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QUESTION DEPENDENCY IN TERMS OF COMPLIANCE AND EROTETIC IMPLICATION*

Abstract. The dependency relation between questions is discussed in terms of compliance (developed within inquisitive semantics – INQ) and erotetic implication (developed within Inferential Erotetic Logic – IEL). I show that INQ approach to questions' dependency is more narrow and strict than the one offered by IEL.

Keywords: question-response; dependent question; inferential erotetic logic; inquisitive semantics; erotetic implication; compliance

Introduction

The main aim of this paper is to compare two ways of modelling a semantic relation between questions in the situation when a question is given as a response to a question. These ways are characterised by the following notions: *compliance* (developed within inquisitive semantics – INQ, cf. [5, 7]) and *erotetic implication* (developed within Inferential Erotetic Logic – IEL, cf. [16, 18]). What is interesting, INQ and IEL represent different approaches to questions itself, however these both frameworks share a similar treatment of questions' dependency.[1] After [19] I present a method of interpreting INQ interrogative formulae within IEL framework. I also show that INQ approach to questions' dependency is more narrow and strict than the one offered by IEL.

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[1]It is worth to mention other approaches here, like topicality [15], question elaboration [8, 1] or KoS [4, 10].

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The rationale behind dependent questions may be summarised as follows [3, p. 123] (see also [4, p. 57]): initial question Q depends on question given as a response Q_1, if discussion of Q_1 will necessary bring about the provision of information about Q. This allows one to say that Q_1 may be used to answer Q — in other words Q_1 is an acceptable response to Q. The following natural language dialogue examples illustrate this idea.

Example 1. BETTY: *Want a cup of coffee?*
ANN: **Well, were you making one now?**
BETTY: He's just making one, yeah.
ANN: Oh, half a one please, Paul!
[*KB8 8370–8373*][2]

In this dialogue Ann is responding to Betty's question with a question. One may observe that the answer to Betty's question depends on the answer provided to Ann's question. Whether Ann wants a cup of coffee depends on whether Betty is making one at the moment (which is visible in the way the conversation goes further — see fragment typeset with grey colour). The same relation between questions may be observed in next dialogue:

UNKNOWN: *Any other questions?*
ANON: **Are you accepting questions on the statement of faith at this point?**
[*F85, 70–71*]

This time whether more questions exist depends on whether Unknown is accepting questions on the statement of faith at the point. Similarly for the following fragment (whether Gail wants to buy the thing, depends on how much does it cost):

UNKNOWN: *Do you want to buy them?*
GAIL: **How much are they?**
[*KC5 1389–1394*]

A recent corpus study of responding to questions by means of questions revealed that dependent questions are commonly used as question-responses [10]. In the British National Corpus, dependent questions provided as responses are the second largest class among recognised

[2] This notation indicates the British National Corpus file (KB8) together with the sentence numbers (8370–8373).

question-responses (after clarification requests).[3] Understanding such a dependency between questions gives us an insight on the sub-questionhood relation (i.e. replacing the issue raised by the initial question by an easier to answer sub-issue raised by a dependent question-response). It is also important for successful modelling of goal directed dialogues. This paper offers a comparison of INQ and IEL approaches to dependency of questions. I present a method of interpreting INQ interrogative formulae within IEL framework. I show that INQ approach to questions' dependency is more narrow and strict than the one offered by IEL.

The paper is structured as follows: in the first section I introduce necessary concepts of IEL and definitions of erotetic implication (hereafter e-implication) and so called pure e-implication. In the second section, I present INQ approach to interrogative formulae and provide the definition of compliance. Third section covers the issue of interpreting INQ formulae in IEL framework. Here also examples of question — question-responses are presented and analysed using compliance and e-implication. In this section I also prove that compliance is stronger than pure e-implication. The section ends with and analysis of extended approach to compliance offered within INQ. In summary I provide links to computational resources developed for compliance and e-implication and I present future work ideas.

1. Question dependency in IEL

In what follows I will use the formal language $\mathcal{L}_?$. $\mathcal{L}_?$ is First-order Logic language enriched with the question-forming operator ? and brackets $\{$, $\}$. Well formed formulae of FoL (defined as usual) are *declarative well-formed formulae* of $\mathcal{L}_?$ (d-wffs for short). Expressions of the form $?\{A_1, \ldots, A_n\}$ are *questions* or erotetic formulae of $\mathcal{L}_?$ (e-formulae) provided that $A_1, \ldots, A_n$ are syntactically distinct d-wffs and that $n > 1$. The set $\mathbf{d}Q = \{A_1, \ldots, A_n\}$ is the set of all *direct answers* to the question $Q = ?\{A_1, \ldots, A_n\}$. The question $?\{A_1, \ldots, A_n\}$ might be read as 'Is it the case that A_1 or is it the case that A_2, ..., or is it the case that A_n?'.

[3]The following question-responses taxonomy is a result of the mentioned corpus study: clarification responses, dependent questions, questions about an underlying motivation behind asking the initial question, questions aimed at avoiding answering the initial question, questions considering the way of answering the initial question, questions with a presupposed answer, responses ignoring the initial question — for more details see [10, p. 355].

For brevity, I will adopt a different notation for one type of questions. So called (*binary*) *conjunctive questions*[4], namely $?\{A \wedge B, A \wedge \neg B, \neg A \wedge B, \neg A \wedge \neg B\}$ will be written as $? \pm |A, B|$ ('Is it the case that A and is it the case that B?') [17, p. 399].

It is worth mentioning that, with a richer language, possibilities for modelling natural language questions grow (see [18]). However, for my present purposes the language $\mathcal{L}_?$ will be sufficient.

In IEL the dependency relation is modelled in terms of the erotetic implication (e-implication for short).

A definition of the e-implication may be formulated as follows (for simplicity only finite questions and sets of declarative premises will be considered):

Definition 1 ([18, p. 67]). A question Q_1 is *e-implied* by a question Q on the basis of a set X of declarative formulae (in symbols $Q_1, \triangleright_X, Q$) iff:

1. for each direct answer A to the question Q : $X \cup \{A\}$ entails the disjunction of all the direct answers to the question Q_1, and
2. for each direct answer B to the question Q_1 there exists a non-empty proper subset Y of the set of direct answers to the question Q such that $X \cup \{B\}$ entails the disjunction of all the elements of Y.

Intuitively, erotetic implication ensures the following: (i) if Q is sound[5] and X consists of truths, then Q_1 has a true direct answer as well ('transmission of soundness and truth into soundness' [17, p. 401]), and (ii) each direct answer to Q_1, if true, and if all elements of X are true, narrows down the class at which a true direct answer to Q can be found ('open-minded cognitive usefulness' [17, p. 402]).

If a set X of declarative formulae is empty, an e-implication is called a *pure* e-implication.

Definition 2 ([18, p. 76]). A question Q_1 is *purely e-implied* by a question Q (in symbols $Q_1 \triangleright Q$) iff:

1. for each direct answer A to the question Q, A entails the disjunction of all the direct answers to the question Q_1, and
2. for each direct answer B to the question Q_1 there exists a non-empty proper subset Y of the set of direct answers to the question Q such that B entails the disjunction of all the elements of Y.

[4]For a generalised definition of conjunctive questions see [13, p. 76].

[5]A question Q is *sound* iff it has a true direct answer (with respect to the underlying semantics).

Let us now consider several examples of e-implication, starting from the pure one.

Example 2. The following pure e-implication holds:

$$?\{A, B, C\} \triangleright ?\{A, B \vee C\}.$$

Here Q is $?\{A, B \vee C\}$ and Q_1 is $?\{A, B, C\}$. The first condition for pure e-implication is met. The same applies to the second condition. One may observe that the proper subset Y of the set of direct answers to the question Q is the following: (i) for the direct answer A to question Q_1 it is $\{A\}$, (ii) when it comes to the answer B it is $\{B \wedge C\}$, and (iii) for the answer C it is also $\{B \wedge C\}$.

After [14], we may consider e-implication example, with declarative premises involved.

Example 3. My initial question is;

(Q) Who stole the tarts?

Suppose that I manage to establish the following evidence:

(E_1) It is one of the courtiers of the Queen of Hearts attending the afternoon tea-party who stole the tarts.

Thus my initial question together with the evidence implies the question:

(Q_1) Which of the Queen of Hearts' courtiers attended the afternoon tea-party?

If moreover I know that:

(E_2) Queen of Hearts invites for a tea-party only these courtiers who made her laughing the previous day.

then Q_1 and E_2 imply the question:

(Q_2) Which courtiers made the Queen of Hearts laughing the previous day?

E-implication allows also for modelling dialogues retrieved from language corpora, like those presented in Example 1.

Example 4. Let us consider the following dialogue:

ANN: Do you want me to <*pause*> push it round?
BILL: Is it really disturbing you?
[*FM1, 679–680*]

Bill's question-response in this case is certainly a dependent question (*Whether I want you to push it depends on whether it really disturbs you*). In order to explain why Bill introduced such a reply to Ann's question I will use e-implication. However, to do this I have to assume a premise which Bill probably accepts in this dialogue situation (but is not explicitly given by Bill). One of possibilities in this case might be: 'I want you to push it round if and only if it is disturbing you'. If we accept this premise, our example will appear as follows:

ANN: Do you want me to <*pause*> push it round?
(BILL): *I want you to push it round iff it is disturbing you.*
BILL: Is it really disturbing you?

Its logical structure may be reconstructed using the language $\mathcal{L}_?$

ANN: $?p$
(BILL): $q \leftrightarrow p$
BILL: $?q$

Because the following holds: $?p \rhd_{q \leftrightarrow p} ?q$ we may say that Bill's question-response in this case is e-implied (on the basis of the introduced declarative premise).

An interested reader might find more e-implication examples in [18, pp. 77–83] and more natural dialogue analysis in terms of e-implication [9, 10, 12].

One may notice that e-implication enables to capture the dependency of questions on the basis of a set of declarative premises (e.g. earlier contributions of dialogue participants) as well as pure sub-questionhood (in the case of a pure e-implication).

2. Question dependency as expressed in terms of compliance

In the framework of inquisitive semantics the dependency relation is analysed in terms of *compliance*. The intuition behind the notion of compliance is to provide a criterion to 'judge whether a certain conversational move makes a significant contribution to resolving a given issue' [7, p. 167]. If we take two conversational moves: the initiative A and the response B, there are two ways in which B may be compliant with A (cf. [7, p. 168]):

(a) B may partially *resolve* the issue raised by A (answerhood).
(b) B may replace the issue raised by A by an easier to answer sub-issue (subquestionhood).[6]

Here I will be interested only in the case when we are dealing with subquestionhood. Before I will be able to give the definition of compliance, first I will introduce (after [19, pp. 6–12]) the necessary concepts of INQ, especially the notion of question used in this framework.

2.1. INQ basic concepts

Firstly (after [19]) let us introduce a language $\mathcal{L}_{\mathcal{P}}$. It is a propositional language over a non-empty set of propositional variables $\mathcal{P}$, where $\mathcal{P}$ is either finite or countably infinite. The primitive logical constants of the language are: $\bot, \vee, \wedge, \rightarrow$. Well-formed formulae (wffs) of $\mathcal{L}_{\mathcal{P}}$ are defined as usual.

The letters A, B, C, and D are metalanguage variables for wffs of $\mathcal{L}_{\mathcal{P}}$, and the letters X, Y are metalanguage variables for sets of wffs of the language. The letter $\mathtt{p}$ is used below as a metalanguage variable for propositional variables.

$\mathcal{L}_{\mathcal{P}}$ is associated with the set of suitable possible worlds, $\mathcal{W}_{\mathcal{P}}$, being the *model* of $\mathcal{L}_{\mathcal{P}}$. A possible world is identified with indices (that is valuations of $\mathcal{P}$). $\mathcal{W}_{\mathcal{P}}$ is the set of all indices.

A *state* is a subset of $\mathcal{W}_{\mathcal{P}}$ (states are thus sets of possible worlds). I will use the letters σ, τ, and γ to refer to states.

The most important semantic relation between states and wffs is that of *support*. In the case of INQ support, $\succ$, is defined by:

DEFINITION 3 ([19, p. 6]). Let $\sigma \subseteq \mathcal{W}_{\mathcal{P}}$.

1. $\sigma \succ \mathtt{p}$ iff for each $w \in \sigma$: $\mathtt{p}$ is true in w,[7]
2. $\sigma \succ \bot$ iff $\sigma = \emptyset$,
3. $\sigma \succ (A \wedge B)$ iff $\sigma \succ A$ and $\sigma \succ B$,
4. $\sigma \succ (A \vee B)$ iff $\sigma \succ A$ or $\sigma \succ B$,
5. $\sigma \succ (A \rightarrow B)$ iff for each $\tau \subseteq \sigma$: if $\tau \succ A$ then $\tau \succ B$.

[6] In the inquisitive semantics also combinations of (a) and (b) are possible, i.e. B may partially resolve the issue raised by A and replace the remaining issue by an easier to answer sub-issue.

[7] "$\mathtt{p}$ is true in w" means "the value of $\mathtt{p}$ under w equals $\mathbf{1}$".

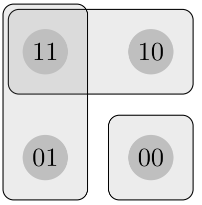


Figure 1. The diagram of possibilities for the formula $(p \vee q) \vee \neg(p \vee q)$

For our analysis we will use also the notion of the *truth set* of a wff A (in symbols: $|A|$). It is the set of all the worlds from $\mathcal{W}_{\mathcal{P}}$ in which A is true, where the concept of truth is understood classically.

Now we can introduce the concept of a *possibility* for a wff A. Intuitivelly it is a maximal state supporting A. This might be expressed as follows:

Definition 4 ([19, p. 9]). A possibility for wff A is a state $\sigma \subseteq \mathcal{W}_{\mathcal{P}}$ such that $\sigma \succ A$ and for each $w \notin \sigma : \sigma \cup \{w\} \not\succ A$.

I will use $\lfloor A \rfloor$ to reffer to the set of all possibilities for a wff A.

In INQ we may divide all wffs to assertions and inquisitive wffs. The latter are the most interesting from our point of view, because they raise an issue to be solved. When a wff is inquisitive, the set of possibilities for that formula comprises at least two elements. (When a formula has only one possibility it is called assertion.)

Let us now consider a simple example of an inquisitive formula:

$$(p \vee q) \vee \neg(p \vee q) \tag{1}$$

The set of possibilities for (1) is:

$$\{|p|, |q|, |\neg p| \cap |\neg q|\} \tag{2}$$

and its union is just $\mathcal{W}_{\mathcal{P}}$.

We can also represent the possibilities for the formula (1) in a form of a diagram specially designed for this purpose (see Figure 1). In these diagrams 1 (11) is the index in which both p and q are true, (10) is the index in which only p is true, etc. [6].

Observe that the language $\mathcal{L}_{\mathcal{P}}$ does not include a separate syntactic category of questions. However, some wffs are regarded as *having the property of being a question*, or $\mathcal{Q}$-property for short.

Definition 5 ([19, p. 11]). A wff A of $\mathcal{L}_{\mathcal{P}}$ has the $\mathcal{Q}$-property iff $|A| = \mathcal{W}_{\mathcal{P}}$.

Where $\mathcal{W}_{\mathcal{P}}$ stands for the model of $\mathcal{L}_{\mathcal{P}}$, and $|A|$ for the truth set of wff A in $\mathcal{W}_{\mathcal{P}}$. An example of a formula having the $\mathcal{Q}$-property is the formula (1). Hence a wff A is (i.e. has the property of being) a question just in case when A is true in each possible world of $\mathcal{W}_{\mathcal{P}}$, the wffs having the $\mathcal{Q}$-property are just classical tautologies.

2.2. Compliance

Let Q be an initiative and Q_1 a response to the initiative. We also assume that Q and Q_1 are inquisitive formulae and that they have the $\mathcal{Q}$-property (further on I will just call them questions for simplicity). $\lfloor Q \rfloor$ denotes the set of possibilities for Q. We may formally express the definition of compliance given in [5, p. 22] as following:

Definition 6. Let $\lfloor Q \rfloor = \{|A_1|, \ldots, |A_n|\}$ and $\lfloor Q_1 \rfloor = \{|B_1|, \ldots, |B_m|\}$. Q_1 is compliant with Q (in symbols $Q_1 \propto Q$), iff

1. For each $|B_i|$ $(1 \leqslant i \leqslant m)$ there exist $k_1, \ldots, k_l$ $(1 \leqslant k_p \leqslant n;$ $1 \leqslant p \leqslant l)$ such that $|A_{k_1}| \cup \cdots \cup |A_{k_l}| = \bigcup_{p=1}^{l} |A_{k_p}| = |B_i|$.
2. For each $|A_j|$ $(1 \leqslant j \leqslant n)$ there exists $|B_k|$ $(1 \leqslant k \leqslant m)$, such that $|A_j| \subset |B_k|$.

As it might be noticed — in the case of compliance — we cannot say anything about declarative premises involved in going from a question to question response. Relation captured by the compliance is a pure sub-questionhood relation. A simple example of question–question response where the reply is compliant to the initiative illustrates this idea.

Example 5. Q is 'Is John coming to the party and can I come?' while Q_1 is 'Is John coming to the party'. Q may be expressed in INQ as $(p \vee \neg p) \wedge (q \wedge \neg q)$ and Q_1 as $p \wedge \neg p$. $\lfloor Q \rfloor = \{|p \wedge q|, |\neg p \wedge q|, |p \wedge \neg q|, |\neg p \wedge \neg q|\}$ and $\lfloor Q_1 \rfloor = \{|p|, |\neg p|\}$. It is the case that $Q_1 \propto Q$, because both conditions for compliance are met. For the first condition observe that $|p| = |p \wedge q| \cup |p \wedge \neg q|$ and $|\neg p| = |\neg p \wedge q| \cup |\neg p \wedge \neg q|$. For the second condition let us observe that: $|p \wedge q| \subset |p|$; $|p \wedge \neg q| \subset |p|$; $|\neg p \wedge q| \subset |\neg p|$; $|\neg p \wedge \neg q| \subset |\neg p|$.

3. Compliace vs pure e-implication

3.1. Translation

In order to compare presented approaches we need to provide a method of interpretation of formulae having the $\mathcal{Q}$-property in INQ in terms of questions in IEL. I will use the method presented by [19].

I will refer to formulae having the $\mathcal{Q}$-property as Q_{INQ} and to questions in IEL as Q_{IEL}. The procedure is the following:

1. Compute all the possibilities for a given Q_{INQ}.
2. For each possibility choose exactly one wff such that the possibility is just the truth set of the wff.
3. Each such wff is a possible answer for Q_{IEL}.

Let us consider some examples here.

Example 6. Formula in INQ is: $Q_{\mathrm{INQ}} = (p \vee q) \vee \neg(p \vee q)$. Its set of possibilities is the following: $\lfloor Q_{\mathrm{INQ}} \rfloor = \{|p|, |q|, |\neg p| \cap |\neg q|\}$, thus its IEL counterpart might be formulated as follows: $Q_{\mathrm{IEL}} = ?\{p, q, \neg p \wedge \neg q\}$.

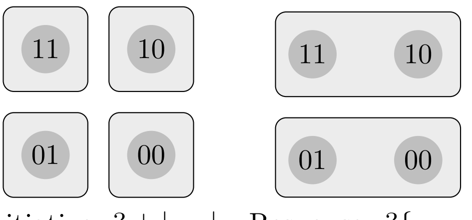
Example 7. Formula in INQ is: $Q_{\mathrm{INQ}} = (p \vee \neg p) \wedge (q \vee \neg q)$. So: $\lfloor Q_{\mathrm{INQ}} \rfloor = \{|p| \cap |q|, |p| \cap |\neg q|, |\neg p| \cap |q|, |\neg p| \cap |\neg q|\}$, thus its IEL counterpart might be formulated as follows: $Q_{\mathrm{IEL}} = ?\{(p \wedge q), (p \wedge \neg q), (\neg p \wedge q), (\neg p \wedge \neg q)\}$ (or using abbreviation according to mentioned convention: $Q_{IEL} = ?\pm|p, q|$).

In what follows I will use questions in the IEL notation.

3.2. Examples of question responses

Let us now consider some examples of question responses in the light of pure e-implication and compliance.

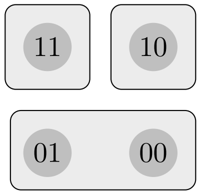
Example 8. Let us take the following initiative: $? \pm |p, q|$ and response: $?\{p, \neg p\}$:



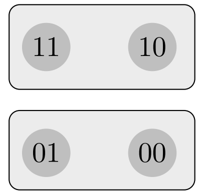
Initiative: $? \pm |p, q|$ Response: $?\{p, \neg p\}$

- $?\{p, \neg p\} \propto ? \pm |p, q|$
- $?\{p, \neg p\} \rhd ? \pm |p, q|$

Example 9. Now let us consider the following case. Initiative: $?\{p \wedge q, p \wedge \neg q, \neg p\}$; response: $?\{p, \neg p\}$:



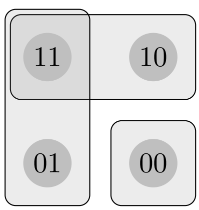
Initiative: $?\{p \wedge q, p \wedge \neg q, \neg p\}$



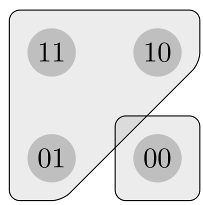
Response: $?\{p, \neg p\}$

- $?\{p, \neg p\} \propto ?\{p \wedge q, p \wedge \neg q, \neg p\}$
- $?\{p, \neg p\} \triangleright ?\{p \wedge q, p \wedge \neg q, \neg p\}$

Example 10. E-implication and compliance hold also for the following initiative and response; initiative: $?\{p, q, \neg p \wedge \neg q\}$; response: $?\{p \vee q, \neg p \wedge \neg q\}$:



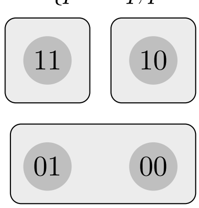
Initiative: $?\{p, q, \neg p \wedge \neg q\}$



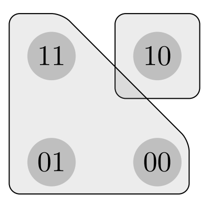
Response: $?\{p \vee q, \neg p \wedge \neg q\}$

- $?\{p \vee q, \neg p \wedge \neg q\} \propto ?\{p, q, \neg p \wedge \neg q\}$
- $?\{p \vee q, \neg p \wedge \neg q\} \triangleright ?\{p, q, \neg p \wedge \neg q\}$

Example 11. And also for the following initiative: $?\{\neg p, p \wedge q, p \wedge \neg q\}$ and response: $?\{p \rightarrow q, p \wedge \neg q\}$:



Initiative: $?\{\neg p, p \wedge q, p \wedge \neg q\}$



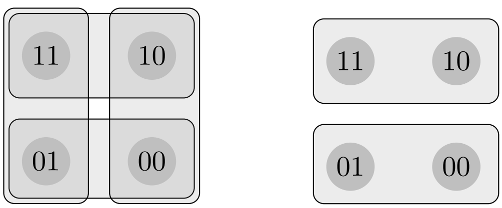
Response: $?\{p \rightarrow q, p \wedge \neg q\}$

- $?\{p \rightarrow q, p \wedge \neg q\} \propto ?\{\neg p, p \wedge q, p \wedge \neg q\}$
- $?\{p \rightarrow q, p \wedge \neg q\} \triangleright ?\{\neg p, p \wedge q, p \wedge \neg q\}$

An interesting observation is that when we analyse the diagrams one might notice that they allow for easy tracing the second condition for e-implication, i.e. (in simplified form, see page 360): *for each direct answer B to the question Q_1 there exists a non-empty proper subset Y of the set of direct answers to the question Q such that B entails the disjunction of all the elements of Y.* We may notice that the states in the response, which are formed by summing up some states from the initiative point out the required subset (namely the states that are forming the sum in the response). Consider Example 11 for $p \to q$ we have the subset of the initiative is $\{\neg p, p \wedge q\}$ and for $p \wedge \neg q$ we have $\{p \wedge \neg q\}$.

There are, however cases where e-implication holds, while the response might not be treated as a compliant one.

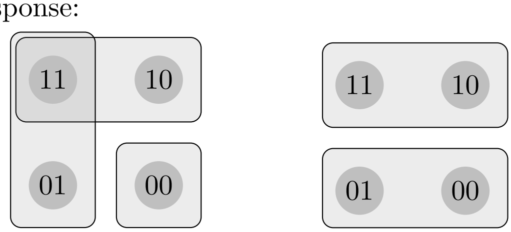
Example 12. For the following initiative: $?\{p, \neg p, q, \neg q\}$ and response: $?\{p, \neg p\}$ we have:



Initiative: $?\{p, \neg p, q, \neg q\}$ Response: $?\{p, \neg p\}$

- $?\{p, \neg p\} \not\propto ?\{p, \neg p, q, \neg q\}$
  The response is not compliant here because the second condition for the compliance is not met. Two of the possibilities for the initiative are not contained in any possibility for the response (q, $\neg q$).
- $?\{p, \neg p\} \rhd ?\{p, \neg p, q, \neg q\}$
  E-implication holds in this case because we still can treat the response as preserving cognitive usefulness.

Example 13. Similarly for $?\{p, q, \neg p \wedge \neg q\}$ as the initiative, and $?\{p, \neg p\}$ as the response:



Initiative: $?\{p, q, \neg p \wedge \neg q\}$ response: $?\{p, \neg p\}$

- $?\{p, \neg p\} \not\propto ?\{p, q, \neg p \wedge \neg q\}$
- $?\{p, \neg p\} \rhd ?\{p, q, \neg p \wedge \neg q\}$

Example 14. Both — compliance and e-implication — do not hold in the case when we want the question response to be more detailed than the initiative, like in the following case.

- $?\{p, \neg p, q, \neg q\} \not\propto ?\{p, \neg p\}$
- $?\{p, \neg p, q, \neg q\} \not\triangleright ?\{p, \neg p\}$

3.3. Compliance is stronger than pure e-implication

In all the analysed examples we were dealing with a pure e-implication (i.e. e-implication between questions where no declarative premises are involved). Let us now formulate:

Theorem 1 ([11, p. 112]). *If* $Q_1 \propto Q$ *then* $Q_1 \triangleright Q$.

Proof. Suppose that $Q_1 \propto Q$. We should show that both conditions of e-implication for $Q_1 \triangleright Q$ are met.

The first condition for e-implication is met for obvious reasons: as only classical tautologies have $\mathcal{Q}$-property in INQ, Q_1 must be a safe question and thus a sound one (see [18, p. 77, Corollary 7.22]).

Now consider the second condition for e-implication, which states that for each direct answer B to the question Q_1 there exists a proper non-empty subset Y of the set of direct answers to the question Q such that B entails the disjunction of all the elements of Y. Compliance demands that for each answer B to the question Q_1 there exists a non-empty subset Y of the set of direct answers to question Q such that the truth set for B is equivalent to the truth set for the disjunction of all formulae in Y. In other words Y is such that B entails the disjunction of all the elements of Y and the disjunction of all the elements in Y entails B. When the stronger condition for compliance will hold also the condition for e-implication will be satisfied. □

We may observe this asymmetry between compliance and e-implication in Example 13. The reason why $?\{p, \neg p\} \not\propto ?\{p, q, \neg p \wedge \neg q\}$ is that there exists the answer to Q_1 — namely $\neg p$ — for which one cannot point the subset Y, which will met the first condition for compliance. At the same time e-implication holds because the second condition for e-implication is fulfilled.

When we take a closer look on the Example 12 we will notice that also the second condition of compliance definition (called the restriction clause) makes it stronger than pure e-implication. The intuition behind

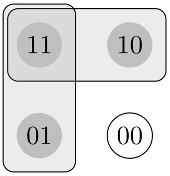


Figure 2. Example of a hybrid sentence in INQ

this clause is that — while proposing Q_1 — we cannot rule out a possible answer without providing any information. We may say that the level of information while passing form Q to Q_1 remains the same — the information needed to answer the Q is always enough to answer the Q_1 (cf. [2, p. 12]). For e-implication it is enough that for each direct answer A to question Q, A entails disjunction of all answers to Q_1. For compliance for each A (which is answer to question Q) there should exist an answer B to question Q_1 such that A entails B. If we consider Example 12 this condition for compliance is not met for the following answers to Q: q and $\neg q$.

3.4. Hybrids

As one might noticed in the previous section the expressive power of wffs having the $\mathcal{Q}$-property in INQ is rather restricted. One of the main problems is that we cannot show how a question-response might be formulated on a basis of the question-initiative and some declarative premises (like it is done for e-implication, e.g. in Example 3).

What I propose here is to extend the introduced idea of being a question in INQ. In order to do this let us take a closer look at the so called hybrid sentences in INQ.

Let us consider a formula $p \vee q$ in INQ. It has two possibilities $\lfloor p \vee q \rfloor = \{|p|, |q|\}$, so it is inquisitive. On the other hand the formula also proposes to exclude an index in which both p and q are false, and in this sense it is also informative. Concluding, $p \vee q$ is inquisitive and informative at the same time — that is why it is called a hybrid sentence. The formula is presented in a form of a diagram in Figure 2. The excluded index is marked as an empty circle.

I will interpret such a hybrid formula as expressing both a question and a piece of information. I assume that if — like in the example — index $\neg p \wedge \neg q$ is excluded it is known that $\neg(\neg p \wedge \neg q)$. As for question it expresses the following $?\{p, q\}$.

Because a hybrid formula is inquisitive and informative at the same time we have to use a slightly modified definition of compliance to grasp the fact that some possibilities might be excluded by the response.

We assume that Q and Q_1 are hybrids.

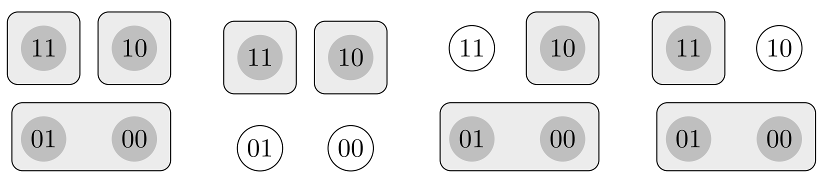
Definition 7 ([5, p. 21]). Let $\lfloor Q \rfloor = \{|A_1|, \dots, |A_n|\}$ and $\lfloor Q_1 \rfloor = \{|B_1|, \dots, |B_m|\}$. Q_1 is compliant with Q, $Q_1 \propto Q$, iff

1. For each $|B_i|$ $(1 \leqslant i \leqslant m)$ there exist $k_1, \dots, k_l$ $(1 \leqslant k_p \leqslant n;\ 1 \leqslant p \leqslant l)$ such that $|A_{k_1}| \cup \dots \cup |A_{k_l}| = \bigcup_{p=1}^{l} |A_{k_p}| = |B_i|$.
2. For each $|A_j|$ $(1 \leqslant j \leqslant n)$ there exists $|B_k|$ $(1 \leqslant k \leqslant m)$, such that $|A_j| \cap |Q_1| \subset |B_k|$.

In this definition the second condition is changed. It states that every possibility in $\lfloor Q \rfloor$, restricted to $|Q_1|$, is contained in a possibility in $\lfloor Q_1 \rfloor$. Intuitively it says that it is allowed to remove possibilities from the initiative by providing information.

Example 15. Let us consider the following example. Our initiative is: $?\{p \wedge q, p \wedge \neg q, \neg p\}$, which is a formula having the $\mathcal{Q}$-property.

The possible compliant hybrid-responses are: $?\{p \wedge q, p \wedge \neg q\}$, $?\{p \wedge \neg q, \neg p\}$, $?\{p \wedge q, \neg p\}$.



(a) $?\{p \wedge q, p \wedge \neg q, \neg p\}$ (b) $?\{p \wedge q, p \wedge \neg q\}$ (c) $?\{p \wedge \neg q, \neg p\}$ (d) $?\{p \wedge q, \neg p\}$

Figure 3. Diagrams for Example 15

In the case of $?\{p \wedge q, p \wedge \neg q\} \propto ?\{p \wedge q, p \wedge \neg q, \neg p\}$ we may notice that the possibilities $\neg p \wedge q$ and $\neg p \wedge \neg q$ are excluded. When we consider e-implication in this case we see that:

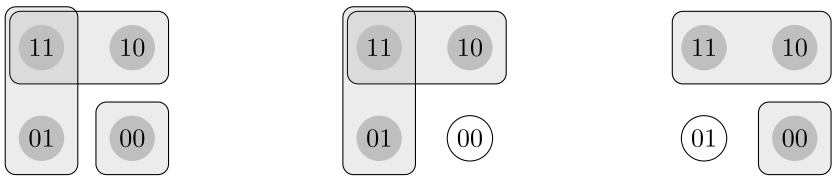
- $?\{p \wedge q, p \wedge \neg q\} \not\triangleright ?\{p \wedge q, p \wedge \neg q, \neg p\})$, however when we take the excluded indices into account (as declarative premises) we have
- $?\{p \wedge q, p \wedge \neg q\} \triangleright_{\neg(\neg p \wedge q), \neg(\neg p \wedge \neg q)} ?\{p \wedge q, p \wedge \neg q, \neg p\})$.

Consequently, if we have excluded indices in the question-response we may use negation of them as declarative premises when establishing e-implication.

- $?\{p \wedge \neg q, \neg p\} \propto ?\{p \wedge q, p \wedge \neg q, \neg p\}$ eliminates $p \wedge q$; $?\{p \wedge \neg q, \neg p\} \rhd_{\neg(p \wedge q)} ?\{p \wedge q, p \wedge \neg q, \neg p\}$ holds.
- $?\{p \wedge q, \neg p\} \propto ?\{p \wedge q, p \wedge \neg q, \neg p\}$ eliminates $p \wedge \neg q$; $?\{p \wedge q, \neg p\} \rhd_{\neg(p \wedge \neg q)} ?\{p \wedge q, p \wedge \neg q, \neg p\}$ holds.

Example 16. Initiative: $?\{p, q, \neg(p \vee q)\}$
Compliant responses:

1. $?\{p, q\}$ (eliminates $\neg p \wedge \neg q$);
2. $?\{p, \neg(p \vee q)\}$ (eliminates $\neg p \wedge q$)



(a) $?\{p, q, \neg(p \vee q)\}$ (b) $?\{p, q\}$ (c) $?\{p, \neg(p \vee q)\}$

Figure 4. Diagrams for Example 16

The following hold:

- $?\{p, q\} \rhd_{\neg(\neg p \wedge \neg q)} ?\{p, q, \neg(p \vee q)\}$
- $?\{p, \neg(p \vee q)\} \rhd_{\neg(\neg p \wedge q)} ?\{p, q, \neg(p \vee q)\}$

Example 17. Initiative: $?\{p \wedge q, p \wedge \neg q, \neg p \wedge q\}$
Compliant responses:

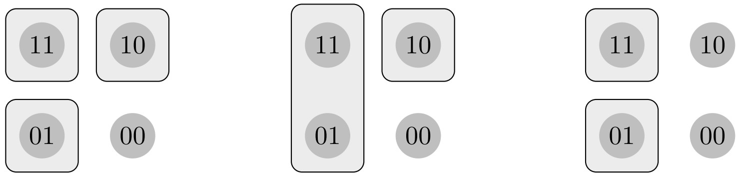
- $?\{q, p \wedge \neg q\}$
- $?\{p \wedge q, \neg p \wedge q\}$

The following hold:

- $?\{q, p \wedge \neg q\} \rhd_{\neg(\neg p \wedge \neg q)} \{p \wedge q, p \wedge \neg q, \neg p \wedge q\}$
- $?\{p \wedge q, \neg p \wedge q\} \rhd_{\neg(\neg p \wedge \neg q), \neg(p \wedge \neg q)} \{p \wedge q, p \wedge \neg q, \neg p \wedge q\}$

Summary

In the paper I have introduced and compared approaches to question dependency — based in IEL and INQ framework. The first observation is that — despite many differences between IEL and INQ frameworks — they share a very similar approach to question dependency. When we consider compliance and e-implication in context where no declarative premises



(a) $?\{p \wedge q, p \wedge \neg q, \neg p \wedge q\}$ (b) $?\{q, p \wedge \neg q\}$ (c) $?\{p \wedge q, \neg p \wedge q\}$

Figure 5. Diagrams for Example 17

are involved), the result is that compliance constitutes a stronger relation than pure e-implication. This is the consequence of an intuition behind compliance, saying that a compliant question response should not rule out any possibilities form the initiative. As such compliance might serve as a good source of inspiration for strengthening e-implication. I have also presented that when we will extend the notion of formulae having the question-property in INQ we will gain the possibility to express sub-questionhood based on declarative premises (like in the case of e-implication). To do this we have to use so called hybrid formulae in INQ and accept the proposed convention of ruling out some possibilities. However, even with this solution it seems that expressive power of e-implication when it comes to question's dependency is better (especially when we will consider another tool from IEL involving e-implication, namely erotetic search scenarios — see [17]).

It is worth to mention that an interested reader may use useful computational tools available for presented concepts:

- Computing compliance:
  `https://sites.google.com/site/inquisitivesemantics/` (Computational Tools).
- Computing e-implication:
  `http://kognitywistyka.amu.edu.pl/intquestpro/` (Resources).

The future works will focus on comparing the notion of usefulness of a sub-question in the presented approaches. In IEL the definition of e-implication states how we should understand usefulness of a sub-question (see the second condition of the definition). When it comes to INQ a similar solution is offered by the concept of *ultimate compliance* [5, p. 25]. In both, IEL and INQ the idea of usefulness is focused on the most optimal information gathering in order to answer the initiative (initial question).

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