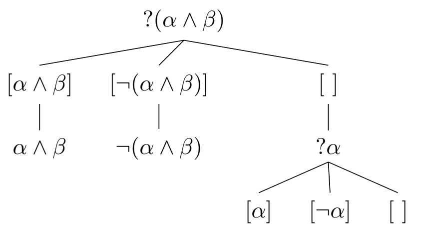
The rightmost node $[\ ]$ is not deductively expandable. We check the possibility of applying an erotetic move. Since $\Gamma([\ ]) = \emptyset$, we need a question which is (purely) e-e-implied or (at least) e-e-implied by default with respect to $\Gamma([\ ])$ in the current epistemic state. There are more candidates satisfying the conditions in Definition 9 (in particular relevance)[9] but it is in some sense most efficient to ask $?\alpha$ or $?\beta$. It is easy to see that the original question e-e-implies each of these by default (see (5)):

$$(?(\alpha \wedge \beta) \xrightarrow[|\Gamma([\ ])|]{} ?\alpha) \text{ as well as } (?(\alpha \wedge \beta) \xrightarrow[|\Gamma([\ ])|]{} ?\beta).$$

If we decide for $?\alpha$, the resulting structure is the following:

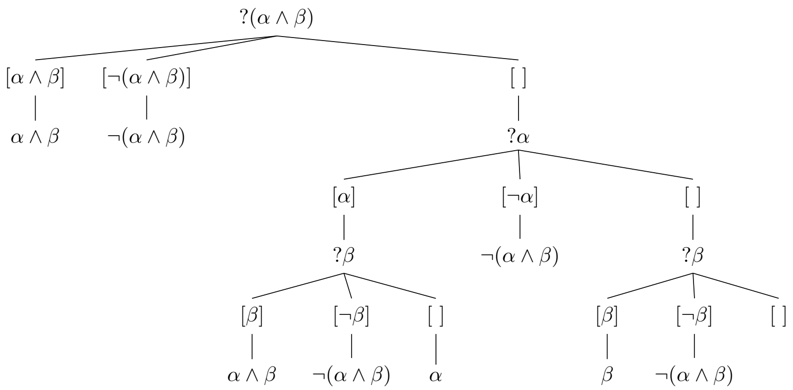


As we have reached a direct answer to the initial question on two leftmost nodes, there is no need for further expansion. For the node $?\alpha$ we can employ the basic epistemic scenario:



[9] For example, the question $?(\alpha \vee \beta)$ as was pointed to us by an anonymous referee.

The newly-introduced basic epistemic scenario for the question $?\alpha$ can be extended in an analogous way to the steps described above. The $[\alpha]$-node can be expanded deductively: we infer α from $\Gamma([\alpha])$. Then (directly and effectively) we can use erotetic expansion by $?\beta$. The $[\neg\alpha]$-node can be expanded deductively and we obtain a direct answer to the initial question $\Gamma([\neg\alpha]) \models \neg(\alpha \wedge \beta)$. It terminates this branch. The $[\,]$-node can be extended erotetically by $?\beta$. The $?\beta$-nodes are managed similarly. The complete structure is presented below.



The leaves of the tree contain direct answers to the initial question or direct answers to auxiliary questions or $[\,]$ which means that we have no information to answer any question in the tree.

A structure of the type presented above (i.e., with the empty set of initial premises Γ) is called a *pure* e-e-scenario, see, e.g., [22, p. 128].

Because we work in epistemic $\mathsf{S5}$, an empty set of initial premisses does not mean that the agent does not know anything. She knows at least all logical truths. As we have mentioned above, the role of $\Gamma(x)$ and $|\Gamma(x)|$ is rather to represent explicitly a part of the agent's knowledge/ignorance related to a particular (initial or auxiliary) question.

The labelled tree presented in this section is built for the initial question of the form $?(\alpha \wedge \beta)$. We can introduce similar structures for other simple yes-no questions based on formulas with other connectives as the main ones. By analogy to the erotetic search scenarios in IEL we will refer to them as *standard e-e-scenarios*. They are presented on figures 2–5.

It is quite clear that questions askable with respect to a non-empty set of initial premises lead to more interesting e-e-scenarios.

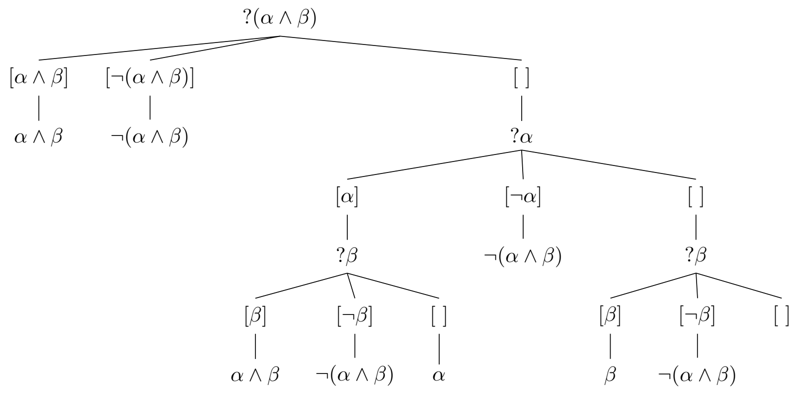


Figure 2. Standard e-e-scenario for conjunction

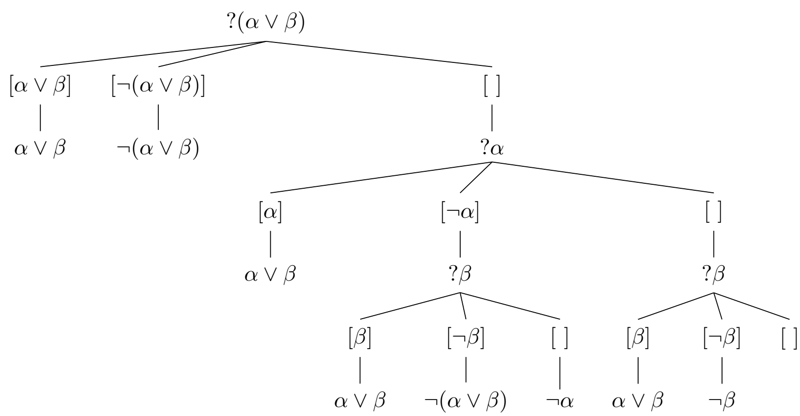


Figure 3. Standard e-e-scenario for disjunction

Let us suppose that our agent wants to know whether it is the case that a piece of software is an open-source one. We know that it would be so if and only if the source code of this software was publicly available and permission to modify the code was granted. This situation may be modelled within our approach.

We will represent the main question as $?o$. This question becomes the initial question of the agent's e-e-scenario. Moreover she knows that:

> The piece of software is an open-source one (o) if and only if its source code is publicly available (p) and permission to modify the code is granted (g).

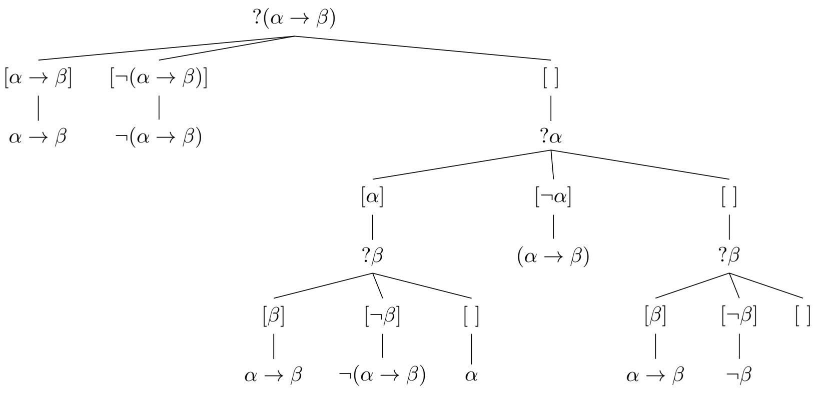


Figure 4. Standard e-e-scenario for implication

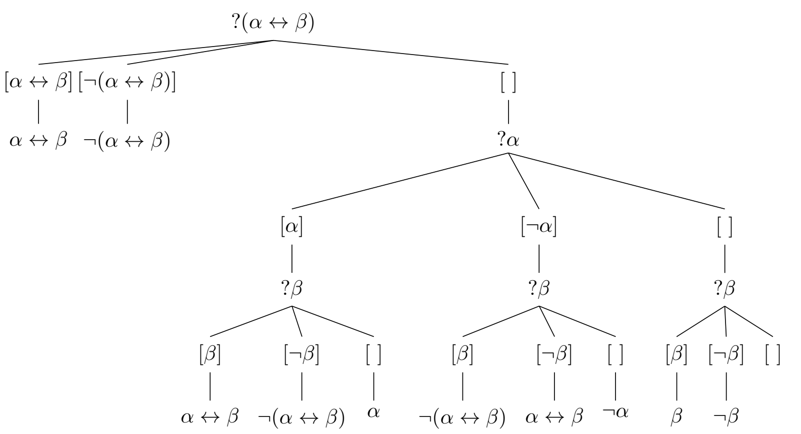


Figure 5. Standard e-e-scenario for equivalence

We represent this piece of knowledge as $\Gamma = \{o \leftrightarrow (p \wedge g)\}$. Now we can present a model of a questioning agenda for our agent as the e-e-scenario presented in Figure 6.

Again, the scenario consists of paths representing the possible ways in which the questioning might unfold (depending on the answers obtained). The leftmost path and the path next to it represent the situation when after our agent asks a question, a direct answer is provided immediately (which is represented by $[o]$, $[\neg o]$ respectively). The rightmost path is more interesting. It is the case where after the initial question is asked but no answer is provided (which is represented by $[\ ]$). Our agent then

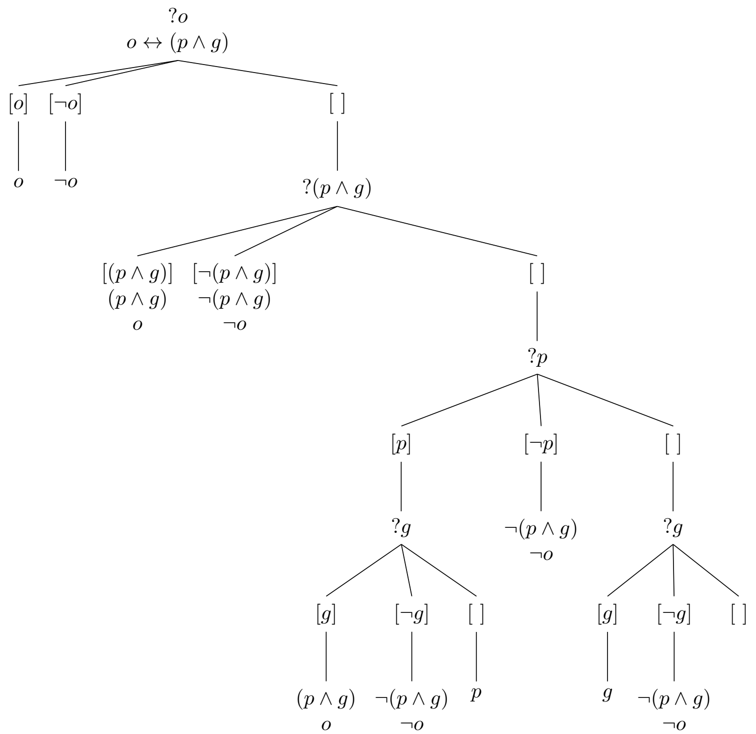


Figure 6. An example of e-e-scenario with a non-empty set of premises

decomposes the initial question with respect to her knowledge and gets a query of the form $?(p \wedge g)$; if no one knows the answer to $?o$, maybe it is the case when the new question might be resolved. One may observe that the answer to the new auxiliary question will provide our agent with the answer to the initial question (see the leaves of the e-e-scenario that are produced by deductive expansion moves).[10]

An epistemic erotetic search scenario will be defined as a labelled tree structure that is composed of some prototypical parts. These parts are connected *via* expansion moves taking into consideration the effectiveness of a particular move. The starting point is clear: it is a basic scenario. Nodes with direct answers in square brackets are deductively expandable. This move is considered effective whenever the appropriate direct answers are inferred immediately. Other expansions are more

[10] In this example (Figure 6), we did not draw all lines (edges) to save space.

free. Nonetheless, we can also follow some patterns as can be seen in our previous examples.

Deductive expansion moves (Definition 7) are preferred, especially if they provide a direct answer to an initial question. They bear answers to the questions that have appeared so far and the main aim is to progress at nodes with answers to the initial question. We have to avoid repetitions with the same results.

In the case of erotetic expansion moves (Definition 9) we require them to be 'relevant' to the solution of the initial question. Fortunately, we do not have too many options as for which questions can be considered relevant. In Section 3.2 we introduced some prototypical e-e-implications, see (2)–(5). In particular, (3) says that any question Q entails yes-no questions based on direct answers to Q. The work with yes-no questions is crucial for our approach. In all the examples we used variants of (5) where we moved from a compound yes-no question to more simple yes-no questions. These variants of (5), listed on figures 2–5, will be important components of the scenarios.

Definition 10 (Epistemic erotetic search scenario). A labelled tree structure Σ is an *epistemic erotetic search scenario* for a question Q wrt a set Γ of formulas iff the following conditions hold:

- The root is labelled by Q.
- If a node x is labelled by a question, then there are n+1 successors of the root, n of which are labelled by $[\alpha_i]$, $i = 1, \dots, n$ respectively, and the $n + 1$st is labelled by [ ] (cf. item 2 in Definition 5 – basic scenario).
- If a node x is labelled by a direct answer to Q without square brackets, then it has no child node.
- If a node x is not labelled by any question and there is x's child node y, then y is labelled in compliance with either
  - deductive expansion move (Definition 7), or
  - erotetic expansion move (Definition 9).

Definition 10 defines which labelled tree structures are e-e-scenarios. Nonetheless, some of them need not to be in a correspondence with our requirement of effectiveness. For example, it is useful to require that deductive expansions be preferred over erotetic ones and that expansions do not repeatedly produce the same formulas. In the following pseudocode we define a procedure that describes the effective formation of an e-e-scenario for Q and Γ.

ProcEESS

- Input: Q, Γ
- Form basic scenario for Q (and Γ) according to Definition 5.
- Until all leafs (nodes without successors) are labelled by CLOSED, do
  - Start at the leftmost leaf x without CLOSED-label.
  - **ProcExpand** Form $\Gamma(x)$.
    * If x is deductively expandable by $\alpha \in dQ'$ such that Q' is the label of some $y \in \pi(x)$ (Definition 7), then form a new node $x+1$ as a direct successor of x and label it $\alpha \in dQ'$ for some $Q' \in \Gamma(x)$.
      ProcTest.
      Move to **ProcStep**.
    * If x is eroteticaly expandable by Q^* according to Definition 9, then form a new node $x+1$ as a direct successor of x and label it Q^*. Prefer pure e-e-implication over e-e-implication and prefer e-e-implication over default e-e-implication. If a default e-e-implication is applied for a compound yes-no question, use standard e-e-scenario (figures 2–5).
      ProcTest.
      Move to **ProcStep**.
  - **ProcStep** If there is a leaf to the right of the actual one without the CLOSED-label, move to it and do **ProcExpand**.

ProcTest If $(x+1)$-label is the same as y-label, for $y \in \pi(x)$, then label $(x+1)$ CLOSED.

The basic step is always to create a basic scenario for an initial question (and a set of premises). **ProcEESS** ends whenever all the leaves are labelled as CLOSED. This label is placed on nodes which are not labelled with a new label if we consider all the previous labels of the path **ProcTest**. In such a case, a deductive or erotetic expansion move produces formulas that have already occurred on the path. We always start at the leftmost non-CLOSED node and after one-step expansion the procedure moves to the next non-CLOSED node on the right (**ProcStep**). In every non-CLOSED node the deductive expansion move is always preferred. If the erotetic expansion move is applied, then valid e-e-implications are preferred, or else the scenarios for $?(\alpha \wedge \beta)$, $?(\alpha \vee \beta)$, $?(\alpha \rightarrow \beta)$, and $?(\alpha \leftrightarrow \beta)$ from figures 2–5 (**ProcExpand**) are used.

5. Summary and further research

In this paper we have proposed an epistemic interpretation of erotetic search scenarios with an epistemic logic of questions as the underlying framework. Our approach is based on the idea that a communication in a group, in order to be effective requires a certain strategy (questioning agenda). This idea was already behind the erotetic search scenarios developed in the framework of IEL. In our approach, however, we deal with communication in the way dynamic epistemic logic does (see [17]); we consider the knowledge update of an agent as the main feature of communication. Epistemic erotetic search scenarios employ the key notions of epistemic erotetic logic — askability and erotetic epistemic implication. Scenarios are supposed to represent the questioning agenda of a single agent. We can see them as instructions for an agent allowing her to gain maximum information with respect to the initial question (and some auxiliary knowledge). Our scenarios are in fact composed of modules — basic scenarios and inferential expansions. The resulting questioning agenda has the form of tree which represents all the situations the agent can face when she reasonably (in some well-specified sense) tries to get an answer to the initial question.

Our background theory — an epistemic logic of questions — deals also with questions in a group of agents and provides us with group counterparts of all the basic notions. An extension of the presented framework to a multi-agent setting is our plan for future work. This extension comes even closer to dynamic epistemic logic and will allow us to interpret group scenarios in public announcement logic with questions (cf. [13]).

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