Małgorzata Doman, Ryszard Doman*

The Dynamics and Strength of Linkages between the Stock Markets in the Czech Republic, Hungary and Poland after their EU Accession**

Abstract. We analyze the dynamics and strength of linkages between the Czech, Hungarian and Polish stock markets after the EU accession of the corresponding countries. In addition, we examine linkages between each of the markets and developed markets (European and US). The analysis is based on the daily quotations of the main representative stock indices (PX, BUX, WIG20, DAX, S&P 500) and includes the period from May 5, 2004 to July 20, 2012. The dynamics of dependencies is modeled by means of Markov-switching copula models, and the applied measures of the strength of the linkages are dynamic Spearman’s rho and tail dependence coefficients. The results show that dependencies between the considered emerging markets are very sensitive on market situation, but the linkages of these markets with the developed ones are stable.

Keywords: Central European stock market, conditional dependence, Markov-switching copula model, Spearman’s rho, tail dependence, model confidence set, stock index.

JEL Classification: G15; G01; C32; C58.

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Introduction

In the paper, we analyze the dynamics and strength of linkages between the Czech (Prague Stock Exchange, PSE), Hungarian (Budapest Stock Exchange, BSE) and Polish (Warsaw Stock Exchange, WSE) stock markets after the EU accession of the corresponding countries. In addition, we examine the linkages between each of the markets and developed markets (European and US). The analysis is based on the daily quotations of the main representative stock indices (PX, BUX,WIG20, DAX, S&P 500). The dynamics of dependencies is modeled by means of Markov-switching copula models and the applied measures of the strength of the linkages are dynamic Spearman’s rho and tail dependence coefficients. We investigate the period from May 5, 2004 to July 20, 2012. It begins shortly after the three considered Central European countries joined the European Union and includes subperiods of two recent financial crises (the US subprime and European debt crises).

The Czech Republic, Hungary and Poland are often perceived by investors as one block of very similar markets. In particular, bad news concerning one of these countries strongly impacts all the markets. The history of the three economies looks similar. In 1989 all they started the transformation process, in May 2004 joined the European Union, and now all they are getting ready to adopt the euro. Nevertheless, their specific paths of development differ from each other. The impact of the recent crisis on the economies, measured with the evolution of the GDP annual growth rate, was dissimilar to that exerted on the stock markets (see Federation of European Securities Exchange, 2013; Trading Economics, 2013). The stock markets themselves are unlike each other as well. The Warsaw Stock Exchange plays the special role in the region as the largest stock exchange market (see Federation of European Securities Exchange, 2013). A comparison of the quotations of the representing stock indices with the evolution of GDP annual growth rate in the considered countries (see Federation of European Securities Exchange, 2013; Trading Economics, 2013) shows that during the subprime crisis of 2007–2009 all the markets experienced significant decline, and the same applies to their GDP growth rate, though the Polish GDP growth rate remained positive.

There exists a quite extensive literature on linkages between the three Central European stock markets. Many investigations were performed for the period before the recent crises (see for instance Scheicher, 2001; Gilmore et al., 2008; Caporale, Spagnolo, 2011). Some newer papers on the subject, mostly concerning the impact of the recent crises, can also be found.
instance, Ülkü and Demirci (2012) present a thorough analysis of the joint
dynamics of foreign exchange and stock markets in emerging European
countries for the period 2003–2010 and document the key role of global
developed market returns in driving the stock market–exchange rate interac-
tion in emerging economies. Their study uses impulse response functions
from a cointegrating structural vector autoregressive model as the research
tool. Another methodology, involving a dynamical conditional correlation
model, has been applied by Kiss (2011) who examines the transmission
mechanism of interbank, stock and currency market crashes for the Czech
Republic, Hungary and Poland as well as for the US and Euro area, using
daily data from January 2002 to October 2010.

The main novelty in our analysis is a detailed description of the dynam-
ics of two types of dependence between the considered markets: an average
dependence measured by Spear-man’s rho, and tail dependence estimated by
means of upper and lower tail dependence coefficients. Unlike Pearson’s
correlation, which is invariant only under increasing affine trans-formations
of the margins of a bivariate random vector, the dependence measures ap-
plied by us are invariant with respect to strictly increasing transformations of
the margins. Hence, the measures are more suitable in detecting dependen-
cies of non-linear type between analyzed stock returns. In fact, we use
a copula regime-switching approach to model the conditional bivariate dis-
tributions of the returns. So, the aforementioned properties of the applied
dependence measures can be drawn from the fact that the measures are cop-
ula-based. What is more, using copulas models allows to leave behind the
class of elliptical distributions. This is because, in this approach, description
of the behavior of the marginals is separated from model-ing their depend-
ence structure. The advantages of our approach stem also from the fact that
the tail dependence coefficients reflect the dependence in extreme values,
which is very fruitful in describing the market linkages during crisis periods.
Additionally, a Markov regime switching mechanism we use provides
a possibility to model the temporal fluctuation of the strength and character-
istics of the modeled dependencies. We also compare the estimates of the
measures of the dependencies between the Czech, Polish and Hungarian
stock markets with the corresponding quantities obtained for the dependen-
cies between these markets and Western European and American stock mar-
kets represented by the stock indices DAX and S&P 500. Our results show
that the dynamics of dependencies in the two cases are completely different,
and that the dependencies between the emerging markets are more sensitive
to new information flow. However, the linkages of these markets with the
main world markets are rather stable.
1. Markov-Switching Copula Models

Modeling the dynamics of dependencies between financial returns is not an easy task because of interaction between the volatility of the individual returns and the dependence structure. The interaction is especially apparent during the periods of financial crises and turbulence in financial markets (Ang, Bekaert, 2002; Forbes, Rigobon, 2002). In such circumstances, the investigation of the dynamics of the linkages becomes more difficult due to different types of asymmetries and structural breaks which are likely to arise (Ang, Bekaert, 2002; Ang, Chen, 2002; Patton, 2004). From the point of view of the choice of a suitable multivariate statistical model for the conditional distribution of the returns, this means that elliptical distributions should hardly be assumed. As a consequence, multivariate GARCH models (see a survey by Bauwens et al., 2006) can be an inappropriate tool for modeling the dynamics of linkages. In such a situation, a concept of copula can provide an alternative solution.

A bivariate copula is a mapping \( C: [0,1] \times [0,1] \rightarrow [0,1] \) from the unit square into the unit interval, which is a distribution function with standard uniform marginal distributions.

Assume that \((X,Y)\) is a 2-dimensional random vector with joint distribution \(H\) and marginal distributions \(F\) and \(G\). Then, by a theorem by Sklar (1959), \(H\) can be written as:

\[
H(x, y) = C(F(x), G(y)).
\]

If \(F\) and \(G\) are continuous then the function \(C\) is uniquely given by the formula:

\[
C(u,v) = H(F^{-1}(u), G^{-1}(v)),
\]

for \(u,v \in [0,1]\), where \(F^{-1}(u) = \inf \{ x : F(x) \geq u \} \) (Nelsen, 2006). In this case, \(C\) is called the copula of \(H\) or of \((X,Y)\). Since the marginal distributions in the decomposition (1) are separated, it makes sense to interpret \(C\) as the dependence structure of the vector \((X,Y)\). We refer to Patton (2009) and references therein for an overview of applications of copulas in analysis of financial time series.

The simplest copula is defined by \(C^{\Pi}(u,v) = uv\) and it corresponds to independence of marginal distributions. In the empirical part of this paper we will also use the Gaussian, Clayton, and Joe-Clayton copulas. They are defined as follows:
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\[ C^\text{Gauss}_{\rho}(u,v) = \Phi_{\rho}\left(\Phi^{-1}(u), \Phi^{-1}(v)\right), \]

\[ C^\text{Clayton}_{\gamma}(u,v) = (u^{-\gamma} + v^{-\gamma} - 1)^{-1/\gamma}, \]

\[ C^\text{Joe-Clayton}_{\kappa,\gamma}(u,v) = 1 - \left(1 - (1 - (1 - u)^{\kappa})^{-\gamma} + [1 - (1 - v)^{\kappa}]^{-\gamma} - 1\right)^{-1/\gamma}, \]

where \( \Phi_{\rho} \) denotes the distribution of a 2-dimensional standardized normal vector with the linear correlation coefficient \( \rho \), \( \Phi \) stands for the standard normal distribution function, and \( \kappa \geq 1, \gamma > 0 \). The Clayton copula \( C^\text{Clayton}_{\gamma} \) is a special case of the Joe-Clayton copula \( C^\text{Joe-Clayton}_{\kappa,\gamma} \) for \( \kappa=1 \). In the limit case \( \gamma=0 \), the Clayton copula approaches the independent copula \( C^\Pi \) (Nelsen, 2006).

The density associated to an absolutely continuous copula \( C \) is a function \( c \) defined by:

\[ c(u,v) = \frac{\partial^2 C(u,v)}{\partial u \partial v}. \]

For an absolutely continuous random vector, the copula density \( c \) is related to its joint density function \( h \) by the following canonical representation:

\[ h(x,y) = c(F(x),G(y))f(x)g(y), \]

where \( F \) and \( G \) are the marginal distributions, and \( f \) and \( g \) are the marginal density functions.

In the case of nonelliptical distributions, the most well-known copula-based dependence measures, which are more appropriate than the linear correlation coefficient, are Kendall’s tau and Spearman’s rho (Embrechts et al., 2002). Since the dynamics of Spearman’s rho is exploited in this paper, we recall a suitable definition. If \( (X,Y) \) is a random vector with marginal distribution functions \( F \) and \( G \), then Spearman’s rho for \( (X,Y) \) can be defined as:

\[ \rho_S(X,Y) = \rho(F(X),G(Y)), \]

where \( \rho \) denotes the usual Pearson correlation. For continuous marginal distributions, Spearman’s rho, \( \rho_S(X,Y) \), depends only on the copula \( C \) linking \( X \) and \( Y \), and, in particular, is given by the formula:

\[ \rho_S(X,Y) = \rho_C = 12\iint_{[0,1]^2} C(u,v)du dv - 3 \]
(Nelsen, 2006). It follows from (9) that if a copula $C$ is a mixture of copulas $C_1$ and $C_2$: $C = \alpha C_1 + (1-\alpha)C_2$, $0 \leq \alpha \leq 1$, then:

$$\rho_C = \alpha \rho_{C_1} + (1-\alpha)\rho_{C_2}. \quad (10)$$

For the Gaussian copula, $C_{Gauss}^\rho$, Spearman’s rho equals $\frac{6}{\pi} \arcsin^2 \rho$. For the Clayton and Joe-Clayton copulas it can be computed numerically using (9).

Another concept of dependence, which depends only on the copula of a random vector $(X,Y)$, is tail dependence. It measures the dependence between extreme values of the variables. If $F$ and $G$ are the cumulative distribution functions of $X$ and $Y$, then the coefficient of upper tail dependence is defined as follows:

$$\lambda_U = \lim_{q \rightarrow 1^-} P(Y > G^{-1}(q) | X > F^{-1}(q)), \quad (11)$$

provided a limit $\lambda_U \in [0,1]$ exists. Analogously, the coefficient of lower tail dependence is defined as:

$$\lambda_L = \lim_{q \rightarrow 0^+} P(Y \leq G^{-1}(q) | X \leq F^{-1}(q)), \quad (12)$$

provided that a limit $\lambda_L \in [0,1]$ exists. If $\lambda_U \in (0,1]$ ( $\lambda_L \in (0,1)$), then $X$ and $Y$ are said to exhibit upper (lower) tail dependence. Upper (lower) tail dependence quantifies the likelihood to observe a large (low) value of $Y$ given a large (low) value of $X$. The coefficients of tail dependence can be expressed in terms of the copula $C$ of $X$ and $Y$ in the following way:

$$\lambda_L = \lim_{q \rightarrow 0^-} \frac{C(q,q)}{q}, \quad (13)$$

$$\lambda_U = \lim_{q \rightarrow 0^+} \frac{C(q,q)}{q}, \quad (14)$$

where $\tilde{C}(u,v) = u + v - 1 + C(1-u,1-v)$. For the Gaussian copula it holds $\lambda_U = \lambda_L = 0$ (see Embrechts et al., 2002), meaning asymptotic independence in the tails. The Clayton copula $C_{Clayton}^\gamma$ has lower tail dependence: $\lambda_L = 2^{-1/\gamma}$. In the Joe-Clayton copula case, $\lambda_U = 2 - 2^{1/\kappa}$ and $\lambda_L = 2^{-1/\gamma}$ for $\gamma > 0$ (Patton, 2006). Thus both upper and lower tail dependence can be nonzero, and, moreover, they can change independently of each other.

In applications of copulas to financial time series the notion of conditional copula introduced by Patton (2004) is usually employed. It allows to
model the joint conditional distribution $\mathbf{r}_t | \Omega_{t-1}$, where $\mathbf{r}_t = (r_{1,t}, r_{2,t})$ is a bivariate vector of financial returns, and $\Omega_{t-1}$ is the information set available up to time $t-1$. In this paper we consider the following general conditional copula model:

$$r_{1,t} | \Omega_{t-1} \sim F_1(\cdot), \quad r_{2,t} | \Omega_{t-1} \sim G_2(\cdot),$$

(15)

$$\mathbf{r}_t | \Omega_{t-1} \sim C_1(F_1(\cdot), G_2(\cdot) | \Omega_{t-1}),$$

(16)

where $C_1$ is the conditional copula linking the marginal conditional distributions, and the information set $\Omega_{t-1}$ remains the same for the copula and marginals. Further, we assume that:

$$\mathbf{r}_t = \mu + y_t, \quad \mu = E(\mathbf{r}_t | \Omega_{t-1}),$$

(17)

$$y_{it} = \sigma_{it} \varepsilon_{it}, \quad \sigma_{it}^2 = \text{var}(r_{it} | \Omega_{t-1}),$$

(18)

$$\varepsilon_{it} \sim \text{iid} \text{Skew-t}(0, 1, \xi, \eta),$$

(19)

where $\text{Skew-t}(0, 1, \xi, \eta)$ denotes the standardized skewed Student $t$ distribution with $\eta > 2$ degrees of freedom, and skewness coefficient $\xi > 0$ (Lambert, Laurent, 2001). Moreover, the marginal return series $r_{it}, i = 1, 2$, are modeled as ARMA-GARCH processes.

When the conditional copula $C_1$ is allowed to fluctuate over time, some model for its evolution has to be specified. Commonly, the functional form of the conditional copula is fixed but its parameters evolve through time (Patton, 2004, 2006). In this paper, however, we apply an alternative approach (Okimoto, 2008; Garcia, Tsafack, 2011) assuming the existence of regimes where a fixed copula prevails, which are switched according to some homogeneous Markov chain. Thus, in the applied Markov-switching copula model (MSC model) the joint conditional distribution has the following form:

$$\mathbf{r}_t | \Omega_{t-1} \sim C_S(F_1(\cdot), G_2(\cdot) | \Omega_{t-1}),$$

(20)

where $S_t$ is a homogeneous Markov chain with state space \{1,2\}. The parameters of the model are the parameters of the univariate models for the marginal distributions, the parameters of the copulas $C_1$ and $C_2$, and the transition probabilities:
of the Markov chain.

We estimate the MSC models by the maximum likelihood method. The main by-product of the estimation are the probabilities $P(S_t = j \mid \Omega_{t-1})$, $P(S_t = j \mid \Omega_j)$, $j = 1, 2$, which are calculated by means of Hamilton’s filter (Hamilton, 1994):

\[
P(S_t = j \mid \Omega_{t-1}) = \sum_{i=1}^{2} p_{ij} P(S_{t-1} = i \mid \Omega_{t-1}),
\]

\[
P(S_t = j \mid \Omega_j) = \frac{c_j(u_t \mid S_t = j, \Omega_{t-1}) P(S_t = j \mid \Omega_{t-1})}{\sum_{i=1}^{2} c_i(u_t \mid S_t = i, \Omega_{t-1}) P(S_t = i \mid \Omega_{t-1})},
\]

where:

\[
p_{12} = P(S_t = 2 \mid S_{t-1} = 1) = 1 - p_{11}, \quad p_{21} = P(S_t = 1 \mid S_{t-1} = 2) = 1 - p_{22},
\]

\[
u_t = (u_{1,t}, u_{2,t}), \quad u_{1,t} = F_t(r_{1,t}), \quad u_{2,t} = G_t(r_{2,t}),
\]

and $c_j(\cdot \mid S_t = j, \Omega_{t-1})$ is the density of the conditional copula coupling the conditional marginal distributions in regime $j$. By the arguments of Hamilton (1994), the maximized log-likelihood function has the form:

\[
L = \sum_{t=1}^{T} \ln \left( \sum_{j=1}^{2} c_j(u_t \mid S_t = j, \Omega_{t-1}; \theta) P(S_t = j \mid \Omega_{t-1}; \theta) \right) + \sum_{t=1}^{T} \ln \left( f_t(r_{1,t} \mid \Omega_{t-1}; \theta_1) + \sum_{j=1}^{2} \ln \left( g_j(r_{2,t} \mid \Omega_{t-1}; \theta_2) \right) \right),
\]

where $f_t$ and $g_j$ are the density functions corresponding to $F_t$ and $G_t$, fitted using ARMA-GARCH models.

In the empirical results presented in this paper we apply the so-called smoothed probabilities which can be obtained from the probabilities (23) and (24) by using the backward recursion:

\[
P(S_t = j \mid \Omega_T) = P(S_t = j \mid \Omega_T) \sum_{i=1}^{2} p_{ij} \frac{P(S_{t+1} = i \mid \Omega_F)}{P(S_{t+1} = i \mid \Omega_i)},
\]

$t = T - 1, \ldots, 1$.
The series $P(S_t = j | \Omega_t)$ indicates which regime prevails at each date, using all the information in the sample.

2. The Model Confidence Set Methodology

The Model Confidence Set (MCS) methodology, introduced by Hansen et al. (2003), is usually applied to select in a set of forecasting models a subset of models which are better than the others in terms of forecasting ability, with a given level of confidence. More precisely, for some collection $\mathcal{M}^0$ of models for which the subset of superior models, $\mathcal{M}$, is defined, the MCS procedure produces a subset $\hat{\mathcal{M}}$ of $\mathcal{M}^0$ that contains the set $\mathcal{M}$ with a given confidence level $1 - \alpha$. The collection $\mathcal{M}^0$ can be any finite set of objects indexed by $i = 1, \ldots, m_0$ and evaluated in terms of a loss function that assigns to the object $i$ in period $t$ the loss $L_{i,t}$, $t = 1, \ldots, n$. If the relative performance is defined as $d_{i,j,t} = L_{i,t} - L_{j,t}$ for $i, j \in \mathcal{M}^0$ and the mean $\mu_j = E(d_{i,j,t})$ is finite and does not depend on $t$, then the set of superior objects is defined by:

$$\mathcal{M}^* = \{ i \in \mathcal{M}^0 : \mu_j \leq 0 \text{ for all } j \in \mathcal{M}^0 \}.$$

The MCS procedure is based on an equivalence test $\delta_M$ that tests the hypothesis:

$$H_{0,M} : \mu_j = 0 \text{ for all } i, j \in \mathcal{M},$$

at level $\alpha$ for any $\mathcal{M} \subset \mathcal{M}^0$. When $H_{0,M}$ is rejected, an elimination rule $e_M$ is applied to identify the object of $\mathcal{M}$ that is to be removed from $\mathcal{M}$. The MCS algorithm proposed by Hansen et al. (2003) is as follows:

- Step 0: Set $\mathcal{M} = \mathcal{M}^0$.
- Step 1: Test $H_{0,M}$ using $\delta_M$ at significance level $\alpha$.
- Step 2: If $H_{0,M}$ is not rejected define, $\hat{\mathcal{M}}_{1-\alpha} = \mathcal{M}$, otherwise use $e_M$ to eliminate an object from $\mathcal{M}$ and repeat the procedure from Step 1.

It is proved in Hansen et al. (2011) that under some standard requirements for the equivalence test and the elimination rule it holds that:

$$\liminf_{n \to \infty} P(\mathcal{M}^* \subset \hat{\mathcal{M}}_{1-\alpha}) \geq 1 - \alpha, \quad (27)$$
\begin{align*}
\lim_{n \to \infty} P(i \in \hat{M}_{1-\alpha}^*) &= 0 \text{ for all } i \not\in M^*, \\
\lim_{n \to \infty} P(M^* = \hat{M}_{1-\alpha}^*) &= 1 \text{ if } M^* \text{ is a singleton.}
\end{align*}

If \( P_{H_0,M} \) denotes the p-value associated with the null hypothesis \( H_{0,M} \) (with convention that \( P_{H_0,M_0} = 1 \)), then the MCS p-value for model \( e_{M,j} \in M^0 \) is defined as \( \hat{p}_{e_{M_j}} = \max_{i \in j} P_{H_0,M}. \)

It is shown in Hansen et al. (2011) that \( H_{0,M} \) can be tested using traditional quadratic-form statistics or multiple t-statistics. In this paper we apply the t-statistic \( T_{R,M} \) defined as:

\[
T_{R,M} = \max_{i,j \in M} |t_{ij}|
\]

where:

\[
t_{ij} = \frac{\hat{d}_{ij}}{\sqrt{\text{var}(\hat{d}_{ij})}} \quad \text{for } i, j \in M,
\]

and \( \hat{d}_{ij} = n^{-1} \sum_{\tau=1}^{n} d_{ij,\tau} \). The asymptotic distribution of the statistic \( T_{R,M} \) is nonstandard. The bootstrap implementation of the MCS procedure involving this statistic is described in Appendix B in the paper by Hansen et al. (2003). The estimated bootstrap distribution of \( T_{R,M} \), under the null hypothesis, is given by the empirical distribution of:

\[
T_{b,R} = \max_{i,j \in M} \frac{|\hat{d}_{b,ij} - \hat{d}_{ij}|}{\sqrt{\text{var}(\hat{d}_{ij})}},
\]

where \( \hat{d}_{b,ij} = \frac{1}{B} \sum_{\tau=1}^{n} d_{ij,\tau_b} \) is the bootstrap resample average, and \( \text{var}(\hat{d}_{ij}) = \frac{1}{B} \sum_{b=1}^{B} \left( \hat{d}_{b,ij} - \hat{d}_{ij} \right)^2 \). For generating \( B \) resamples, the block bootstrap is used. The p-value of \( H_{0,M} \) is given by:

\[
P_{H_{0,M}} = \frac{1}{B} \sum_{b=1}^{B} 1\{T_{b,R}^* > T_{R,M}\}.
\]

Even though the MCS methodology was originally designed in Hansen et al. (2003) to select the best volatility forecasting models, its applications are not limited to comparisons of models. It can be used as well to compare...
the means of two or more populations. In this paper, we apply the MCS procedure to establish rankings of strength of conditional dependencies between the investigated stock markets, calculated by means of the discussed copula-based measures of dependence.

3. The Data

The dataset used in the analysis includes closing quotations of the following stock indices: the Hungarian BUX, the Czech PX, the Polish WIG20, the DAX, and the S&P 500, from the period May 5, 2004 to July 20, 2012. The period under scrutiny begins after the Czech Republic, Hungary and Poland joining the European Union. This allows us to avoid a possible structural break in the data caused by that event. The quotations of the PX index were obtained from the website of the Prague Stock Exchange, and those of the other indices come from the website Stooq.pl.

Figure 1. Time plots of daily quotations of BUX/10, PX, WIG20, DAX/5 and S&P500 from May 5, 2004 to July 20, 2012. Scaling in the case of BUX and DAX is performed for easier comparison

Since the patterns of non-trading days in the national stock markets differ, for the purposes of modeling the dependencies, the dates of observations for all the indices were checked and observations not corresponding to ones in the other index quotation series were removed. The plots of the adjusted series of quotations are presented in Figure 1. The modeled time series are percentage logarithmic daily returns calculated by the formula:
where \( P_t \) denotes the closing index value on day \( t \).
In Figure 2, we show time plots displaying the evolution of GDP annual growth rate in the Czech Republic, Hungary, Poland, the UE, and the USA during the period under scrutiny. The market capitalization of the analyzed stock exchanges is presented in Figure 3 and shows the leading position of the Warsaw Stock Exchange in the region. From the plots in Figures 1–3, it is evident that during the subprime crisis of 2007–2009 all the markets experienced significant decline, and that the same applies to their GDP growth rate, though the Polish GDP growth rate remained positive.

As in many papers dealing with dynamic copula models, we separate the estimation of models for marginal distributions from the estimation of the model for the dependence structure. The resulting multi-stage maximum likelihood estimation method, known as „inference functions for margins” (Joe, Xu 1996; Joe, 1997), is not fully efficient but there are simulation results that motivate this approach (Patton, 2009). To describe marginal distributions, we use ARMA-GARCH models. The estimation of the parameters of the marginal distributions (ARMA-GARCH) was performed using G@RCH6.2 package (Laurent, 2009; Doornik, 2006).

Figure 4. Volatility estimates for the indices DAX, S&P 500, BUX, WIG20, and PX for the period May 5, 2004 – July 20, 2012

In Table 1, the parameter estimates of best fitted models are presented. In the case of the WIG20, the best fitted model is GARCH(3,1) with the innovation term following a standardized Student $t$ distribution. The result indicates lack of asymmetry in volatility dynamics. For all the other indices, the chosen volatility models are ARMA-GJR-GARCH models specified as:
\[ r_t - \mu = \sum_{i=1}^{q} a_i (r_{t-i} - \mu) + \sum_{i=1}^{q} b_i y_{t-i} + y_t, \]  
(35)

\[ y_t = \sigma \varepsilon_t, \]  
(36)

\[ \sigma_t^2 = \omega + \sum_{i=1}^{q} (\alpha_i y_{t-i}^2 + \gamma_i I(y_{t-i} < 0) y_{t-i}^2) + \sum_{j=1}^{q} \beta_j \sigma_{t-j}^2, \]  
(37)

where \( I \) is the indicator function.

**Table 1. Parameter estimates for fitted GJR-GARCH models**

<table>
<thead>
<tr>
<th>Parameters</th>
<th>BUFX</th>
<th>PX</th>
<th>WIG20</th>
<th>DAX</th>
<th>S&amp;P500</th>
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<td>0.0542</td>
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<td>0.0563</td>
<td>0.0313</td>
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<td>-0.005</td>
<td>(0.0236)</td>
<td>(0.097)</td>
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<td>0.0622</td>
<td>0.0309</td>
<td>0.0146</td>
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</tr>
<tr>
<td>( \omega )</td>
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<td>(0.0053)</td>
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<tr>
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<td>0.0214</td>
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<td>(0.0153)</td>
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<td>( \gamma_2 )</td>
<td>(0.0202)</td>
<td>(0.032)</td>
<td>(0.0376)</td>
<td>(0.0317)</td>
<td></td>
</tr>
<tr>
<td>( \gamma_3 )</td>
<td>0.0859</td>
<td>0.0898</td>
<td>0.1784</td>
<td>0.0952</td>
<td></td>
</tr>
<tr>
<td>( \beta_1 )</td>
<td>(0.0255)</td>
<td>(0.032)</td>
<td>(0.0376)</td>
<td>(0.0317)</td>
<td></td>
</tr>
<tr>
<td>( \ln \xi )</td>
<td>-0.0842</td>
<td>-0.1475</td>
<td>-0.1644</td>
<td></td>
<td></td>
</tr>
<tr>
<td>DF</td>
<td>10.2987</td>
<td>6.9791</td>
<td>8.8613</td>
<td>8.7099</td>
<td>7.0120</td>
</tr>
</tbody>
</table>

Figure 4 presents the plots of volatility (conditional variance) estimates from fitted models for the period from May 5, 2004 to July 20, 2012. The plots allow to easy identify periods of turmoil in financial markets. In the considered period, usually the PX index exhibits the highest volatility level. An explosion of volatility visible in Figure 4 is the result of a panic after...
bankruptcy of Lehman Brothers. The peaks in the volatility of the BUX and PX indices appearing just after October 22, 2008 are likely to be a reaction to Hungary’s request for financial support directed to the International Monetary Fund.

4. The Analysis of Dependencies

Modeling the dependencies between the European and the US market one meets the problem of non-synchronous observations due to different trading hours. There is no a really good solution to this difficulty. The most frequently applied approaches include using weekly data (thus losing much of the available information), interpolating the quotations beyond the trading hours to get simultaneous observations (thus introducing additional noise), or analyzing the observations that are coincident but locally come from different phases of a trading day (thus approving of noise caused by microstructure effects). In our research, we decided to use the close to close returns, but in the cases involving the S&P 500 index we investigate two types of dependencies. In the first case the analysis is performed for the returns corresponding to the same date, and in the second – for the returns of the S&P 500 one-day lagged with respect to the returns on the European indices, denoted as S&P 500(−1). A part of the results concerning the WIG20 index was first published in Polish (Doman, Doman, 2013). We quote them here for the reader’s convenience.

Tables 2–4 show parameter estimates of the MSC models fitted to the returns of each of the considered pairs of the indices. A large set of copula specifications was considered during the fitting procedure. Taking into account the results by Doman and Doman (2012), we considered MSC models with 3 and 2 regimes, and static copula models. The best models are selected on the basis of information criteria and the results of likelihood ratio tests. Apart from the pair (WIG20, S&P 500), in all cases we apply 2-regime MSC models with the first regime described by the Gaussian copula and the second characterized by the Clayton or the Joe-Clayton copula. So, in the first regime there is no dependence in tails, and in the second at least dependence in lower tail is observed.

The plots of estimated dynamic dependence measures are presented in Figures 5–8. The first impression from the pictures is that the dependencies within the group of the Central European markets are stronger and exhibit stronger dynamics, in comparison with their linkages with the considered developed markets. It means that the linkages within the group of the Central European markets are more sensitive on new information.
Table 2. Linkages of the BUX: parameter estimates for the MSC models. Standard errors in parentheses.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>PX</th>
<th>WIG20</th>
<th>DAX</th>
<th>S&amp;P 500</th>
<th>S&amp;P 500(-1)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$p_{11}$</td>
<td>0.9960</td>
<td>0.9895</td>
<td>0.9994</td>
<td>0.9982</td>
<td>0.9991</td>
</tr>
<tr>
<td>$p_{22}$</td>
<td>0.8957</td>
<td>0.9702</td>
<td>0.9867</td>
<td>0.9964</td>
<td>0.9991</td>
</tr>
<tr>
<td>Copula in regime 1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\rho$</td>
<td>0.6658</td>
<td>0.6799</td>
<td>0.5937</td>
<td>0.4260</td>
<td>0.3089</td>
</tr>
<tr>
<td>Spearman’s rho</td>
<td>0.6482</td>
<td>0.6624</td>
<td>0.5756</td>
<td>0.4100</td>
<td>0.2962</td>
</tr>
<tr>
<td>Copula in regime 2</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\kappa$</td>
<td>1.3925</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\gamma$</td>
<td>0.4769</td>
<td>0.4144</td>
<td>0.3533</td>
<td>0.1680</td>
<td>0.1382</td>
</tr>
<tr>
<td>Spearman’s rho</td>
<td>0.2843</td>
<td>0.4265</td>
<td>0.2230</td>
<td>0.1159</td>
<td>0.0968</td>
</tr>
<tr>
<td>$\lambda_L$</td>
<td>0.2338</td>
<td>0.1878</td>
<td>0.1406</td>
<td>0.0162</td>
<td>0.0066</td>
</tr>
<tr>
<td>$\lambda_U$</td>
<td>0.3549</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Figure 5. Dependencies between the BUX and the other indices. Estimates of Spearman’s rho.
The first observation concerning models describing the linkages of the BUX with the other indices (Table 2) is that in each case the value of Spearman’s rho is higher in the regime with the Gaussian copula. Figure 5 indicates an increase in the strength of connection between the BUX and DAX, which can be interpreted as an effect of the EU joining. During the period of the subprime crisis we observe a drop in the strength of dependencies between the returns on the BUX and lagged returns on the S&P 500. At the same time, however, there is an increase in the strength of dependencies between the returns with the same date. Taking into account different impact of the Hungarian and the US market on the global economy, we can formulate a conjecture that during the crisis period the dependencies between the markets were affected by a common factor driving the prices’ dynamics.

In Figure 6, the plots of the conditional lower tail dependence coefficients are presented. This type of dependence is observed for the pairs (BUX, PX) and (BUX,WIG20). It does not occur when we analyze the linkages for the pair (BUX, S&P 500(–1)). Some very weak dependence is visible for the pairs (BUX, DAX) and (BUX, S&P 500), but it disappears at the beginning of 2006.

In the case of the PX index, the results are similar to those for the BUX in the part dealing with the emerging markets, but different when considering the DAX and S&P 500 (Table 3). First observation is that the dependence between the PX and DAX, measured by means of Spearman’s rho (Figure 7), is very strong during almost all the period under scrutiny (from
Some decrease in the strength of dependencies is visible from July 2009 to February 2010. The dependence between the returns on the PX and S&P 500 is quite significant for both the returns from the same day and in the case of lagged returns on the S&P 500. The pair (PX, S&P 500) is the only one for which the value of Spearman’s rho in the regime with tail dependence is higher than in that with the Gaussian copula. Lower tail dependence coefficient for (PX, S&P 500) is higher than for (PX, DAX). The periods of stronger dependencies in lower tail for the pair (PX, DAX) occur in the beginning of the sample period and in late 2009.

Table 3. Linkages of the PX: parameter estimates for the MSC models. Standard errors in parentheses

<table>
<thead>
<tr>
<th></th>
<th>PX and WIG20</th>
<th>DAX</th>
<th>S&amp;P500</th>
<th>S&amp;P 500(-1)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\rho_{11}$</td>
<td>0.9589</td>
<td>0.9960</td>
<td>0.9903</td>
<td>0.9932</td>
</tr>
<tr>
<td>$\rho_{22}$</td>
<td>0.9702</td>
<td>0.9849</td>
<td>0.9909</td>
<td>0.9916</td>
</tr>
<tr>
<td>Copula in regime 1</td>
<td>$C_{\rho}^{\text{Gauss}}$</td>
<td>$C_{\rho}^{\text{Gauss}}$</td>
<td>$C_{\rho}^{\text{Gauss}}$</td>
<td>$C_{\rho}^{\text{Gauss}}$</td>
</tr>
<tr>
<td>$\rho$</td>
<td>0.6796</td>
<td>0.6028</td>
<td>0.2305</td>
<td>0.3209</td>
</tr>
<tr>
<td>Spearman’s rho</td>
<td>0.6621</td>
<td>0.5648</td>
<td>0.2206</td>
<td>0.3078</td>
</tr>
<tr>
<td>Copula in regime 2</td>
<td>$C_{\lambda}^{\text{Clay}}$</td>
<td>$C_{\gamma}^{\text{Clay}}$</td>
<td>$C_{\kappa}^{\text{Clay}}$</td>
<td>$C_{\lambda}^{\text{Clay}}$</td>
</tr>
<tr>
<td>$\kappa$</td>
<td>1.2935</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\gamma$</td>
<td>0.6991</td>
<td>0.3991</td>
<td>0.4484</td>
<td>0.2832</td>
</tr>
<tr>
<td>$\lambda_L$</td>
<td>0.3710</td>
<td>0.1761</td>
<td>0.2132</td>
<td>0.0865</td>
</tr>
<tr>
<td>$\lambda_U$</td>
<td></td>
<td></td>
<td></td>
<td>0.2911</td>
</tr>
<tr>
<td>Spearman’s rho</td>
<td>0.3779</td>
<td>0.2465</td>
<td>0.4013</td>
<td>0.1848</td>
</tr>
</tbody>
</table>

Table 4 and Figures 9–10 present the estimation results for dependencies with the WIG20. The estimates for (WIG20, DAX) are very similar to those for (WIG20, BUX) and (WIG20, PX). The linkages between the WIG20 and DAX, however, seem to be stronger than for (PX, DAX) or (BUX, DAX). The results concerning dependencies between the returns on the WIG20 and S&P 500 (Table 4) could seem surprising, but are in agreement with practitioners’ beliefs. The best models describing the dependencies are in this case the static Gaussian copula for the returns from the same day, and the static Joe-Clayton copula in the case of lagged S&P 500 returns. This supports the established opinion about the stability of the US market impact on the Warsaw Stock Exchange. The plots in Figure 9 show that the estimates of dyna-
Figure 7. Dependencies between the PX and the other indices. Estimates of Spearman’s rho

Figure 8. Dependencies between the PX and the other indices. Estimates of lower tail dependence coefficient

Spearman’s rho for the pairs (WIG20, BUX) and (WIG20, PX) are changing between 0.4 and 0.7. In the case of linkages with the DAX, in principle only values 0.34 (until July 2006) and 0.63 (from December 2006) are observed. Tail dependence for the WIG20 and DAX has disappeared since December 2006, and tail dependence between the returns on the WIG20 and lagged S&P 500 is present during all the sample period, but is very weak.
Table 4. Linkages of the WIG20: parameter estimates for the MSC models. Standard errors in parentheses

<table>
<thead>
<tr>
<th></th>
<th>WIG20 and DAX</th>
<th>S&amp;P 500</th>
<th>S&amp;P 500(-1)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P_{21}$</td>
<td>0.9995</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$P_{22}$</td>
<td>0.9962</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Copula in regime 1</td>
<td>$C^\rho_{\text{Gauss}}$</td>
<td>$C^\rho_{\text{Gauss}}$</td>
<td></td>
</tr>
<tr>
<td>$\rho$</td>
<td>0.6529 (0.0127)</td>
<td>0.3936 (0.0177)</td>
<td></td>
</tr>
<tr>
<td>Spearman’s rho</td>
<td>0.6531</td>
<td>0.3783</td>
<td></td>
</tr>
<tr>
<td>Copula in regime 2</td>
<td>$C^\gamma_{\text{Clayton}}$</td>
<td>$C^\lambda_{\text{Clayton}}$</td>
<td></td>
</tr>
<tr>
<td>$\kappa$</td>
<td>1.0406 (0.0205)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\gamma$</td>
<td>0.6132 (0.0737)</td>
<td>0.1895 (0.0291)</td>
<td></td>
</tr>
<tr>
<td>$\lambda_L$</td>
<td>0.3229</td>
<td>0.0258</td>
<td></td>
</tr>
<tr>
<td>$\lambda_U$</td>
<td>0.0534</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Spearman’s rho</td>
<td>0.3440</td>
<td>0.1570</td>
<td></td>
</tr>
</tbody>
</table>

As it was mentioned earlier, we used the Model Confidence Set methodology, described in Section 3, to compare the average strength of dependencies for the considered pairs of indices. Table 5 presents the comparison results for the strength of dependencies in the form of a ranking, where a higher score means stronger average dependence. The MCS procedure is applied sequentially – after each run, the series constituting the MCS obtain the subsequent score and are excluded. In the case where the MCS consists of at least two elements, the arithmetic mean of the corresponding scores is assigned to each of them. The procedure is applied to the series of the smoothed values of Spearman’s rho and tail dependence coefficients. All the MCSs have been estimated (with the significance level $\alpha = 0.1$) using the package MulCom of Hansen and Lunde (2010).

The ranking results for Spearman’s rho differ from those concerning the lower tail dependence coefficient. Generally, however, we can say that in average the linkages between the BUX, PX and WIG20 are stronger than the dependencies between each of them and the DAX or S&P 500. The only exception is for the PX index where the mean levels of the lower tail dependence coefficient for (PX, WIG20) and (PX, S&P 500) are statistically indistinguishable. The results indicate the importance of the WIG20 as far as we measure average dependence using Spearman’s rho. The pattern for de-
Dependencies in lower tail is more complicated. In the case of the BUX, the highest score is assigned to the pair (BUX, PX). Accordingly, for the PX this is so for the pairs (PX, WIG20) and (PX, S&P 500), and for the WIG20 – for the pair (WIG20, BUX).

Figure 9. Dependencies between the WIG20 and the other indices. Estimates of Spearman’s rho

Figure 10. Dependencies between the WIG20 and the other indices. Estimates of lower tail dependence coefficient
Table 5. Rankings of average strength of dependencies

<table>
<thead>
<tr>
<th></th>
<th>Spearman’s rho</th>
<th>Lambda_L</th>
<th></th>
<th>Spearman’s rho</th>
<th>Lambda_L</th>
<th></th>
<th>Spearman’s rho</th>
<th>Lambda_L</th>
</tr>
</thead>
<tbody>
<tr>
<td>BUX &amp; PX</td>
<td>4</td>
<td>5</td>
<td>BUX &amp; WIG20</td>
<td>4</td>
<td>3</td>
<td>BUX &amp; PX</td>
<td>5</td>
<td>3</td>
</tr>
<tr>
<td>WIG20 &amp; PX</td>
<td>5</td>
<td>4</td>
<td>WIG20 &amp; PX</td>
<td>5</td>
<td>4.5</td>
<td>WIG20 &amp; PX</td>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>DAX &amp; PX</td>
<td>3</td>
<td>3</td>
<td>DAX &amp; WIG20</td>
<td>3</td>
<td>1.5</td>
<td>DAX &amp; PX</td>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>S&amp;P 500 &amp; PX</td>
<td>2</td>
<td>2</td>
<td>S&amp;P 500 &amp; PX</td>
<td>2</td>
<td>4.5</td>
<td>S&amp;P 500 &amp; PX</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>S&amp;P 500 &amp; (-1)</td>
<td>1</td>
<td>1</td>
<td>S&amp;P 500 &amp; (-1)</td>
<td>1</td>
<td>1.5</td>
<td>S&amp;P 500 &amp; (-1)</td>
<td>1</td>
<td>2</td>
</tr>
</tbody>
</table>

Figure 11. Smoothed probabilities of the regime with tail dependence for pairs (BUX, PX), (BUX, WIG20) and (PX, WIG20)

Usually in literature, the presence of tail dependence is linked with the turmoil periods in markets. The plots presented in Figures 11–13 allow to determine the moments of switching on the regime with tail dependence. First observation is that the changes of regimes are very frequent as far as we consider the three pairs of the analyzed emerging markets (Figure 9). If tail dependence is present for the pair (BUX, WIG20) it is usually present for the two other pairs. However, for the pairs (BUX, PX) and (PX, WIG20), there occur some additional periods with tail dependence. Tail dependence between the considered emerging markets and the German market disappeared in 2006. In the case of the pair (PX, DAX), it appeared again during a short period from July 2009 to February 2010. For the pair (BUX, S&P 500(−1)), tail dependence has occurred since September 2008 (the crisis effect), and for (WIG20, S&P 500(−1)), it is present during all the sample period. In the
case of (PX, S&P 500(−1)), the pattern is more complicated because periods with tail dependence appear and disappear cyclically. Tail dependence in the case of (WIG20, S&P 500) is absent. For the pair (BUX, S&P 500) it occurs in pre-crisis period, and for (PX, S&P 500) it is present during most of the time in the recent crisis years.

Figure 12. Smoothed probabilities of the regime with tail dependence for pairs (BUX, DAX), (PX, DAX) and (WIG20, DAX)

Our findings can be sum up as follows. Dependencies between the three considered emerging markets are stronger than the one of each of them with the German or the US markets, regardless of the applied dependence measure. In almost all cases the dynamics of the dependencies is rich, indicating sensitivity on information process. The periods with tails dependence occur very often and they are not necessary connected with crisis periods, though this type of dependence is present during these periods too. The linkages between the considered emerging markets and the developed ones are very stable and mostly do not change during the crises. The extreme example involves the WIG20 index for which the dependencies with contemporary and lagged S&P 500 returns are described by means of static copulas. As regards the dynamics of dependencies between the Czech, Hungarian and Polish markets, the crucial relation is that between the BUX and WIG20. The importance of the influence of major European markets and the US market on the analyzed emerging markets can be perceived as comparable if one takes into account that due to the no-synchronization effect the impact of the US market is divided between two days.
Conclusions

In the paper we analyzed the dynamics and strength of linkages between the Czech, Hungarian and Polish stock markets. In addition, we examined linkages between each of these markets and developed markets (European and US). The analysis was based on the daily quotations of the main representative stock indices (PX, BUX, WIG20, DAX, S&P 500). The period
under scrutiny was from May 5, 2004 to July 20, 2012. So, it starts after the EU accession of the corresponding countries and includes recent financial crises. Our tool to model the dynamics of dependencies were Markov-switching copula models. Due to this choice, we were able to use two kinds of measure of the strength of the linkages: dynamic Spearman’s rho and tail dependence coefficients. Our results show that the dependencies between the three considered emerging markets are stronger than those between each of them and the German or the US markets, regardless of the applied measure. The dynamics of dependence in almost all cases is rich, clearly indicating sensitivity on information process. The tail dependence appears and disappears cyclically and is not necessary connected with the crisis periods, though this type of dependence is present during these periods too. The linkages of the examined emerging markets with the considered developed markets are very stable, not sensitive on present information, and mostly do not change during the crises. The crucial relation for the considered region is the dependence between the Hungarian and Polish markets. The importance of the impact of the developed European market and the US market on the analyzed emerging markets can be viewed as comparable.

References


Dynamika i siła zależności pomiędzy rynkami giełdowymi Czech, Węgier i Polski po ich wejściu do Unii Europejskiej

Za r y s t r e ś c i. W pracy analizowana jest dynamika i siła powiązań pomiędzy rynkami giełdowymi Czech, Węgier i Polski. Badanie obejmuje okres po wejściu wymienionych krajów do Unii Europejskiej. Ponadto oceniamy powiązania pomiędzy każdym z wymienionych rynków a wybranymi rynkami rozwiniętymi (europejskim i amerykańskim). Analiza oparta jest na dziennych notowaniach głównych indeksów giełdowych (PX, BUX, WIG20, DAX, S&P 500) i obejmuje okres od 5 maja 2004 r. do 20 lipca 2012 r. Dynamika zależności jest opisywana za pomocą modeli kopuli z przełączaniem typu Markowa, a stosowanymi miarami siły powiązań są dynamiczne współczynniki rho Spearmana i dynamiczne współczynniki zależności w ogonach rozkładów. Uzyskane wyniki pokazują, że zależności pomiędzy rozważanymi rynkami wschodzącymi są bardzo wrażliwe na sytuację rynkową, natomiast powiązania z rynkami rozwiniętymi są stabilne.

Słowa kluczowe: Środkowoeuropejski rynek giełdowy, zależność warunkowa, modele kopuli z przełączaniem typu Markowa, współczynnik rho Spearmana, zależność w ogonie rozkładu, zbiór ufności modeli, indeks giełdowy.