Microeconometric Analysis of Telecommunication Services Market with the Use of SARIMA Models

Abstract. The paper presents the results of testing the effectiveness of the multi sectional model in the short-term forecasting of hourly demand for telephone services. The model was based on the integration of the linear regression model with dichotomous independent variables and the SARIMA model. The regression was used as a filter of modelled variability of the demand. The SARIMA was applied to model residual variability. The research shows that the proposed integration provides a greater possibility of approximation and prediction in comparison to the non-supported linear regression model. The results of the study provide support for operational planning of telecommunications operator.

Keywords: Decision Support System; dichotomous regression; SARIMA model, forecasting.

JEL Classification: C53; L86; L96.

Introduction

The level of competition in the telecommunication market is getting higher. The number of operators is increasing and the division of telecommunication markets is increasingly greater. It originated from the execution of measures, which were assumed in the Lisbon Strategy (Lisbon European Council, 2000) and its current continuation and extension – Europe 2020 Strategy (Begg, 2010; European Commission 2010). According to these strategies the telecommunication market should be primarily liberalised (the
abolition of restrictions, monopolies and discriminations) and harmonised (common regulations to create fair activity conditions for all telecommunications operators). The premises of the conceptions involve the initiative of the Information Society building.

In connection with increasingly higher level of competition in the telecommunications market, the problem of effective modelling and forecasting of demand for electronic communication services has gained significantly greater importance.

Sales forecasts of electronic connection services play a particularly important role in the management of a telecommunications operator. These forecasts support planning policy of the operator, because they are the basis for operational planning. Within the framework of operational planning of the telecommunications enterprise, decisions relating to price calculation and network management are made that are connected with achievement of operational (short and medium-term) objectives. In terms of literature, this level of planning is defined as the key decision-making field of managers, because the operational planning can strengthen the effectiveness of growth of the enterprise value. Due to the fact that operational planning also involves the means to achieve operational objectives, these means may be considered as analytical tools. Therefore the use of effective analytical techniques that improve operational management is a source of increase of enterprise value.

In order to rationalise the operational planning and finally to strengthen the market position, managerial staff of telecoms enterprises are interested in effective Prediction Systems (PS) application (Dittman, 2004) that belong to one of the Decision Support Systems (DSS) subclasses.

1. The Purpose and Thesis of the Research

The literature study leads to statements that PS can function alone, or as a part of a broader (multifunction) DSS. The effect of the PS work is the prospective information about the external (micro and macro) environment of an enterprise as well as internal characteristics of an enterprise. PS is built the following components: prognostic database, statistical data preprocessing methods, statistical data analysis methods, forecasting methods, computer programs and forecast monitoring system.

Within the framework of the literature on electronic communications, the contents, which refer to modelling and forecasting of demand for telecommunications services, are not popular. Furthermore, it can be noticed that there is a lack of such contents as effectiveness descriptions of the telecom-
Communications market data mining techniques and methods (including econometric techniques) applied by operators. It is caused by the existence of the previously mentioned significant competition between operators. In practice the transfer of used knowledge does not exist in the field of telecommunications data mining, because operators protect their experience. They treat the knowledge acquired, by using data mining methods, as a part of their competitive advantage (Muraszkiewicz, 2000).

The study of the predictive potential, which is implemented in available commercial Decision Support Systems (e.g. PROHIKS, KOBAT-SAIR, KOBAT-SAD), encourages one to conduct research into other approaches to forecasting of the demand for telephone services, and to assess their implementation techniques. Commercial software to conduct data mining calculation is not always effective in solving tasks, which are important for telecommunications operators. It is ineffective particularly in solving problems where there are more complex data structures and temporal dependences, i.e. sequences of events (Muraszkiewicz, 2000).

The results of the conducted research, which has been described in this article, relate to one of the PS components, i.e. the internal characteristics forecasting techniques, namely sales expectations techniques. The purpose of this study is to verify the effectiveness of the constructed model in short-term forecasting of the demand for telephone services. The linear regression model, including dichotomous (binary) explanatory variables, was integrated with the SARIMA model. This integration was based on the assumption that the regression model was supported by the SARIMA model. Thus the regression model is used as a filter of modelled variability of demand for telephone operator services (response variable). In turns the SARIMA model is used to reflect the remaining volatility of the demand for electronic communications services, i.e. received after the filtration of the origin variability of the modelled demand.

The author formulated the thesis, with regard to approximation and prediction, supported linear regression model enables better results in comparison to non-supported linear regression model. Effectiveness comparison of the above mentioned two techniques (integrated and non-integrated model) was verified by means such obtained values as: fit coefficients, autocorrelation coefficients, partial autocorrelation coefficients, and the average errors of expired forecasts ex-post.

The calculation study was carried out on the basis of data provided by one of the telecommunications network operators. The range of empirical material consisted of hourly counted seconds of outgoing calls within the framework of: given subscriber group, particular day (e.g. working or non-
working), and specific category of connection (7 categories of connections were taken into account).

2. The Theoretical Conception of the Prepared Model

The configuration of the forecasting model of demand for electronic connection services depends on the forecasting horizon. If the purpose of constructing the model is long-term prediction, apart from obvious quantitative changes, qualitative changes should also be considered. Quantitative changes are based on changes in the value of the response variable according to the detected regularity, e.g., the regression function. On the other hand, qualitative changes are transformations of the essential features of the phenomenon, such as the transformation of the existing regularity, which is expressed by the change of parameters or function type of the model (Nadolny, 2011).

If the purpose of the modelling is short-term and medium-term prediction, the qualitative changes mentioned above do not occur or occur in trace dimension. Therefore, it is not necessary to include them in the prognostic process. When a model is created for short and medium forecast horizon, the following factors should be considered: the type of day (typical working day, Saturday, Sunday, high days, and holidays) hour of the day, category of connection, the type of subscribers, promotions (Kaczmarczyk, 2016).

The author applied the approach consisting of several segments. The approach is based on the fusion of the results obtained with the use of two different models, i.e. the linear multiple regression model and the SARIMA (p,d,q)(P,D,Q) model. The first one is used to isolate the linear relationship between the dependent variable and independent variables, and the second one is used to model the residual values of the first model. This is shown in Figure 1.

In the first segment, the linear (multiple) regression model is estimated. The regression model enables one to obtain typical demand values for telecommunications services that are generated by the specified subscriber group on the particular hours of the given type of day, within the particular category of connection. In the second segment, the residual values are calculated (i.e. cleaning time series of the response variable). The first and the second segments can only execute their tasks in the proposed sequence. The third segment serves to forecast the demand by using the regression model, and the fourth segment serves to forecast the residual values of the regression model. In the fourth segment the SARIMA model is used to model and forecast a lower variability (after elimination of the multiple relationships
included in regression model). The third and fourth segments can work collaterally. The results received by using the forecasting tools were integrated in the fifth segment, i.e. the forecast values obtained with the use of the regression model are corrected by the prognostic residual values. The econometric analysis of high frequency data was researched by Kufel (2010). Methods of elimination of deterministic components are a very important issue because they have a great impact on the accuracy of forecasts (Box et al., 1994; Makridakis and Wheelwright 1989; Makridakis et al., 1998).

Figure 1. The integration of regression and SARIMA model

3. The Research Results

In the conducted empirical analyses, the demand for telephone services (response variable) was considered as the hourly call time measurements (sec.) of outgoing connections of the telecommunications operator network. As the classification factors of the demand were assumed: hour within 24 hours, type of 24 hours, connection category, and the kind of subscribers group. Within the framework of every mentioned classification factor were defined particular levels. Therefore the following 35 variables were defined: BUS – business subscribers, IND – individual subscribers, MN – mobile networks, LC – local calls to the same network, LCO – local calls to other networks, TC – trunk calls, IC – international calls, OC – other connections,
W – working 24 hours, SAT – Saturday, SUN – Sunday, and 24 variables to describe particular hours during the day: from 12AM – 00:00:00–01:00:00 to 11PM – 23:00:00–00:00:00.

Hourly averages of the demand for telephone services in 24-hours cycles, within the selected working 24 hours (Wednesdays) and generally non-working 24 hours (Sundays) during a year and generated by business or individual subscribers, are presented in Figure 2.

![Figure 2. The average time (sec.) of outgoing calls generated by business or individual customers in hours of working and non-working 24 hours](image)

The courses of the demand for telecommunications services are different due to the category of connection, subscriber group and type of 24 hours. The analytical sections have different location of the demand extreme.

The structure ( categorised histogram) of demand values (hourly counted seconds of outgoing calls) generated by business customers, within working and non-working 24 hours, is shown in Figure 3. Figure 3 was drawn on the basis of the same statistical material that was used to present daily course of demand (generated by business customers) in Figure 2. Visual analysis leads to the remark that the variables distributions within the framework of business subscribers are different, and the variable LCO has the highest observa-
tions in the both types of days. The lowest values were observed in the case of the variable IC and OC. A relatively high value may be observed also within LC variable and may be noticed within working 24 hours as well as within Sundays.

Figure 3. The structure of observations (hourly counted sec.) of outgoing calls generated by business customers during working and non-working 24 hours

In the next figure (Figure 4), categorised histogram of hourly combined seconds of outgoing calls generated by individual subscribers also during
working and non-working 24 hours is presented. It can be noticed in both types of day that the highest observations there are within the framework of local connection to other network and these observations are even higher than in the case business subscribers within this category and during the same type of day. The demand value within remaining categories of connection during both type of 24 hours are significantly lower.

Figure 4. The structure of observations (hourly counted sec.) of outgoing calls generated by individual customers during working and non-working 24 hours.
The intervals of hourly counted seconds of outgoing calls are left-open and right-closed. Therefore, the intervals \((-50000, 0]\) or \((-5000, 0]\) include the observations that equal to 0. This assumption enables isolation of the observations that equal to 0 and consequently it allows for better overview of the structure (categorised histogram) of the demand for telecommunication services generated by particular subscribers group, within defined type of day. In this analysis, the volumes of demand that equal to 0 constitute significant group of observations and these observations can be isolated in separated interval.

The regression model has included 35 dichotomous independent variables, which were specified at the beginning of this study section. Independent variables take value 0 or 1. The dependent variable was set by hourly measurements of seconds of outgoings calls of the operator network. Multiple regression parameters were estimated from data for the period from 1st January to 20th February of the selected year (14688 cases). The data, which was used to construct the tested models, was very complex. The constructed regression model was based on data relating to six categories of connections and two subscriber groups (12 separated analytical sections of demand). Due to the fact that the period from 1th January to 20th February consist of 1224 hours, the modelling of full variability of demand for telecommunication connections services needed to involve 14688 cases. Therefore, the number of cases reflects joint analysis of demand in terms of various analytical sections at the same time.

Due to the fact that ridge regression was estimated, it was necessary to optimise parameter \(\lambda\). The optimal value of the parameter \(\lambda\) was set at 0.0570. At this value the ridge regression model was the best fitted to the modelled data (\(R^2 = 0.4748\); std. error of the estimate = 59524.1568; calculated F statistic = 378.4818 on 35 and 14,652 df, \(p < 0.05\)). In the obtained regression model only one of the structural parameters, i.e. parameter standing by the variable 09PM, was statistical insignificant – see Table A1 in appendix.

Then the object of the research was autocorrelation function and the partial autocorrelation function of the regression model residuals. The residuals of the regression model are characterised by transparent repetitions in 24-hour cycles. The result of the autocorrelation analysis is presented in Figure 5. The regression model was constructed for almost full variability (many levels of applied classification factors) of demand for telecommunication connections services. The estimated model possibilities are too low to model such complex variability. This was the reason that autocorrelation was in the error term.
Forecasting procedure (by the use of regression and constructed integrated model) was conducted with the use of the period from 21st to 28th of February (2304 forecasts). The predictive activity was conducted in all the assumed analytical section. Therefore the demand course was forecasted in section of: 24 hours, subscribers groups and connections categories.

The autocorrelation function

Figure 5. The autocorrelation function and the partial autocorrelation function of the regression model residuals

The demand forecasts accuracy was verified by using the mean absolute error (MAE) and the root mean square error (RMSE) according to the following formulas:

\[
MAE = \frac{1}{T - n} \sum_{t=n+1}^{T} |y_t - y_t^*| \\
RMSE = \sqrt{\frac{1}{T - n} \sum_{t=n+1}^{T} (y_t - y_t^*)^2}
\]

where \(T\) – forecast horizon, and \(n\) – a number of observations, which were used to estimated model.

The research results, of the forecasting effectiveness of the regression model by the use of the above errors (in sec.), were 43148.92 and 57409.18 for \(MAE\) and \(RMSE\) respectively.

The analysis of the regression residuals shows that they are characterised by seasonality in daily courses and in particular analytical section. In connection with the seasonality, the thesis can be formulated that the use of the SARIMA model to reflect residuals variability allows for improving results in terms of approximation and forecasting of the analysed demand. Moreover, the analysis of the calculated Cook’s distances and the obtained standardised residuals indicate that there are unusual observations, i.e. influence
observations or outliers (see Figure A1 in appendix). Due to the risk of oblitera-
tion of real patterns occurring in the studied phenomenon, the unusual
observation were not eliminated and not replaced by their estimates
(Dittmann et al., 2011). General overview of residuals course for business
and then individual subscribers is presented in Figure 6.

![Figure 6. The regression residuals course](image)

The data used to estimate the regression model included patterns from
each analytical section of the demand, so the regression residuals course is
different in particular intervals of observations. The highest residuals values
can be observed in the interval of the local connection to other network gen-
erated by individual subscribers (the interval of observations 8089–8832 in
January and the interval of observations 12289–12768 in February). The
high values of residuals were also noticed in the interval of the local connec-
tion to other networks generated by business subscriber, but these observa-
tions were obviously lower than in the case of the individual subscribers.
These results came from the highest values of the demand for telephone
services within the framework of this category of connection in both groups
of customers. The analysis of the regression residuals confirms the remarks
on categorised histogram and interaction plot (Figure 1–3).

Several SARIMA models were tested. The maximum likelihood estima-
tion (MLE) was applied. Two approaches i.e. MLE according to Melard
(1984), also known as exact likelihood, and MLE according to McLeod and
Sales (1983) were used. The maximum likelihood estimation according to
Salad and Macleod (1984) was also used. The criterion for assessing the
model fit were squared errors of the SARIMA model. The initial sum of
squared errors (ISS), final sum of squared errors (FSS) and mean of squared
errors (MS) were taken into account. The goodness of fit was also assessed
due to the percentage relation of these errors (RSS = FSS/ISS). In the all
experiments the estimation process was stopped when the convergence criterion (required accuracy) was reached. Thus, it was assumed that the changes in the SARIMA parameters over consecutive iterations should be less than the value of the convergence criterion. The optimal model was SARIMA (1,0,3)(1,0,4)\textsubscript{24}. The goodness of the best SARIMA model fit is presented in Table 1.

Table 1. Summary of the SARIMA (1,0,3)(1,0,4)\textsubscript{24}

<table>
<thead>
<tr>
<th>Coefficient</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>ISS</td>
<td>49.7153</td>
</tr>
<tr>
<td>FSS</td>
<td>4.5122</td>
</tr>
<tr>
<td>RSS</td>
<td>9.0761</td>
</tr>
<tr>
<td>MS</td>
<td>0.0003</td>
</tr>
</tbody>
</table>

Note: The value of ISS, FSS, and MS was presented in trillions and RSS in per cent. Convergence criterion was set at 0.0001. The estimation process reached convergence criterion after cumulatively 46 iterations. The parameters was obtained by the use of MLE according to McLeod and Sales (9 iterations) and then MLE according of Melard (37 iterations).

All of the model parameters were statistically significant. The values and standard errors of the parameters were juxtaposed in Table 2.

Table 2. Parameters and their errors of SARIMA (1,0,3)(1,0,4)\textsubscript{24}

<table>
<thead>
<tr>
<th>Coeff.</th>
<th>Non-seasonal parameters</th>
<th>Seasonal parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>p(1)</td>
<td>q(1)</td>
</tr>
<tr>
<td>Parameter</td>
<td>0.75</td>
<td>-0.19</td>
</tr>
<tr>
<td>ASE</td>
<td>0.01</td>
<td>0.01</td>
</tr>
<tr>
<td>(A t(14684))</td>
<td>82.99</td>
<td>-15.06</td>
</tr>
<tr>
<td>LLC</td>
<td>0.73</td>
<td>-0.21</td>
</tr>
<tr>
<td>ULC</td>
<td>0.77</td>
<td>-0.16</td>
</tr>
</tbody>
</table>

Note: Non-seasonal parameters and also seasonal parameters are statistically significant at significance level \(p = 0.05\). Explanation of the abbreviations: ASE – asymptotic standard error, \(A t(14684)\) – asymptotic \(t(14684)\), LLC – lower limit of confidence interval (95%), ULC – upper limit of confidence interval (95%).

The course of SARIMA (1,0,3)(1,0,4)\textsubscript{24} residuals (for both subscribers groups) shows that their values are much lower than in the case of the regression model and the variability reflecting of demand for telephone services was improved (Figure 7).

Then the estimated value of the regression residuals (that were obtained by using the SARIMA model) were applied to correction estimated value of the demand (determined values of regression). The goodness of the fit to data for the final model, which was verified by means of R square, was 0.9480. Therefore the level of fit was clearly higher in comparison to the regression, which was not supported by the SARIMA model.
The analysis of obtained values of $Q$ Box and Ljung coefficients and also partial correlation coefficients (Figure 8) indicate that they are much lower than the values of these coefficients, which were calculated in the analysis of regression model residuals (Figure 5). However they can be considered as not fully satisfactory. It is noticeable that there are still repetitions in 24 cycles (but smaller than previously). The results provide rationales to further research to reduce correlation in error term. The reduction of the correlation in terms of error could be probably achieved by reduction of such high numbers of the analytical section included in the regression model.

Then the forecasting effectiveness of the supported regression model was verified and compared with the previous regression model. Average forecasting errors for the forecast period (the same as in the case of non-
supported regression), accounted according to (1) and (2) formulas, amount to 8488.65 and 15758.65 respectively. These results can be objectively considered as much better than the result obtained by using the previous regression model, i.e. the forecasting effectiveness (in the mean of forecast accuracy) was significantly higher.

Conclusions

In the light of the obtained research, the thesis can be confirmed that the supported regression model enables higher efficiency of approximation and prediction of demand for telecommunications services in comparison with the non-supported regression model.

The results encourage further research in the explored field. The fit and forecasts accuracy could probably be higher by the volatility reflecting within the framework of lower number of the analytical sections, for example by the volatility modelling only within the business group, or even only within the business group and working 24-hours. The separation of particular types of day is especially important, because cycles of repetitions of daily demand during the same day in different categories of connection are similar in terms of the phases of the cycles.

It is also interesting to try to improve the fit of the constructed model by creating demand patterns in particular analytical sections, i.e. by using averages for the particular hours within the sections. This could contribute to a better fit of the regression model and consequently better fit of the overall model. In this approach one could also use separate information for separate models.

Further research could concern another realisation of the fourth segment. Thus future work may relate to the use of other modelling and forecasting methods for seasonality, i.e. the residuals of the regression model.

References

Europe 2020. A strategy for Smart, Sustainable and Inclusive Growth. Communication from the Commission, European Commission, Brussels 3.3.2010,
Microeconometric Analysis of Telecommunication Services Market...

http://www.buildup.eu/sites/default/files/content/com2010_2020en01.pdf
(22.07.2017).


**Mikroekonometryczna analiza rynku telekomunikacyjnego z wykorzystaniem modeli SARIMA**

Za r y s t r e ś c i. W artykule przedstawiono wyniki testów efektywności wieloprzekrojowego modelu w krótkookresowym prognozowaniu cogodzinnego zapotrzebowania na usługi telefoniczne. Model został oparty na integracji zero-jedynkowego modelu regresji liniowej i modelu SARIMA. Model regresji spełnia rolę filtra modelowanej zmienności popytu na usługi telefoniczne. Model SARIMA służy do modelowania pozostałej zmienności. Badania wykazały, że proponowana integracja zapewnia wyższe możliwości aproksymacyjne i predykcyjne w porównaniu z niezintegrowanym modelem regresji liniowej. Wyniki badań stanowią dla operatora wsparcie procesu planowania operacyjnego.

Słowa kluczowe: System Wspomagania Decyzji, regresja zero-jedynkowa, model SARIMA, prognozowanie.
### Appendix

Table A1. The results of the multiple regression estimation

<table>
<thead>
<tr>
<th>Variable</th>
<th>Standardised parameters and errors</th>
<th>Non-standardised parameters and errors</th>
</tr>
</thead>
<tbody>
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<td></td>
<td>$\beta$</td>
<td>Std. error of $\beta$</td>
</tr>
<tr>
<td>BUS</td>
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<td>0.01798</td>
</tr>
<tr>
<td>IND</td>
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</tr>
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<td>MN</td>
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<tr>
<td>OC</td>
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</tr>
<tr>
<td>W</td>
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<tr>
<td>11PM</td>
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<td>0.01443</td>
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* denote significance at 5% level.
Figure A1. Normal probability plot of the regression residuals