ON THE DISCUSSIVE CONJUNCTION IN THE PROPOSITIONAL CALCULUS FOR INCONSISTENT DEDUCTIVE SYSTEMS*

Two-valued discussive systems (cf. [1]) of the propositional calculus $D_2$ can be enlarged by means of the discussive conjunction $\land_d$. To this end instead of the definition $M_2$ def. 1 from [1] we need to posit the following definition:

\[ p \land_d q := p \land q. \]

After this emendation we can simplify the definition of the discussive equivalence by replacing $M_2$ def. 2 by the following:

\[ p \leftrightarrow_d q := (p \rightarrow_d q) \land_d (q \rightarrow_d p) \]

The metalogical theorem 1 (cf. [1], p. 68) remains valid in the following generalized form: Each thesis $A$ of the two-valued classical calculus $L_2$ containing no other symbols than $\rightarrow$, $\leftrightarrow$, $\lor$ or $\land$ is transformed into thesis of the discussive calculus $D_2$ by replacing in $A$ functors $\rightarrow$ by $\rightarrow_d$, $\leftrightarrow$ by $\leftrightarrow_d$, and $\land$ by $\land_d$, respectively.

The proof of the theorem contains no essential change in comparison with the proof of the metalogical theorem 1 from my original paper [1]. We must only use theorems 5–7 of \( M_2 \) (cf. [1], p. 68) plus a new thesis of \( M_2 \):

\[
M_2 \quad 7.1 \quad \Diamond (p \land q) \leftrightarrow (\Diamond p \land \Diamond q).
\]

The law of the inconsistency for the discussive conjunction is the following thesis of \( D_2 \):

\[
D_2 \quad 4.1 \quad \neg (p \land \neg p),
\]

whereas the refuted conjunctive form [i.e., Duns Scotus Law – J.P.] is

\[
(non\; D_2) \quad 3.1 \quad (p \land \neg p) \rightarrow a q
\]

despite the fact that previously we had an analogous theorem for the usual [classical – J.P.] conjunction, which in my previous paper [1] is denoted by \( D_2 \) 5 (cf. [1], p. 69).

References


(translated by Jerzy Perzanowski)

Comments of the translator

1. The main result of this very short, but quite important, note is its main metatheorem that \( D_2 \) in fact contains the full positive part of the classical logic plus observation (\( M_2 \) 7.1) that with the new notion of discussive conjunction Jaśkowski’s basic transformation is remarkably simplified, becoming a common homomorphism.
Moreover, on the ground of a modified $D_2$ we have quite a lot of nice new
theorems, such as the law of inconsistency ($D_2$ 4.1). Indeed, on the basis of
$M_2$ (i.e., $S5$) we have that:

\[ \neg(p \land \neg p) \implies \lozenge \neg \neg(p \land \lozenge \neg p) \]
\[ \neg \lozenge (p \rightarrow \square p) \]
\[ \neg \lozenge (\square p \rightarrow \lozenge \square p). \]

3. It is clear that on the ground quite close to the modified $D_2$ we can define
quite a lot of new discussive connectives, including discusive negation:

\[ (\neg_d) \]
\[ \neg_d p := \lozenge \neg p. \]

Indeed, in $S5$ it is easy to verify that

\[ \neg_d p \iff \lozenge \neg p \]
\[ \iff ((p \rightarrow p) \land \lozenge \neg p) \]
\[ \iff ((p \rightarrow p) \land_d \neg p). \]

Also reversely,

\[ (p \land_d q) \iff (p \land \lozenge q) \]
\[ \iff (p \land \lozenge \neg q) \]
\[ \iff (p \land \neg_d \neg q). \]

Discusive conjunction and discussive negation are thereby interdefinable on
the ground $S5$, hence they are closely interconnected in the modified version
of $D_2$.

4. Of course, we have

\[ \neg \neg_d p \rightarrow p \]
\[ \neg p \rightarrow \neg_d p, \]
\[ p \rightarrow \neg_d \neg p, \]
\[ \neg_d \neg_d p \rightarrow p. \]

But not reversely. For in $S5$ we easily obtain

\[ \lozenge p \iff \neg_d \neg p, \]

whereas

\[ \square p \iff \neg \neg_d p, \]
\[ \iff \neg_d \neg_d p. \]

J.P.