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## Analysis of $\beta$ -Convergence. From Traditional Cross-Section Model to Dynamic Panel Model

**A b s t r a c t.** The aim of the paper is to discuss the course of development of methodology of economic convergence analyses, which points up the necessity of taking into consideration spatial connections among regions in regional growth models. It presents empirical models of  $\beta$ -convergence concerning the economic growth of European regions using various methodological conceptions. In the paper the models offered by spatial econometrics are recommended. The empirical data refer to per capita GDP across the European Union regions at a NUTS-2 level over the period 1995–2009 (annual data).

**Key words:** economic convergence, spatial effects, connectivity matrix, spatial panel models.

**J E L Classification:** C52.

### Introduction

The empirical analyses of  $\beta$ -convergence described in the growth literature can be divided into two parts. The first part contains the analyses which use cross-sectional data, while the second one refers to the analyses based on pooled time series and cross-sectional data. Among other things the works of Mankiw, Romer and Weil (1992), Barro and Sala-i-Martin (1995) can be found within the classic literature on cross-sectional data models of convergence, while Islam (1995) is a representative for the second part of the growth literature. The theoretical framework for these analyses of growth

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and convergence is the neo-classical Solow-Swan model (Solow, 1956; Swan, 1956). The Solow-Swan model also forms the base of many modern analyses in this domain.

The spatial and spatio-temporal econometrics points out that for more precise explanation of economic growth it is necessary to take into consideration spatial connections among economics as the connected economies' income levels are interdependent.

Elhorst, Piras and Arbia (2010) emphasize that the hypothesis that the relative location of an economy affects economic growth has been underpinned by theoretical extensions of the Solow-Swan model and confirmed by numerous and vast empirical analyses using mostly cross-sectional data. Exemplifying literature is as follows: Le Gallo, Ertur and Baumont (2003), López-Bazo, Vayá and Artis (2004), Abreu, de Groot and Florax (2005), Rey and Janikas (2005), Arbia (2006), Bode and Rey (2006), Fingleton and López-Bazo (2006), Ertur and Koch (2007), Rey and Le Gallo (2009). Up to now there have been fewer empirical analyses which support the hypothesis using panel data. The examples are: Badinger, Müller and Tondl (2004), Elhorst, Piras and Arbia (2010).

This paper presents a review of fundamental conceptions of verifying the hypothesis of  $\beta$ -convergence, starting with the traditional regional cross-sectional data model, through the models which include the spatial connections among the regions and the panel data models without the spatial effects, concluding on the spatial panel data models. The considerations presented are a continuation of the previous works by Górna, Górna and Szulc (2013, 2014).

The contents of the successive sections of the paper are as follows: in Section 2 the subject and range of the investigation are defined as well as the aim of the paper and the research hypothesis are formulated. Section 3 characterizes the data used in the investigation. Section 4 presents the methodology. In this section the theoretical models of  $\beta$ -convergence in formulation of the cross-section regressions and the regressions for the pooled time series and cross-sectional data are presented. Moreover, in Section 4 the diagnostic tests for verification of the empirical models are pointed out. The results of the analysis are presented in Section 5. Recapitulation contains final conclusions and indicates further investigations.

## 1. Subject and Range of the Investigation

The paper concerns the phenomenon of economic convergence. It presents changes of spatial differentiation of per capita incomes across the Eu-

ropean Union regions at a NUTS-2 level over the period 1995–2009. The question is about validity of economic convergence hypothesis in the area of the European countries in the investigated period. In the paper the methods of verification of the hypothesis are considered.

As a result of the analysis the empirical models of  $\beta$ -convergence concerning the economic growth of the European regions were obtained. To achieve it various methodological conceptions, especially the models offered by spatial econometrics, have been used. The attention was paid to the so-called absolute  $\beta$ -convergence (Baumol, 1986; De Long, 1988; Arbia, 2006). In the constructed models of convergence apart from per capita GDP in the initial (basic) period no additional variable explaining the state of economies was taken into consideration. On the so-called conditional  $\beta$ -convergence in classic version, see e.g. Bal-Domańska (2010, 2011). On the contrary Elhorst, Piras and Arbia (2010) is an example of the spatial approach.

The aim of the investigation is to show that the empirical spatial models of convergence for cross-sectional data as well as the spatial panel data models have better statistical properties than the models which ignore the spatial and spatio-temporal connections among the regions. In addition, they allow more precise interpretation of the model parameters. The models presented are used to verify the hypothesis that relative location of a region affects the economic growth rate of the region.

## 2. Data

In the investigation the data on per capita GDP for 261 regions of 27 European countries were used. The data refer to the period of 1995–2009, i.e. to 15 years. They describe the per capita GDP spatial distributions and dynamics of incomes in the European Union and come from Eurostat database ([ec.europa.eu/eurostat/](http://ec.europa.eu/eurostat/)). All calculations were prepared with the use of R (versions 2.7.2 and 3.0.1).

Figure 1 presents the spatial distribution of per capita GDP values (expressed by log terms) at the beginning of 1995–2009 period, i.e. in the year 1995 (Figure 1a) and the analogical distribution in the year 2009 (Figure 1b). It can be noticed that the spatial regularities of the distributions did not change in the period investigated. However, comparing the spatial trends, which have been fitted to the data and presented in Figure 2, it can be seen that the surface of the trend for 2009 is flatter than the surface for 1995. This fact seems to confirm the supposition that economic convergence of the European regions occurs in the investigated period.

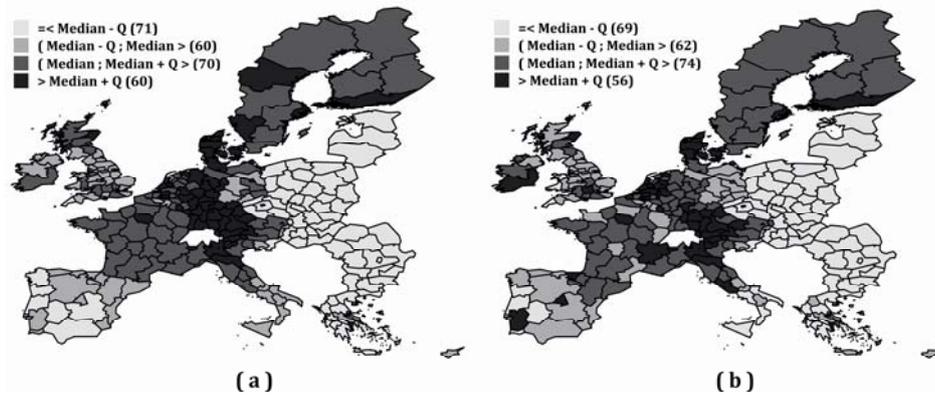


Figure 1. Distribution of the per capita GDP in the European regions: (a) in the year 1995, (b) in the year 2009

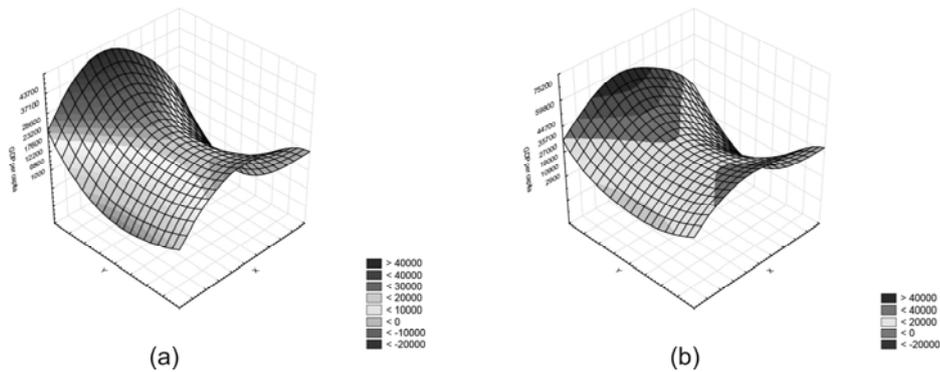


Figure 2. Trend surfaces of per capita GDP for the European regions: (a) in the year 1995, (b) in the year 2009

Then, Figure 3 compares the spatial distribution of per capita GDP in the year 1995 with the growth rates of GDP across the regions during the period 1995–2009. This comparison shows that the poorer, at the beginning, regions have faster growth rates than the richer ones. Thus, the economic convergence of the regions in the period considered is probable. This conclusion is conformable to the one formulated above. Moreover, the next Figure 4 which presents the surfaces of regions' per capita GDP (expressed by log terms) and of regions' per capita GDP growth rates (also expressed by log terms), seems to substantiate the supposition that the convergence is possible as well. The tendencies in Figure 4 in parts a) and b) are inverted. Additionally, Figure 5 presents the average annual growth rate of GDP across the

regions of the EU over the investigated period (the map of spatial distribution of the growth rates and the trend surface of them respectively).

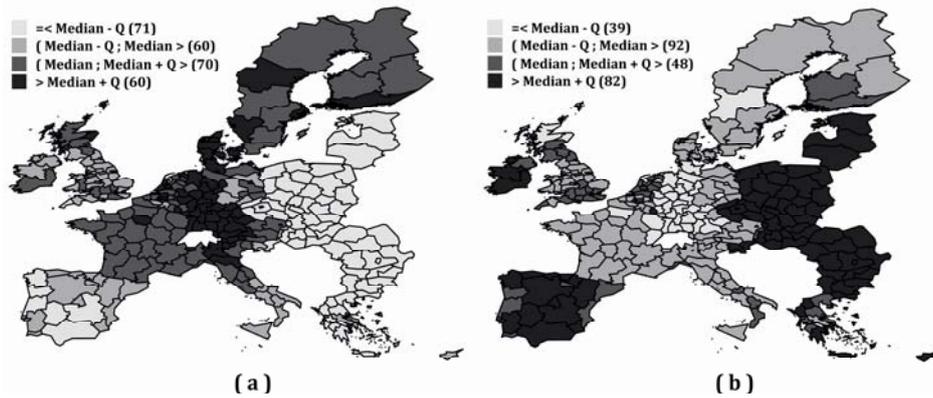


Figure 3. Distributions of the per capita GDP: (a) in the year 1995, (b) growth rates during the period 1995–2009, in the European regions

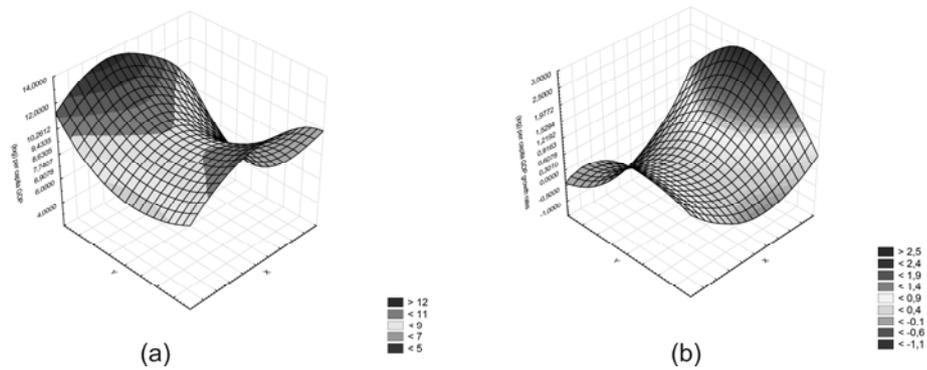


Figure 4. Spatial distributions – trend surfaces: (a) of regions' (log) per capita GDP in the year 1995, (b) of regions' (log) per capita GDP growth rates during the period 1995–2009

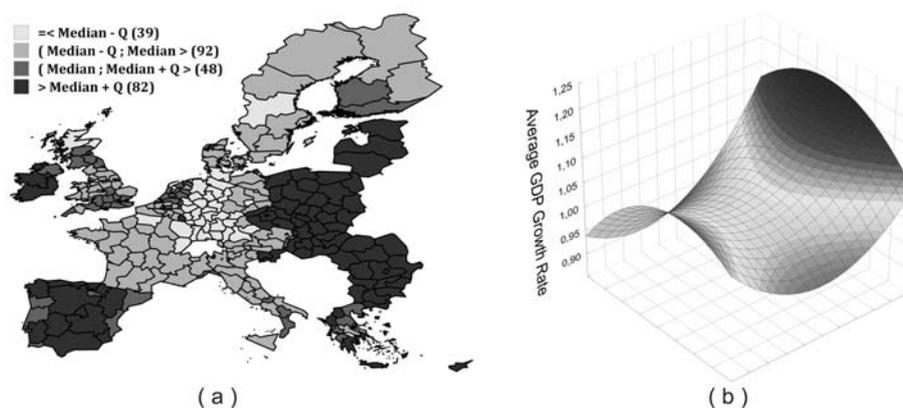


Figure 5. Map and trend surface of average annual growth rate of GDP across the European regions

### 3. Methodology

The values of per capita GDP in the area established are treated as realizations of spatial stochastic process  $Z(\mathbf{s}_i)$ , where:  $\mathbf{s}_i = [x_i, y_i]$  – location coordinates on the plane,  $i = 1, 2, \dots, N$  – spatial units (regions) or of spatio-temporal stochastic process  $Z(\mathbf{s}_i, t)$ , where  $i$  – as above and  $t = 1, 2, \dots, T$  – the successive years in the period considered. The cross-sectional data or the pooled time series and cross-sectional data are used. Each observation is connected with the established location on the plane within some structure of connections among the spatial units. The structure is quantified by spatial connectivity matrix  $\mathbf{W}$ . The matrix  $\mathbf{W}$  has as many rows and columns as there are the regions. Each row of the matrix contains non-zero elements in columns which correspond to the connected regions (the so-called neighbours), according to the established criterion ( $w_{ij} \neq 0$ ). Furthermore, the given region cannot be connected to itself, i.e. it cannot be a neighbour of itself, so  $w_{ij} = 0$  for all  $i = j$ . Thus, the diagonal elements of  $\mathbf{W}$  are zeros.

The successive specifications of  $\beta$ -convergence models are considered. The classical model of  $\beta$ -convergence using data in cross-section takes the form:

$$\ln \left[ \frac{GDP_{iT}}{GDP_{i1}} \right] = \alpha + \beta \ln [GDP_{i1}] + \varepsilon_i. \quad (1)$$

The spatial cross-sectional data model of  $\beta$ -convergence which contains the spatially lagged dependent variable (Spatial Lag Model – SLM) can be written as follows:

$$\ln\left[\frac{GDP_{iT}}{GDP_{i1}}\right] = \alpha + \beta \ln[GDP_{i1}] + \rho \sum_{j \neq i} w_{ij} \ln\left[\frac{GDP_{jT}}{GDP_{j1}}\right] + \varepsilon_i. \quad (2)$$

Model (2) belongs to the class of the spatial autoregressive models (SAR). In turn, the cross-section model of  $\beta$ -convergence with the spatially autocorrelated error component (Spatial Error Model – SEM) takes the form:

$$\ln\left[\frac{GDP_{iT}}{GDP_{i1}}\right] = \alpha + \beta \ln[GDP_{i1}] + \eta_i, \quad \eta_i = \lambda \sum_{j \neq i} w_{ij} \eta_j + \varepsilon_i. \quad (3)$$

The use of cross-section regressions (1)–(3) for the investigation of economic convergence is connected with the loss of the information on variability of economies in time. This also means that in the analysis the individual economies’ features are omitted. Besides, the model (1) ignores the spatial connections among the economies which appear important for describing the economic growth (see empirical characteristics of models (2)–(3)).

Below, the spatial models of  $\beta$ -convergence for pooled time series and cross-sectional data (TSCS) are considered.

Including into the model TSCS a spatial component leads to the following specifications:

1. The spatial autoregressive model (SAR\_pooled)

$$\ln\left[\frac{GDP_{it}}{GDP_{it-1}}\right] = \alpha + \beta \ln[GDP_{it-1}] + \rho \sum_{j \neq i} w_{ij} \ln\left[\frac{GDP_{jt}}{GDP_{jt-1}}\right] + \varepsilon_{it}. \quad (4)$$

2. The model with spatial autoregressive residuals (SE\_pooled)

$$\ln\left[\frac{GDP_{it}}{GDP_{it-1}}\right] = \alpha + \beta \ln[GDP_{it-1}] + \eta_{it}, \quad \eta_{it} = \lambda \sum_{j \neq i} w_{ij} \eta_{jt} + \varepsilon_{it}. \quad (5)$$

Pooled spatial and temporal data create the so-called panel data. Methodology of the panel data analysis suggests including into the model individual or/and time effects which can be taken into consideration as fixed – and then they can be estimated – or as random becoming the part of the error component.

As much as in the cross-sectional data growth models also in the panel data models more and more frequently the spatial connections among the

regions are taken into account. It is caused by the opinion that the growth rate of any region is connected with the growth rates of its neighbours.

The spatial panel models for verification of  $\beta$ -convergence hypothesis are as follows:

1. The spatial autoregressive panel model with individual fixed effects (the spatial autoregressive fixed-effects model) (SAR\_FE\_IND)

$$\ln \left[ \frac{GDP_{it}}{GDP_{it-1}} \right] = \alpha_i + \beta \ln[GDP_{it-1}] + \rho \sum_{j \neq i} w_{ij} \ln \left[ \frac{GDP_{jt}}{GDP_{jt-1}} \right] + \varepsilon_{it}. \quad (6)$$

2. The spatial autoregressive panel model with individual and time fixed effects (SAR\_FE\_TWO-WAY)

$$\ln \left[ \frac{GDP_{it}}{GDP_{it-1}} \right] = \alpha_i + \gamma_t + \beta \ln[GDP_{it-1}] + \rho \sum_{j \neq i} w_{ij} \ln \left[ \frac{GDP_{jt}}{GDP_{jt-1}} \right] + \varepsilon_{it}. \quad (7)$$

3. The spatial error panel model with individual fixed effects (SE\_FE\_IND)

$$\ln \left[ \frac{GDP_{it}}{GDP_{it-1}} \right] = \alpha_i + \beta \ln[GDP_{it-1}] + \eta_{it}, \quad \eta_{it} = \lambda \sum_{j \neq i} w_{ij} \eta_{jt} + \varepsilon_{it}. \quad (8)$$

4. The spatial error panel model with individual and time fixed effects (SE\_FE\_TWO-WAY)

$$\ln \left[ \frac{GDP_{it}}{GDP_{it-1}} \right] = \alpha_i + \gamma_t + \beta \ln[GDP_{it-1}] + \eta_{it}, \quad \eta_{it} = \lambda \sum_{j \neq i} w_{ij} \eta_{jt} + \varepsilon_{it}. \quad (9)$$

5. The spatial autoregressive panel model with individual random effects (SAR\_RE\_IND)

$$\ln \left[ \frac{GDP_{it}}{GDP_{it-1}} \right] = \alpha_0 + \beta \ln[GDP_{it-1}] + \rho \sum_{j \neq i} w_{ij} \ln \left[ \frac{GDP_{jt}}{GDP_{jt-1}} \right] + \zeta_{it}, \quad (10)$$

$\zeta_{it} = \alpha_i + \varepsilon_{it}$ , or  $\zeta_{it} = \alpha_i + \gamma_t + \varepsilon_{it}$  (in the case SE\_RE\_TWO-WAY).

6. The spatial error panel model with individual random effects (SE\_RE\_IND)

$$\ln \left[ \frac{GDP_{it}}{GDP_{it-1}} \right] = \alpha_0 + \beta \ln[GDP_{it-1}] + \zeta_{it}, \quad (11)$$

$$\zeta_{it} = \alpha_i + \eta_{it}, \quad \eta_{it} = \lambda \sum_{j \neq i} w_{ij} \eta_{jt} + \varepsilon_{it},$$

and in the model with individual and time random effects (SE\_RE\_TWO-WAY),  $\zeta_{it} = \alpha_i + \gamma_t + \eta_{it}$ .

Economic convergence is said to be confirmed by the data if the estimates of  $\beta$  coefficients in models (1)–(11) are negative and statistically significant. Furthermore, if parameters  $\rho$  in models: (2), (4), (6), (7), (10) and parameters  $\lambda$  in models: (3), (5), (8), (9), (11) are significantly different from zero, then in the convergence process the spatial connections among economies are important, and the hypothesis that the rate of growth of any economy is related to that of its neighbours is confirmed.

As a result of explicit including the components of spatial dependence into the economic growth model is the evaluation of the convergence phenomena on the ground of  $\beta$  parameter estimated better than in the traditional approach. Then the estimate of the parameter will not be influenced by omitting the dependence and it will more precisely reflect the influence of the per capita GDP in the basic period on the growth rate of incomes.

In order to evaluate the quality of the empirical models in the investigation the following tools were used: the Moran test (Moran's  $I$ ) for spatial independence of the residuals, the Lagrange Multiplier tests (LMlag, LMerr) and their robust versions (RLMlag, RLMerr) as spatial dependence diagnostics, the Likelihood Ratio test (LR) for testing the significance of the spatial dependence, the Breusch-Pagan heteroskedasticity test, the Chow test for spatial invariance of  $\beta$ -convergence parameters and for verifying the need of including into the spatial panel models the fixed effects, the Lagrange Multiplier tests to verify the spatial interactions and the random effects, the Hausman test for choosing between the fixed-effects (FE) model and the random-effects (RE) model (on the tools see e. g. Arbia, 2006; Millo and Piras, 2012; Mutl and Pfaffermayr, 2011; Baltagi et al., 2003; Suhecki (ed.), 2012).

## 5. Results

The successive tables presented below contain the information on the usefulness of various methodological conceptions expressed by the model specifications presented in Section 4. The part of the information which refers to the models estimated on the ground of cross-sectional data is also presented in the works: Góna, Góna and Szulc (2013, 2014), which unlike this paper are limited to the  $\beta$ -convergence analysis by using cross-section regressions.

Table 1 contains the results of estimation and verification of three models: the linear regression model, i.e. the traditional model without the spatial effects, the spatial autoregressive model (SAR) and the spatial error model (SE).

Table 1. The results of estimation and verification of the cross-sectional data models of  $\beta$ -convergence

	Linear regression	Spatial autoregressive model	Spatial error model
Parameters			
$\alpha$	3.8739 (0.0000)	2.8602 (0.0000)	3.6383 (0.0000)
$\beta$	-0.3535 (0.0000)	-0.2618 (0.0000)	-0.3269 (0.0000)
$\rho$	–	0.2860 (0.0000)	–
$\lambda$	–	–	0.4548 (0.0000)
Goodness of fit			
Adjusted R2	0.7642	–	–
AIC	-166.71	-191.86	-195.33
Heteroskedasticity Breusch-Pagan test	11.0877 (0.0009)	13.1741 (0.0003)	4.0292 (0.0450)
Autocorrelation of residuals			
Moran test	5.4531 (0.0000)	2.0380 (0.0416)	-0.0946 (0.0753)
Spatial dependence			
LR	–	27.1540 (0.0000)	30.6230 (0.0000)
LMlag	26.3535 (0.0000)	–	–
LMerr	29.7572 (0.0000)	–	–
RLMlag	–	4.0923 (0.0431)	–
RLMerr	–	–	7.4940 (0.02654)
Speed of convergence	0.0291	0.0202	0.0264
Half-life	23.8477	34.2531	26.2654

Note: numbers in brackets refer to the p-values.

Diagnostics for the models considered suggest that the classical model is the worst of them. This result is conformable to our anticipation because the

assumptions of the model, especially the same variance in the space and independence across residuals for all regions, are usually unrealistic in practice. In this case, the Breusch-Pagan statistic is significant, leading to rejecting the model assumption of homoskedasticity. In addition, on the basis of the Moran's  $I$  test it is necessary to state that the hypothesis of independence of the traditional model residuals should be rejected.

As the Moran test does not admit an explicit alternative hypothesis opposed to the null, the Lagrange Multiplier tests (LM) were used (see Table 1). The LM tests for the linear model used consider the spatial lag model (spatial autoregressive) and the spatial error model as alternatives (LMlag and LMerr, respectively). Table 1 reports the results of using the robust tests (RLMlag, in which  $H_0: \rho = 0$  under the assumption that  $\lambda \neq 0$  and RLMerr, where  $H_0: \lambda = 0$  under the assumption that  $\rho \neq 0$ ) as well. Subsequently, the significance of the spatial effects in SAR and SE models using the Likelihood Ratio test (LR) was confirmed.

Taking into account the spatial connections across the European regions at a NUTS-2 level over the period 1995–2009, in the  $\beta$ -convergence models has removed the problem of autocorrelation of the residuals (at the level of significance  $\gamma = 0.01$ ). However, the problem of variance heteroskedasticity has remained, especially in the case of the spatial autoregressive model. The spatial heteroskedasticity can be caused by omitting the factor responsible for systematic spatial variability. In this connection in Gónna, Gónna and Szulc (2014) the additional analysis searching for the spatial regimes was performed. For this purpose the considered area of the regions has been divided into two sub-areas (see Figure 6). To justify the division the Chow test for verifying the spatial changeability of  $\beta$  parameters was used (see also Arbia, 2006, p. 133). Table 2 contains the results of the test.

Table 2. Results of the test for spatial invariance of the  $\beta$ -convergence parameters

Chow test	Linear regression	Spatial lag model	Spatial error model
Values of test	8.2701	12.1347	18.9393
p-value	0.0407	0.0069	0.0003

The hypothesis of constancy of parameters in  $\beta$ -convergence models estimated in the cited investigation should be rejected. This leads to identification of spatial regimes and means that in the considered area as a whole there are differentials in relationships between regional growth rate and initial per capita GDP. The models of  $\beta$ -convergence according to the sub-areas considered were estimated and verified and the differentiation of the parame-

ters of  $\beta$ -convergence as well as of properties of the empirical models obtained were stated (for the details, see Górna, Górna and Szulc, 2014).

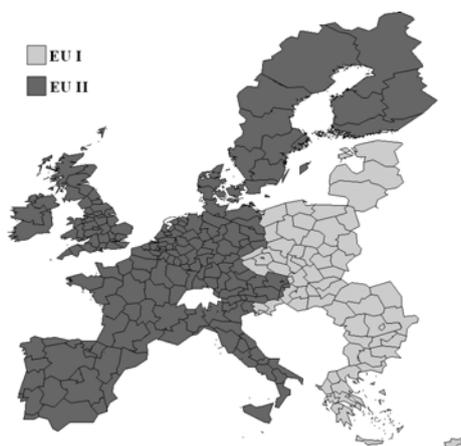


Figure 6. Classification of the EU regions within the two spatial regimes

The classical model estimated with the use of the pooled time series and cross-sectional data like the classical cross-section regression does not satisfy the fundamental criteria of statistical verification.

For the purpose of spatial effects verification in the models for pooled time series and cross-sectional data the Lagrange Multiplier tests analogous to those applied in the case of the cross-section regressions are used. The results of them have confirmed the need of re-specifications towards the models with the spatial effects (see Table 3). Moreover, the significance of the effects with the aid of the LR test was confirmed.

According to the methodology of panel data modeling in the investigation the reasonableness of including into the spatial models the fixed or/and the random effects was considered. For this purpose the Chow test (the spatial model for pooled TSCS data vs. the spatial panel model with fixed effects) and the LM tests (to verify spatial interactions and random effects) were used. The results of the Chow test have pointed out the statistical significance of the fixed effects in the spatial autoregressive panel model but not in the panel spatial error model. On the basis of the joint LM test, which tested the hypothesis:  $H_0: \lambda = \sigma_\alpha^2 = 0$  under the alternative that at least one component was not zero, it was stated that the null should be rejected.

Table 3. Results of estimation and verification of  $\beta$ -convergence models for pooled time series and cross-sectional data

	Linear regression	Spatial autoregressive model	Spatial error model
Parameters			
$\alpha$	0.3558 (0.0000)	0.3132 (0.0000)	0.3334 (0.0000)
$\beta$	-0.0326 (0.0000)	-0.0299 (0.0000)	-0.0303 (0.0000)
$\rho$	-	0.4044 (0.0000)	-
$\lambda$	-	-	0.4250 (0.0000)
Goodness of fit			
Adjusted R2	0.1439	-	-
AIC	-9442	-9974.6	-9981.7
Heteroskedasticity			
Breuch-Pagan test	184.5378 (0.0000)	243.7477 (0.0000)	250.5849 (0.0000)
Autocorrelation of residuals			
Moran test	26.7685 (0.0000)	-2.1504 (0.0158)	-3.333 (0.0304)
Spatial dependence			
LR	-	534.61 (0.0000)	541.73 (0.0000)
LMlag	702.5570 (0.0000)	-	-
LMerr	713.3021 (0.0000)	-	-
RLMlag	-	12.5044 (0.0004)	-
RLMerr	-	-	23.2494 (0.0000)
Speed of convergence			
Half-life	0.0331	0.0304	0.0308
	20.82	22.73	22.43

Note: numbers in brackets refer to the p-values.

Next, in the investigation the so-called marginal LM tests for verification of the hypotheses:  $H_0: \lambda = 0$  (under the assumption that  $\sigma_\alpha^2 = 0$ ) and  $H_0: \sigma_\alpha^2 = 0$  (assuming  $\lambda = 0$ ), were used respectively. In both of the cases the result of the test has pointed out the lack of the bases for rejecting the null. Therefore, the conditional LM tests, which are more useful tests in this framework, because they test for one effect, and are robust against the other ( $H_0: \lambda = 0$

assuming  $\sigma_\alpha^2 = 0$  or  $\sigma_\alpha^2 \neq 0$ ;  $H_0: \sigma_\alpha^2 = 0$  with  $\lambda = 0$  or  $\lambda \neq 0$ ), confirmed that the interactions of spatial and random effects were possible.

The Hausman test was used to choose between the FE and RE models. It has suggested the choice of the spatial panel model with the fixed effects.

Table 4. Selected characteristics of spatial panel models

Parameter	SAR_FE_IND		SAR_RE_IND		SE_FE_IND		SE_RE_IND	
	Estimate of parameter	Statistic t	Estimate of parameter	Statistic t	Estimate of parameter	Statistic t	Estimate of parameter	Statistic t
$\alpha$	–	–	0.319	26.23	–	–	0.335	26.57
$\beta$	-0.064	-16.08	-0.030	-24.10	-0.083	-16.13	-0.030	-23.50
$\rho$	0.340	19.42	0.344	20.24	–	–	–	–
$\lambda$	–	–	–	–	0.349	19.62	0.359	19.60
Speed of convergence	0.0658		0.0308		0.0865		0.0309	
Half-life	10.4832		22.4255		7.9733		22.3506	
Chow test F	121.4659 (0.0000)				-331.9926 (1.0000)			
Hausman test					74.5562 (0.0000)			
LM tests	Joint LM (Random effects and spatial autocorrelation as alternative hypothesis): 460.0374; p-value=0.0000							
	Marginal LM:							
	LM1 (Random effects as alternative hypothesis): -0.0005; p-value =1							
	LM2 (Spatial autocorrelation as alternative hypothesis): 0.0064; p-value=0.9949							
	Conditional LM (assuming $\sigma_\alpha^2 \geq 0$ ; Spatial autocorrelation as alternative hypothesis): 24.5265; p-value=0.0000							

Note: numbers in brackets refer to the p-values.

## Conclusions

On the ground of the analysis the following conclusions can be formulated:

1. The fact of including the components of spatial dependence (the spatially lagged dependent variable or the spatial autoregressive error component) in the economic convergence model is justified and valid for the analyses of per capita incomes across the regions investigated and of income changes in time.
2. As a result of that it is possible to define the influence of the neighbour connections on the economic growth, the estimate of parameter  $\beta$  is more precise, and properties of the model are better.

3. Like in the growth models for the cross-sectional data in the panel data models it is necessary to take into consideration the spatial connections among the regions.
4. The dynamic panel models with the spatial effects are the natural extension of cross-section regressions as the tools of verifying hypothesis of economic convergence.

In further investigations the models of convergence with the additional explanatory variables and also with these variables spatially lagged are to be considered. Additionally, more diagnostic tests for the spatial panel models are expected to be used.

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### Analiza $\beta$ -konwergencji. Od tradycyjnego modelu przekrojowego do dynamicznego modelu panelowego

**Zarys treści.** Celem artykułu jest omówienie kierunku rozwoju metodologii badania konwergencji gospodarczej, wskazującego na potrzebę uwzględniania w modelach wzrostu regionów powiązań przestrzennych między nimi. Artykuł prezentuje empiryczne modele  $\beta$ -konwergencji dotyczące wzrostu gospodarczego regionów Europy, otrzymane przy wykorzystaniu różnych koncepcji metodologicznych. W artykule rekomenduje się modele z zakresu ekonometrii przestrzennej. Dane empiryczne dotyczą PKB per capita w regionach NUTS-2 27 państw europejskich, będących członkami Unii Europejskiej. Zakres czasowy analizy obejmuje lata 1995–2009 (dane roczne).

**Słowa kluczowe:** konwergencja gospodarcza, efekty przestrzenne, macierz sąsiedztwa, przestrzenny model panelowy.

