DYNAMIC ECONOMETRIC MODELS
Vol. 11 – Nicolaus Copernicus University – Toruń – 2011

Katarzyna Bień-Barkowska
Warsaw School of Economics, National Bank of Poland

Distribution Choice for the Asymmetric ACD Models†

A b s t r a c t. In the paper, I generalize the Asymmetric Autoregressive Conditional Duration (AACD) model proposed by Bauwens and Giot (2003) with respect to the generalized gamma and the Burr distribution for an error term. I derive the log likelihood functions for the augmented models and show how to check the goodness-of-fit of the distributional assumptions with the application of the probability integral transforms proposed by Diebold, Gunther and Tay (1998). Moreover, I present an exemplary empirical application of the Asymmetric ACD model for the durations between submissions of market or best limit orders on the interbank trading platform for the Polish zloty. I test the impact of selected market microstructure factors (i.e. the bid-ask spread, volatility) on the time of order submissions.

K e y w o r d s: Asymmetric ACD model, financial durations, probability integral transforms, market microstructure.

Introduction

Econometric models for financial durations (i.e. time spells between selected events of the trading process) have gained an extreme popularity over the last decade. The standard Autoregressive Conditional Duration (ACD) model of Engle and Russell (1998) and its numerous extensions have become a standard tool in modeling irregularly-spaced financial data. The vast survey of different ACD specifications has been recently presented in the study of Pecurar (2008). The models have been used in order to test selected market microstructure hypotheses (Bauwens, Giot, 2001; Zhang et al., 2001; Hautsch, 2004; Nolte, 2008, among others), as well as in order to augment the volatility measures (Ghysels, Jasiak, 1998; Giot, 2000; Kalimipalli, Warga, 2002; Grammig, Wellner, 2002; Giot, 2005). The outline of the ACD models has been

† This work has been financed from the Polish science budget resources in year 2011 as the part of the research project 03/S/0007/11.

covered in many econometric textbooks (Bauwens, Giot, 2001; Gouriéroux, Jasiak, 2001; Tsay, 2002; Osińska, 2006).

In this paper I focus on the asymmetric ACD model (henceforth AACD model) of Bauwens and Giot (2003). It is a flexible model for the conditional density of financial durations that elapse if one of two possible end states (i.e. events pointed on the micro-scale by appropriate "thinning" the data) occurs. As an exemplary application of the model, the authors suggest a dynamic description of mid-price durations for selected stocks traded on the NYSE. They categorize time intervals between subsequent price movements into (1) "a price increase" duration (if price of a stock increases during the duration) and (2) "a price decrease" duration (if a price of a stock decreases during the duration). The AACD model entwines a logarithmic ACD (LACD) specification for the conditional expectations of durations, hence the model accounts for the well-known clustering that characterizes such processes (see Bauwens, Giot, 2000). On the other hand, the AACD model has a major advantage over the standard ACD specification, as it discriminates between end states of the time spells under study. The model allows for a separate description of expected durations ending as a "price increase" and expected durations ending as a "price decrease". Accordingly, it accounts for the different dynamics characterizing each of the two processes. It also allows for an easy inclusion of explanatory factors that – in this particular setup – may exert a totally different impact on the pace according to which one or the other event (end state) occurs.

More recently, the AACD model has been used by Lo and Sapp (2008) to test the impact of different microstructure factors on the choice and timing of market/limit[1] order placement in the Reuters Dealing 2000-2 Spot Matching system[2]. The authors account for various explanatory variables that reflect the state of the order book and some other market conditions (i.e. volatility, time-of-day) and check their influence on the expected time to a market order or a limit order arrival on the ask and bid side of the market. The application of the AACD specification allows for the joint modelling of both: (1) the order choice and (2) the time that elapses between subsequent order submissions.

Both of the aforementioned studies apply the AACD model with the Weibull-distributed error term. Such a setup allows for a parsimonous and tractable specification. Nevertheless, the studies (Bauwens, Giot, 2001; Bauwens et al., 2004) proved the superiority of some more general and flexible distributions (the Burr or generalized gamma distribution) over the exponential or Weibull ones, as far as the goodness-of-fit of the ACD models is concerned. Moreover, a simulation study presented in (Grammig, Mauer, 2000) proved that

[1] A market order is an order to immediately buy or sell an asset at the best prevailing bid or ask prices. A limit order is an order to buy or sell an asset at a specified price (or better) and it is executed if it can be matched with an upcoming order on the opposite market side.

[2] Reuers Dealing 2000-2 Spot Matching System is a fully automated, order-driven, interbank market for currency trading.

the distributional misspecification may lead to serious inference problems: loss of efficiency and bias in the estimated parameters of duration expectations.

The aim of this paper is to generalize the standard AACD model of Bauwens and Giot (2003) with respect to its distributional assumptions. I derive the likelihood function for the AACD model with the Burr distribution (henceforth the B-AACD model) and with the generalized gamma distribution (henceforth the GG-AACD model). Moreover, I show how to derive the probability integral transforms (PIT) of Diebold, Gunther and Tay (1998) for the generalized AACD specifications in order to verify their dynamic and distributional goodness-of-fit. The theoretical issues have been supported with the empirical example. As in the study of Lo and Sapp (2008), I apply the AACD model to durations between the order submissions in the Reuters Dealing 3000 Spot Matching System which is a leading interbank trading platform for the EUR/PLN currency pair.

1. Econometric Models

1.1. Outline of the Asymmetric ACD Model

The Asymmetric ACD model of Bauwens and Giot (2003) describes the marked point process $\{x_i, y_i\}$, where $x_i = t_i - t_{i-1}$ is a duration between moments in which certain events occur: t_i and t_{i-1}, and y_i is a qualitative variable indicating a type of an event: $y_i \in \{a, b\}$. In the primary study of Bauwens and Giot (2003), the states a and b correspond, respectively, to “a price increase” and “a price decrease”; whereas in the study of Lo and Sapp (2008) they refer to “submission of a market order” and “submission of a limit order”. Therefore, at the end of each duration one of two events (states, risks) can be observed. This modeling setup can be perceived as if the two potential risks were competing with each other and only one of them: a or b could be realized at t_i. Accordingly, the model belongs to a class of competing risks models. The duration x_i is treated as an outcome variable of a function $x_i = \min(x_{i,a}, x_{i,b})$, where $x_{i,a}$ and $x_{i,b}$ are durations that would end up in state a and b, respectively. In this modeling framework, only one duration, i.e. $x_{i,a}$ (or $x_{i,b}$), is realized (i.e. the shorter one), which depends on whether an event a or b occurs first. The duration that is not realized is treated as truncated. The conditional bivariate density for a pair x_i, y_i can be described as:

$$\begin{aligned} f(x_i, y_i \mid F_{i-1}) = {} & h_{x_a}(x_i \mid F_{i-1})^{I^a} S_{x_a}(x_i \mid F_{i-1}) \\ & \cdot h_{x_b}(x_i \mid F_{i-1})^{I^b} S_{x_b}(x_i \mid F_{i-1}), \end{aligned} \tag{1}$$

where $h_{x_a}(\cdot)$ ($h_{x_b}(\cdot)$) and $S_{x_a}(\cdot)$ ($S_{x_b}(\cdot)$) denote the hazard and the survival functions for the $x_{i,a}$ ($x_{i,b}$) variable, I^a and I^b are dummy indicators, i.e. $I^a = 1$ ($I^b = 1$) if state $y_i = a$ ($y_i = b$) is observed and, analogously, $I^a = 0$ ($I^b = 0$) if state $y_i = b$ ($y_i = a$) is observed at the end of duration x_i. F_{i-1} denotes a conditioning information set up to time $t-1$ which contains past realizations of x_i and y_i.

For example, if an event a happens at t_i, $x_{i,a}$ is observed and $x_{i,b}$ is truncated. Accordingly, the realized duration $x_{i,a} = x_i$ contributes to the conditional density function given by equation (1) via its density function $f_{x_a}(x_i \mid F_{i-1})$ and the unrealized (truncated) duration $x_{i,b} > x_i$ via its survival function $S_{x_b}(x_i \mid F_{i-1})$:

$$\begin{aligned} f(x_i, y_i = a \mid F_{i-1}) &= h_{x_a}(x_i \mid F_{i-1}) S_{x_a}(x_i \mid, F_{i-1}) S_{x_b}(x_i \mid F_{i-1}) \\ &= f_{x_a}(x_i \mid F_{i-1}) S_{x_b}(x_i \mid F_{i-1}). \end{aligned} \tag{2}$$

As proposed by Bauwens and Giot (2000) or Lo and Sapp (2008) we parameterize the expectations of the conditional density functions of $x_{i,a}$ and $x_{i,b}$ with the application of the Logarithmic ACD model. The conditional duration expectations, $\Psi_{i,a} = E(x_{i,a} \mid F_{i-1})$ and $\Psi_{i,b} = E(x_{i,b} \mid F_{i-1})$, are specified in a dynamic fashion such as both factors (the previously observed states and the past realized durations) can exert an influence on the expected time to an event a or b:

$$\begin{aligned} \psi_{i,a} &= (\omega_{a,a} + \alpha_{a,a} \ln x_{i-1}) I_{i-1}^a \\ &\quad + (\omega_{b,a} + \alpha_{b,a} \ln x_{i-1}) I_{i-1}^b + \beta_a \psi_{i-1,a}, \end{aligned} \tag{3}$$

$$\begin{aligned} \psi_{i,b} &= (\omega_{a,b} + \alpha_{a,b} \ln x_{i-1}) I_{i-1}^b \\ &\quad + (\omega_{b,b} + \alpha_{b,b} \ln x_{i-1}) I_{i-1}^b + \beta_b \psi_{i-1,b}, \end{aligned} \tag{4}$$

where $\psi_{i,a} = \ln(\Psi_{i,a})$.

The specifications for $\psi_{i,a}$ and $\psi_{i,b}$ change with the previously realized state of y_i. Thus, the expected time to a given event varies with the type of previously observed state and with the length of preceding duration. In the AACD framework, the observed duration x_i stems from a mixing process:

$$\begin{aligned} x_i &= [E(x_{i,a} \mid F_{i-1}) \varepsilon_{i,a}] I_i^a + [E(x_{i,b} \mid F_{i-1}) \varepsilon_{i,b}] I_i^b \\ &= [\Psi_{i,a} \varepsilon_{i,a}] I_i^a + [\Psi_{i,b} \varepsilon_{i,b}] I_i^b, \end{aligned} \tag{5}$$

where $\varepsilon_{i,a}$ ($\varepsilon_{i,b}$) is an independent and identically distributed error term with $E(\varepsilon_{i,a})=1$, ($E(\varepsilon_{i,b})=1$). In this setup $\varepsilon_{i,a}$ and $\varepsilon_{i,b}$ can both have the generalized gamma or the Burr distribution (see Appendix 1 for the theoretical outline of these distributions).

In order to parameterize the conditional bivariate density for $\{x_i, y_i\}$ outlined in equation (1), it is crucial to derive the conditional hazard and the conditional survival functions for x_i under given assumptions about the distribution of $\varepsilon_{i,a}$ and $\varepsilon_{i,b}$. If we denote hazard and survival function of an error term $\varepsilon_{i,a}$ ($\varepsilon_{i,b}$) as $h_{\varepsilon_a}(\cdot)$ ($h_{\varepsilon_b}(\cdot)$) and $S_{\varepsilon_a}(\cdot)$ ($S_{\varepsilon_b}(\cdot)$), respectively, under the necessary assumption that $E(\varepsilon_{i,a})=1$ ($E(\varepsilon_{i,b})=1$), the conditional hazard and the conditional survival function of $x_{i,a}=\Psi_{i,a}\varepsilon_a$ (a similar result holds for $x_{i,b}=\Psi_{i,b}\varepsilon_b$) can be given as:

$$h_{x_a}(x_i \mid F_{i-1}) = \frac{1}{\Phi_{i,a}} h_{\varepsilon_a}\left(\frac{x_i}{\Phi_{i,a}}\right), \tag{6}$$

$$S_{x_a}(x_i \mid F_{i-1}) = S_{\varepsilon_a}\left(\frac{x_i}{\Phi_{i,a}}\right), \tag{7}$$

where $\Phi_{i,a}=\Psi_{i,a}/\mu_a$ and μ_a is the expectation of a Burr-distributed (or a generalized gamma-distributed) random variable (see Appendix 1).

1.2. AACD Model with the Burr Distribution

The Burr distribution has two shape parameters κ and σ^2. Lancaster (1990) proves that it can be derived as a gamma mixture of Weibull distributions. It contains exponential (if $\kappa=1, \sigma^2 \to 0$), Weibull (if $\sigma^2 \to 0$) and log-logistic (if $\sigma^2=1$) distributions as its limiting cases. In contrast to a Weibull distribution, the Burr distribution allows for a non-monotonic hazard functions, which can extensively improve the goodness-of-fit of the ACD models (see Grammig, Maurer, 2000; Bauwens et al., 2004).

I apply the formulas for the hazard and survival functions of a Burr-distributed random variable (see Appendix 1) in equations (6) and (7) in order to derive the hazard and the survival functions $h_{x_a}(\cdot)$ ($h_{x_b}(\cdot)$) and $S_{x_a}(\cdot)$ ($S_{x_b}(\cdot)$). Then, it is straightforward to rewrite the conditional bivariate density outlined in (1) as:

$$
\begin{aligned}
f^{(B)}(x_i, y_i \mid F_{i-1}) = &\left(\frac{\kappa_a x_i^{\kappa_a - 1} \Phi_{i,a}^{-\kappa_a}}{\left(1 + \sigma_a^2 x_i^{\kappa_a} \Phi_i^{-\kappa_a}\right)} \right)^{I_i^a} \left(1 + \sigma_a^2 x_i^{\kappa_a} \Phi_{i,a}^{-\kappa_a}\right)^{-\frac{1}{\sigma_a^2}} \\
&\cdot \left(\frac{\kappa_b x_i^{\kappa_b - 1} \Phi_{i,b}^{-\kappa_b}}{\left(1 + \sigma_b^2 x_i^{\kappa_b} \Phi_i^{-\kappa_b}\right)} \right)^{I_i^b} \left(1 + \sigma_b^2 x_i^{\kappa_b} \Phi_{i,b}^{-\kappa_b}\right)^{-\frac{1}{\sigma_b^2}}.
\end{aligned}
\tag{8}
$$

The marginal distribution of x_i in the B-AACD model can be derived as:

$$
\begin{aligned}
f^{(B)}(x_i \mid F_{i-1}) &= f^{(B)}(x_i, y_i = a \mid F_{i-1}) + f^{(B)}(x_i, y_i = b \mid F_{i-1}) \\
&= \left(\frac{\kappa_a x_i^{\kappa_a - 1}}{\Phi_{i,a}^{\kappa} \left(1 + \sigma_a^2 x_i^{\kappa_a} \Phi_i^{-\kappa_a}\right)} + \frac{\kappa_b x_i^{\kappa_b - 1}}{\Phi_{i,b}^{\kappa_b} \left(1 + \sigma_b^2 x_i^{\kappa_b} \Phi_i^{-\kappa_b}\right)} \right) \\
&\cdot \left(1 + \sigma_a^2 x_i^{\kappa_a} \Phi_{i,a}^{-\kappa_a}\right)^{-\frac{1}{\sigma_a^2}} \cdot \left(1 + \sigma_b^2 x_i^{\kappa_b} \Phi_{i,b}^{-\kappa_b}\right)^{-\frac{1}{\sigma_b^2}}.
\end{aligned}
\tag{9}
$$

Bauwens and Giot derive the conditional (with respect to a current duration and the past filtration F_{i-1}) transition probabilities between state a and b for the AACD model with the Weibull distribution (see Bauwens, Giot, 2003). In the case of the B-AACD model, such conditional transition probabilities can be given as:

$$
\begin{aligned}
f^{(B)}(y_i \mid x_i, F_{i-1}) &= \frac{f^{(B)}(x_i, y_i \mid F_{i-1})}{f^{(B)}(x_i \mid F_{i-1})} \\
&= \left(\kappa_a x_i^{\kappa_a - 1} (\Phi_{i,a}^{\kappa_a} + \sigma_a^2 x_i^{\kappa_a})^{-1}\right)^{I_i^a} \left(\kappa_b x_i^{\kappa_b - 1} (\Phi_{i,b}^{\kappa_b} + \sigma_b^2 x_i^{\kappa_b})^{-1}\right)^{I_i^b} \\
&\cdot \left(\frac{\kappa_a x_i^{\kappa_a - 1}}{\Phi_{i,a}^{\kappa} \left(1 + \sigma_a^2 x_i^{\kappa_a} \Phi_i^{-\kappa_a}\right)} + \frac{\kappa_b x_i^{\kappa_b - 1}}{\Phi_{i,b}^{\kappa_b} \left(1 + \sigma_b^2 x_i^{\kappa_b} \Phi_i^{-\kappa_b}\right)} \right)^{-1}.
\end{aligned}
\tag{10}
$$

As $f^{(B)}(y_i \mid x_i, F_{i-1})$ depends on x_i, x_i and y_i are not independent.

The log-likelihood function of the B-AACD model is obtained as a sum of N logarithms of conditional probabilities given in equation (8) and it can be decomposed into two parts:

$$
L(\Theta_1, \Theta_2) = L_1(\Theta_a) + L_2(\Theta_b), \tag{11}
$$

where:

$$
\begin{aligned}
L_1(\Theta_a) = \sum_{i=1}^{N} \Big\{ & I_i^a \left[\ln \kappa_a - \ln x_i^{(\kappa_a - 1)} - \ln \Phi_{i,a}^{\kappa_a} - \ln\left(1 + \sigma_a^2 x_i^{\kappa_a} \Phi_i^{-\kappa_a}\right) \right] \\
& - \frac{1}{\sigma_a^2} \ln\left(1 + \sigma_a^2 x_i^{\kappa_a} \Phi_i^{-\kappa_a}\right) \Big\},
\end{aligned}
\tag{12}
$$

$$L_2(\Theta_b) = \sum_{i=1}^{N} \Big\{ I_i^b \left[\ln \kappa_b - \ln x_i^{(\kappa_b - 1)} - \ln \Phi_{i,b}^{\kappa_b} - \ln\left(1 + \sigma_b^2 x_i^{\kappa_b} \Phi_i^{-\kappa_b}\right) \right] - \frac{1}{\sigma_b^2} \ln\left(1 + \sigma_b^2 x_i^{\kappa_b} \Phi_i^{-\kappa_b}\right) \Big\}, \tag{13}$$

and

$\Theta_a = \left\{\kappa_a, \sigma_a^2, \omega_{a,a}, \omega_{b,a}, \alpha_{a,a}, \alpha_{b,a}, \beta_a\right\}$, $\Theta_b = \left\{\kappa_b, \sigma_b^2, \omega_{a,b}, \omega_{b,b}, \alpha_{a,b}, \alpha_{b,b}, \beta_b\right\}$.

The model can be easily estimated by maximising the joint likelihood given by equation (11). As the two components of the likelihood function (L_1 and L_2) depend on different parameters, they can also be maximized separately, as suggested by Bauwens, Giot (2003) in the case of the AACD model with the Weibull distribution.

1.3. AACD with the Generalized Gamma Distribution

Generalized gamma distribution has two shape parameters γ and ν. As the Burr distribution, it allows for different, non-monotonic shapes of the hazard function. It nests a gamma distribution (if $\gamma = 1$), a Weibull distribution (if $\nu = 1$) and an exponential distribution (if $\gamma = 1$, $\nu = 1$). As the Burr distribution, the generalized gamma distribution is often applied to the ACD models (see Bauwens, Giot, 2001; Bauwens et al., 2004, among others).

Substitution of the hazard and the survival of a generalized gamma distribution for $\varepsilon_{i,a}$ ($\varepsilon_{i,b}$) into equations (6) and (7) results in the hazard and survival function for $x_{i,a}$ ($x_{i,b}$). Accordingly, the conditional bivariate density of the pair $\{x_i, y_i\}$ is given as:

$$f^{(GG)}(x_i, y_i \mid F_{i-1}) = \left(\frac{\gamma_a x_i^{\nu_a \gamma_a - 1} \Phi_{i,a}^{-\nu_a \gamma_a} \exp(-x_i^{\gamma_a} \Phi_{i,a}^{-\gamma_a})}{\Gamma(\nu_a)(1 - \Gamma^i(\nu_a, x_i^{\gamma_a} \Phi_{i,a}^{-\gamma_a})} \right)^{I_i^a} \left(1 - \Gamma^i(\nu_a, x_i^{\gamma_a} \Phi_{i,a}^{-\gamma_a})\right) \cdot \left(\frac{\gamma_b x_i^{\nu_b \gamma_b - 1} \Phi_{i,b}^{-\nu_b \gamma_b} \exp(-x_i^{\gamma_b} \Phi_{i,b}^{-\gamma_b})}{\Gamma(\nu_b)(1 - \Gamma^i(\nu_b, x_i^{\gamma_b} \Phi_{i,b}^{-\gamma_b})} \right)^{I_i^b} \left(1 - \Gamma^i(\nu_b, x_i^{\gamma_b} \Phi_{i,b}^{-\gamma_b})\right), \tag{14}$$

where Γ denotes the gamma function and Γ^i is the incomplete gamma function (see Appendix 1 for details).

The marginal distribution of x_i can be derived as:

$$
\begin{aligned}
&f^{(GG)}(x_i | F_{i-1}) = f^{(GG)}(x_i, y_i = a | F_{i-1}) + f^{(GG)}(x_i, y_i = b | F_{i-1}) \\
&= \left(\frac{\gamma_a x_i^{\nu_a \gamma_a} \Phi_{i,a}^{-\nu_a \gamma_a} \exp(-x_i^{\gamma_a} \Phi_{i,a}^{-\gamma_a})}{x_i \Gamma(\nu_a)(1 - \Gamma^i(\nu_a, x_i^{\gamma_a} \Phi_{i,a}^{-\gamma_a})} \right. \\
&\left. + \frac{\gamma_b x_i^{\nu_b \gamma_b} \Phi_{i,b}^{-\nu_b \gamma_b} \exp(-x_i^{\gamma_b} \Phi_{i,b}^{-\gamma_b})}{x_i \Gamma(\nu_b)(1 - \Gamma^i(\nu_b, x_i^{\gamma_b} \Phi_{i,b}^{-\gamma_b})} \right) \\
&\cdot \left(1 - \Gamma^i(\nu_a, x_i^{\gamma_a} \Phi_{i,a}^{-\gamma_a}) \right) \cdot \left(1 - \Gamma^i(\nu_b, x_i^{\gamma_b} \Phi_{i,b}^{-\gamma_b}) \right),
\end{aligned}
\tag{15}
$$

and the conditional transition probabilities between states a and b are:

$$
\begin{aligned}
&f^{(GG)}(y_i | x_i, F_{i-1}) = \frac{f^{(GG)}(x_i, y_i | F_{i-1})}{f^{(GG)}(x_i | F_{i-1})} \\
&= \left(\frac{\gamma_a x_i^{\nu_a \gamma_a - 1} \Phi_{i,a}^{-\nu_a \gamma_a} \exp(-x_i^{\gamma_a} \Phi_{i,a}^{-\gamma_a})}{\Gamma(\nu_a)(1 - \Gamma^i(\nu_a, x_i^{\gamma_a} \Phi_{i,a}^{-\gamma_a})} \right)^{I_i^a} \\
&\cdot \left(\frac{\gamma_b x_i^{\nu_b \gamma_b - 1} \Phi_{i,b}^{-\nu_b \gamma_b} \exp(-x_i^{\gamma_b} \Phi_{i,b}^{-\gamma_b})}{\Gamma(\nu_b)(1 - \Gamma^i(\nu_b, x_i^{\gamma_b} \Phi_{i,b}^{-\gamma_b})} \right)^{I_i^b} \\
&\cdot \left(\frac{\gamma_a x_i^{\nu_a \gamma_a} \Phi_{i,a}^{-\nu_a \gamma_a} \exp(-x_i^{\gamma_a} \Phi_{i,a}^{-\gamma_a})}{x_i \Gamma(\nu_a)(1 - \Gamma^i(\nu_a, x_i^{\gamma_a} \Phi_{i,a}^{-\gamma_a})} \right. \\
&\left. + \frac{\gamma_b x_i^{\nu_b \gamma_b} \Phi_{i,b}^{-\nu_b \gamma_b} \exp(-x_i^{\gamma_b} \Phi_{i,b}^{-\gamma_b})}{x_i \Gamma(\nu_b)(1 - \Gamma^i(\nu_b, x_i^{\gamma_b} \Phi_{i,b}^{-\gamma_b})} \right)^{-1}.
\end{aligned}
\tag{16}
$$

From that it can be seen that x_i and y_i are not independent.

The log-likelihood of the GG-AACD model can be derived as a sum of N logarithms of conditional probabilities given in (14). In a close analogy to the B-AACD model, the log-likelihood can be decomposed into two components, $L(\Theta_1, \Theta_2) = L_1(\Theta_a) + L_2(\Theta_b)$, where:

$$
\begin{aligned}
L_1(\Theta_a) = \sum_{i=1}^{N} \Big\{ & I_i^a \left[\ln \gamma_a + \ln x_i^{\gamma_a \nu_a - 1} - \gamma_a \nu_a \ln(\Phi_{i,a}) - x_i^{\gamma_a} \Phi_{i,a}^{-\gamma_a} \right] \\
& - \ln(\Gamma(\nu_a)) - \ln(1 - \Gamma^i(\nu_a, x_i^{\nu_a} \Phi_{i,a}^{-\nu_a}) \Big] - \ln(1 - \Gamma^i(\nu_a, x_i^{\nu_a} \Phi_{i,a}^{-\nu_a}) \Big\},
\end{aligned}
\tag{17}
$$

$$
\begin{aligned}
L_2(\Theta_b) = \sum_{i=1}^{N} \Big\{ & I_i^b \left[\ln \gamma_b + \ln x_i^{\gamma_b \nu_b - 1} - \gamma_b \nu_b \ln(\Phi_{i,b}) - x_i^{\gamma_b} \Phi_{i,b}^{-\gamma_b} \right] \\
& - \ln(\Gamma(\nu_b)) - \ln(1 - \Gamma^i(\nu_b, x_i^{\nu_b} \Phi_{i,b}^{-\nu_b}) \Big] - \ln(1 - \Gamma^i(\nu_b, x_i^{\nu_b} \Phi_{i,b}^{-\nu_b}) \Big\},
\end{aligned}
\tag{18}
$$

and

$\Theta_a = \{\gamma_a, \nu_a, \omega_{a,a}, \omega_{b,a}, \alpha_{a,a}, \alpha_{b,a}, \beta_a\}$, $\Theta_b = \{\gamma_b, \nu_b, \omega_{a,b}, \omega_{b,b}, \alpha_{a,b}, \alpha_{b,b}, \beta_b\}$.

The estimation may be performed on the joint log-likelihood function or in two steps, separately for L_1 and L_2.

1.4. Testing the Distribution Choice with the PIT

The goodness-of-fit of the ACD models can be checked with the probability integral transforms (PIT) proposed by Diebold, Gunther and Tay (1998). This testing procedure has been used to check the adequacy of the distribution choice and the quality of the conditional mean specification in numerous studies on financial durations (e.g. Bauwens et al., 2004; Grammig, Mauer, 2000; Hautsch, 2004; Bień, 2006, among others). In a shortcut, this approach can be presented as following. If $\{f_i(x_i | F_{i-1})\}_1^m$ denotes a sequence of one-step-ahead density forecasts of the ACD model and $\{p_i(x_i | F_{i-1})\}_1^m$ is a sequence of conditional densities for the data generating process of financial durations, the ACD model will be correctly specified if the following equation holds:

$$\{f_i(x_i | F_{i-1})\}_1^m = \{p_i(x_i | F_{i-1})\}_1^m \tag{19}$$

Although the sequence $\{p_i(x_i | F_{i-1})\}_1^m$ cannot be observed, Diebold, Gunther and Tay (1998) show that if equation (19) holds true, the sequence of density transforms $\{z_i\}$ for durations $\{x_i\}$ should be *i.i.d.* uniformly distributed on (0,1):

$$z_i = \int_{-\infty}^{x_i} f_i(t)dt, \quad z_i \sim i.i.d.U(0,1). \tag{20}$$

It can be seen from formula (20) that in order to compute the sequence $\{z_i\}$, we need the cumulative distribution function (CDF) for x_i under given ACD specification. The marginal densities of x_i under the B-AACD data generating process (DGP) or under the GG-AACD DGP have been derived in equations (9) and (15), respectively.

The sequence of integrated density transforms for the B-AACD model can be calculated as:

$$\hat{z}_i^{(B)} = 1 - \left(1 + \hat{\sigma}_a^2 x_i^{\hat{\kappa}_a} \hat{\Phi}_{i,a}^{-\hat{\kappa}_a}\right)^{-\frac{1}{\hat{\sigma}_a^2}} \cdot \left(1 + \hat{\sigma}_b^2 x_i^{\hat{\kappa}_b} \hat{\Phi}_{i,b}^{-\kappa_b}\right)^{-\frac{1}{\hat{\sigma}_b^2}}, \tag{21}$$

which can be seen from the *Proof 1* or the *Proof 2*:

Proof 1:

$$\frac{d\hat{z}_i^{(B)}}{dx_i} = \hat{\kappa}_a x_i^{\hat{\kappa}_a - 1} \hat{\Phi}_{i,a}^{-\hat{\kappa}_a} \left(1 + \hat{\sigma}_a^2 x_i^{\hat{\kappa}_a} \hat{\Phi}_{i,a}^{-\hat{\kappa}_a}\right)^{-\frac{1}{\hat{\sigma}_a^2} - 1} \left(1 + \hat{\sigma}_b^2 x_i^{\hat{\kappa}_b} \hat{\Phi}_{i,b}^{-\kappa_b}\right)^{-\frac{1}{\hat{\sigma}_b^2}}$$

$$+ \hat{\kappa}_b x_i^{\hat{\kappa}_b - 1} \hat{\Phi}_{i,b}^{-\hat{\kappa}_b} \left(1 + \hat{\sigma}_b^2 x_i^{\hat{\kappa}_b} \hat{\Phi}_{i,b}^{-\hat{\kappa}_b}\right)^{-\frac{1}{\hat{\sigma}_b^2} - 1} \left(1 + \hat{\sigma}_a^2 x_i^{\hat{\kappa}_b} \hat{\Phi}_{i,a}^{-\kappa_b}\right)^{-\frac{1}{\hat{\sigma}_a^2}}$$

$$= \left(\frac{\hat{\kappa}_a x_i^{\hat{\kappa}_a - 1}}{\hat{\Phi}_{i,a}^{-\hat{\kappa}_a} \left(1 + \hat{\sigma}_a^2 x_i^{\hat{\kappa}_a} \hat{\Phi}_{i,a}^{-\hat{\kappa}_a}\right)} + \frac{\hat{\kappa}_b x_i^{\hat{\kappa}_b - 1}}{\hat{\Phi}_{i,b}^{-\hat{\kappa}_b} \left(1 + \hat{\sigma}_b^2 x_i^{\hat{\kappa}_b} \hat{\Phi}_{i,b}^{-\hat{\kappa}_b}\right)} \right)$$

$$\cdot \left(1 + \hat{\sigma}_a^2 x_i^{\hat{\kappa}_b} \hat{\Phi}_{i,a}^{-\kappa_b}\right)^{-\frac{1}{\hat{\sigma}_a^2}} \left(1 + \hat{\sigma}_b^2 x_i^{\hat{\kappa}_b} \hat{\Phi}_{i,b}^{-\kappa_b}\right)^{-\frac{1}{\hat{\sigma}_b^2}} = \hat{f}_B(x_i \mid F_{i-1})$$

Proof 2:

If $x_i = \min(x_i^a, x_i^b)$, z_i can be computed as:

$\hat{z}_i = C\hat{D}F(x_i | F_{i-1}) = 1 - \hat{S}(x_i \mid F_{i-1}) = 1 - \hat{S}_a(x_i \mid F_{i-1}) \hat{S}_b(x_i \mid F_{i-1})$

Analogously, from *Proof 2*, z_i for the GG-AACD model can be estimated as:

$$\hat{z}_i^{(GG)} = 1 - \left(1 - \Gamma^i(\hat{v}_a, x_i^{\hat{\gamma}_a} \hat{\Phi}_{i,a}^{-\hat{\gamma}_a})\right) \cdot \left(1 - \Gamma^i(\hat{v}_b, x_i^{\hat{\gamma}_b} \hat{\Phi}_{i,b}^{-\hat{\gamma}_b})\right), \qquad (22)$$

Application of the PIT diagnostic procedures often boils down to checking whether $\hat{z}_i$ is i.i.d. and uniformly distributed. Diebold, Gunther and Tay (1998) and Bauwens et al. (2004) emphasize visual inspection of graphs which depict dynamics and distributional properties of $\hat{z}_i$.

2. Empirical Example

As in the study of Lo and Sapp (2008), I apply the AACD model to analyze the order submission process in the Reuters Dealing 3000 Spot Matching System[3] (RDSM). The RDSM system is a fully automated[4] order-driven market where the interbank currency trading takes place. On this trading platform currency dealers can submit two major order types, i.e. market orders or limit orders, to buy or sell a given amount of the base currency[5]. From the viewpoint of the market microstructure, market orders are perceived as liquidity consuming – they are immediately executed against limit orders listed on the

---

[3] The Reuters Dealing 3000 Spot Matching System is an updated version of the Reuters Dealing 2000-2 Matching System described by Lo and Sapp (2008).

[4] Orders are automatically matched if they arrive to opposite market sides and if their prices agree.

[5] In case of the EUR/PLN currency pair, euro is the base (transaction) currency and zloty is the counter (quote) currency.

opposite side of an order book, hence they exhaust liquidity measured as market depth. Limit orders are liquidity supplying – they wait for a possible execution in future, hence they replenish depth on the ask or the bid side of a market.

I use data on order submissions on the EUR/PLN market during four days, from 2nd to 5th January 2007. The vast majority of Polish zloty trading takes place in the offshore market (between London banks) and in Poland. Therefore, in order to account for periods when trading is high, I consider orders placed after 8:00 CET and before 18:00 CET[6]. In the sample there are 10 515 orders, i.e. 4 848 market orders and 5 667 limit orders[7]. Order durations are defined as time intervals between subsequent moments of order submissions.

In the first step I deseasonalized durations as suggested in the literature on ACD models. I assume a multiplicative intraday seasonality factor s_i, such as $x_i = s_i \bar{x}_i$. As suggested by Bauwens and Veredas (2004), the intraday seasonality factor s_i has been estimated with the application of the kernel regression of x_i on a time-of-day variable[8]. Estimation[9] of the AACD models has been conducted on diurnally adjusted durations $\bar{x}_i$.

For the sake of completeness of my study I estimated four specifications of AACD models, the E-AACD model (with the exponential distribution of an error term), the W-AACD model (with the Weibull distribution of an error term), the B-AACD model and the GG-AACD model. The specifications of conditional expectations were always the same – as in equations (3)–(4). The Bayesian Information Criterion (BIC = 2.6197) favorises the B-AACD model over the GG-AACD (BIC = 2.6540), the W-AACD model (BIC = 2.7296) and the E-AACD model (BIC = 2.9146).

Histograms and autocorrelation functions for the probability integral transforms $\hat{z}_i$ are depicted in Figure 1 and 2. As can be seen in Figure 1, neither the exponential, nor the Weibull distribution are proper for the AACD model. Long (but not very long) durations are underrepresented in both specifications and the distribution of the probability integral transforms is far from being uniform.

Additionally, the Weibull distribution demands more observations of a small value than registered in the duration series. Both parsimonious distributions are not flexible enough to reflect the shape of the true data generating process. The generalized gamma distribution does not fit the data as

[6] Similar truncation was performed by Lo and Sapp (2008).

[7] As in (Lo, Sapp 2008) I accounted for the best limit orders only, i.e. orders that are placed within the best ask and bid prices in the order book.

[8] Quartic kernel is used, with the bandwidth computed as $2.78sN^{-1/5}$, where s is the standard deviation of the data. For details of the estimation procedure please refer to (Bauwens, Veredas, 2004).

[9] The whole empirical study has been performed in Gauss 8.0.

well. Just as the Weibull distribution, it gives too much probability mass to small durations and too little probability mass to a medium-sized durations (from the third to fifth quantile of the IPF distribution). Better distribution choice would require less observations in the lower tail of the distribution and more observations of a middle-sized value. Although the visual inspection suggests that the B-AACD has won the competition among the models, the choice of the Burr distribution is not optimal as well. The standard Pearson's goodness-of-fit statistic[10] $\hat{\chi}^2$ for the uniformity of the $\hat{z}_i^{(B)}$ distribution equals 158.53. Because $\hat{\chi}^{2*} = 30.14$ (5% significance level), so the null of uniformity should be rejected.

The dynamic properties of the model are not perfect, as the ACF function for the PIT depicts significant autocorrelation of the first order. Nevertheless, once more B-AACD model seems to provide the best fit among selected specifications. The AACD models were parsimoniously parameterized in terms of the conditional mean functions in order to avoid a burdensome estimation, but the literature on the ACD models shows that it is very difficult to find a satisfying model as far as its dynamic features are concerned (see Bauwens et al., 2004).

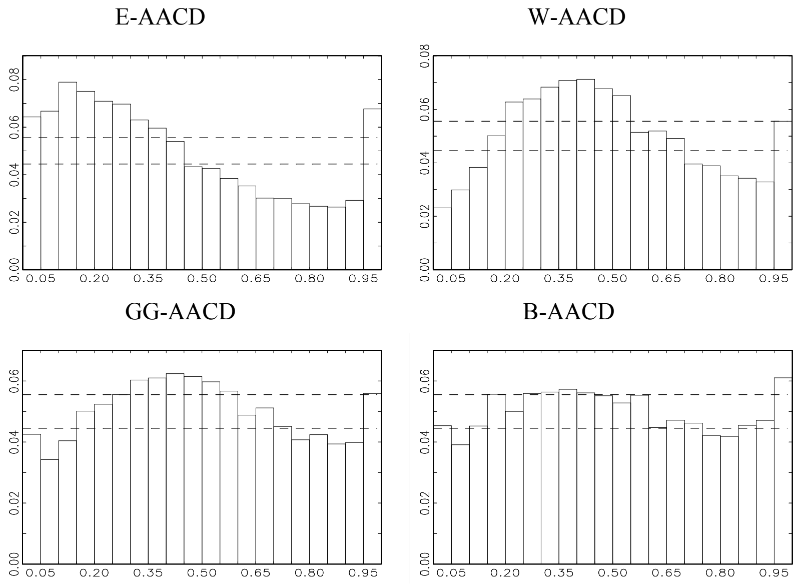

Figure 1. Histogram of the probability integral transforms for the AACD models. Horizontal lines depict the 99% confidence interval

[10] $\chi^2 = \sum_{i=1}^{m} \frac{(n_i - Np)^2}{Np}$, where $m=20$ (number of histogram bins), $p=1/m$.

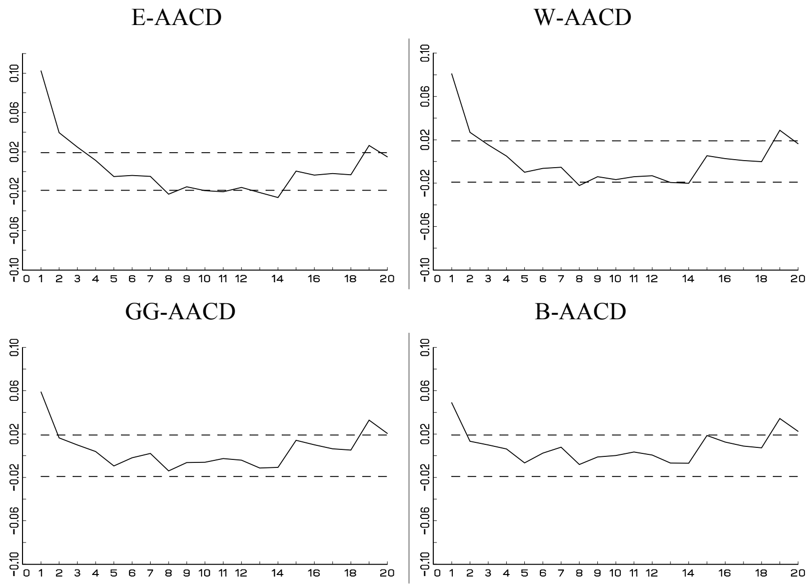

Figure 2. The ACF function of the probability integral transforms for the AACD models. Horizontal lines depict the 99% confidence interval

In the last step of this study, I introduce two explanatory variables into the B-AACD specification. I check the impact of the bid-ask spread and the EUR/PLN volatility – separately – on the expected durations to a market order versus an expected duration to a limit order. There is a large body of theoretical and empirical studies proving that these both factors matter as far as order choice decisions are considered (Foucault, 1999; Hautsch, 2004; Lo, Sapp, 2008; among others). The bid-ask (difference between the best ask and bid price at the moment of order submission) and the volatility (the realized volatility estimate during 10 minutes interval before the order submission) were cleared form the intraday seasonality in the same way as the order durations.

In Table 1, I report the ML estimates and the corresponding p-values (for the robust standard errors) of the B-AACD model. The estimated shape parameter proves that the assumption of the Weibull distribution for the error terms is not proper. As $\hat{\sigma}_a^2 = 0.9425$ and $\hat{\sigma}_b^2 = 1.0777$, the obtained Burr distribution seems to be rather "closer" to a log-logistic distribution (where $\sigma^2 = 1$), than to a Weibull one ($\sigma^2 \to 0$). As far as the dynamic properties of the AACD model are concerned, obtained results agree with the results of Lo and Sapp (2008). For $k = b$, that is for durations ending with a market order I have $\hat{\omega}_{a,a} < \hat{\omega}_{a,b}$ and $\hat{\alpha}_{a,a} > \hat{\alpha}_{a,b}$, hence in result of a previously observed market order, the expected duration to another market order shrinks stronger than directly after a limit order. Such a result seems to agree with a study of Biais et al. (1995) performed for selected stocks traded on the Paris Bourse. It is predicted there that market orders placed on the same side of a market cluster together as traders: (1) may split large orders into small ones to avoid a huge price impact, (2) mimic each other or (3) react similarly to given events. In this study I do not differentiate between ask and bid side of a market, but I suspect that overall clustering of market orders may be caused by similar behaviour patterns. On the other hand, the complicated dynamic structure of the AACD model makes it extremely difficult to capture all information given by distinct parameters and to interpret them accordingly. As far as expected durations that end with a limit orders are concerned ($k = b$), the obtained relations are more unequivocal and easy to interpret. As $\hat{\omega}_{b,a} < \hat{\omega}_{b,b}$ and $\hat{\alpha}_{b,a} < \hat{\alpha}_{b,b}$, the expected duration to a limit order shrinks more considerably after a market order than after a limit order. Market order always "erodes" depth on the ask or bid side of a market which may result in a wider bid-ask spread. Therefore it is more profitable to submit a limit order than a market order, large bid-ask spread makes liquidity consumption more costly. The complete dynamics of the model can be captured in a more detailed way with a simulation of the whole process, as proposed by Bauwens and Giot, (2003).

The bid-ask spread has a significant positive impact on the expected time to a market order and it has a negative impact on the expected time to best limit orders. This result agrees with findings of several studies (Ranaldo, 2004; Verhouven et al., 2003; Ellul et al., 2007; Lo, Sapp, 2008). If the bid-ask spread is large, it becomes more costly for a trader to cross the difference between the best bid and ask prices in order to buy or sell the currency in an immediate way. The bid-ask spread constitutes the cost of such quick transaction. On the other hand, the traders opt for limit orders. As the differences between best bid and ask prices are large, it is much easier for them to compete for the transaction priority by offering a price at least one tick better than the current one ("the tick rule").

An increase in volatility prompts market orders, as it has a negative significant impact on the expected time to a market order submission. A rise in uncertainty about the future movement of the EUR/PLN rate encourages traders to close their open currency positions or to realize their gains quickly. The positive impact of volatility on the expected time to a limit order can be easily understood with the nature of a limit order. The submission of a limit order is the same as writing of the option – if the FX rate moves in the undesirable direction, it can be executed at an unfavourable price (so called "free option risk" of a limit order). As prices in limit orders are frozen during their lifetime, an increased volatility increases the risk of incurring potential losses.

Table 1. ML estimates for the B-AACD model

	$k = a$ (market order)		$k = b$ (limit order)	
parameters	estimate	p – val	estimate	p - val
$\omega_{k,a}$	0.0905	0.0404	0.5388	0.0000
$\omega_{k,b}$	0.3541	0.0000	0.8631	0.0000
β_k	0.7478	0.0000	0.6912	0.0000
$\alpha_{k,a}$	0.1638	0.0000	0.0908	0.0000
$\alpha_{k,b}$	0.1647	0.0000	0.2195	0.0000
κ_k	0.9104	0.0000	1.2104	0.0000
σ_k^2	0.5844	0.0000	0.9840	0.0000
bid-ask spread	1.6472	0.0000	-0.6609	0.0000
volatility	-0.3537	0.0000	0.2304	0.0000
BIC		2.4122		

Conclusions

In the paper I have extended the asymmetric ACD model of Bauwens, Giot (2003) with respect to more general distribution families: the Burr and the generalized gamma. As in the study of Bauwens and Giot (2003), I present the basic properties of the generalized specifications. Additionally, we showed an easy way of testing the goodness-of-fit of such a competing risk models with the probability integral transforms. As Lo and Sapp (2008), I have also presented an exemplary application of the AACD model to the order submission process on the interbank order-driven market for a Polish zloty. The obtained results with regards to the impact of the bid-ask spread and the volatility on the order choice confirm the main results from the empirical literature.

There are many possible extentions to my study. A most natural one would be a more finance-oriented empirical application. Introducing more explanatory factors, reflecting the larger scope of information that can be deduced from the order book or the external market environment would give more insight into the process of market liquidity fluctuation as studied by Lo and Sapp (2008). Second, different functional form for duration expectations or even some more general distributions for the error term would possibly result in a better fit of the model. The PIT diagnostic tool points the Burr distribution as a best one, although this distribution choice does not seem to be the optimal one. The possible solution would be even more general distribution, such as a generalized F distribution advocated by Hautsch (2001) for the ACD models. Third, the assymetric ACD model could be easily extended to more than two competing risks. It could result in a more flexible specification that, in the context of a current empirical study, could also discriminate between orders listed on either bid or ask side of the market.

References

Bauwens, L., Giot, P. (2000), The Logarithmic ACD Model: An Application to the Bid/Ask Quote Process of two NYSE Stocks, *Annales d'Economie et de Statistique*, 60, 117–149.

Bauwens, L., Giot, P. (2001), *Econometric Modelling of Stock Market Intraday Activity*, Kluwer Academic Publishers, Boston.

Bauwens, L., Giot, P. (2003), Asymmetric ACD Models: Introducing Price Information in ACD Models, *Empirical Economics*, 28, 709–731.

Bauwens, L., Giot, P., Grammig, J., Veredas, D. (2004), A Comparison of Financial Duration Models via Density Forecasts, *International Journal of Forecasting* 20, 589–609.

Bauwens, L., Veredas, D. (2004), The Stochastic Conditional Duration Model: A Latent Variable Model for the Analysis of Financial Durations, *Journal of Econometrics* 119, 381–412.

Biais, B., Hillion, P., Spatt, C. (1995), An Empirical Analysis of the Limit Order Book and the Order Flow in the Paris Bourse, *Journal of Finance* 50, 1655–1689.

Bień, K. (2006), Advanced ACD Models – Presentation and the Example of Application, *Statistical Review* 53 (1), 90–108.

Diebold, F.X., Gunther, T.A., Tay, A.S. (1998), Evaluating Density Forecasts with Applications to Financial Risk Management, *International Economic Review* 39, 863–883.

Ellul, A., Holden C., Jain, P., Jennings, R. (2007), Order Dynamics: Recent Evidence from the NYSE, *Journal of Empirical Finance*, 14, 636–661.

Engle, R.F., Russell, J.R. (1998), Autoregressive Conditional Duration; a New Approach for Irregularly Spaced Transaction Data, *Econometrica* 66, 1127–1162.

Foucault, T. (1999), Order Flow Decomposition and Trading Costs in a Dynamic Limit Order Market, *Journal of Financial Markets* 2, 99–134.

Ghysels, E., Jasiak, J. (1998), GARCH for Irregularly Spaced Financial Data: The ACD-GARCH Model, *Studies in Nonlinear Dynamics and Econometrics* 2, 133–149.

Giot, P. (2000), Time Transformations, Intraday Data and Volatility Models, *Journal of Computational Finance*, 4(2), 31–62.

Giot, P. (2005), Market Risk Models for Intraday Data, *European Journal of Finance*, 11, 309–324.

Gouriéroux, C., Jasiak, J. (2001), *Financial Econometrics: Problems, Models and Methods*, Princeton University Press.

Grammig, J., Maurer, K. (2000), Non-monotonic Hazard Functions and the Autoregressive Conditional Duration Model, *Econometrics Journal* 3, 16–38.

Grammig, J., Wellner, M. (2002), Modeling the Interdependence of Volatility and Inter-Transaction Duration Process, *Journal of Econometrics*, 106, 369–400.

Hautsch, N. (2001), The Generalized F ACD Model, *Discussion Paper*, University of Konstanz

Hautsch, N. (2004), *Modelling Irregularly Spaced Financial Data*, Springer, Berlin.

Kalimipalli, M., Warga, A. (2002), Bid-Ask Spread, Volatility and Volume in the Corporate Bond Market, *Journal of Fixed Income*, 3, 31–42.

Lo, I., Sapp, S. (2008), The Submission of Limit Orders or Market Orders: The Role of Timing and Information in the Reuters D2000-2 System, *Journal of International Money and Finance*, 27, 1056–1073.

Nolte, I. (2008), Modeling a Multivariate Trading Process, *Journal of Financial Econometrics*, 6, 143–170.

Osińska, M. (2006), *Financial Econometrics*, PWE, Warszawa.

Pacurar. M. (2008), Autoregressive Conditional Duration Models in Finance: A Survey of Theoretical and Empirical Literature, *Journal of Economic Surveys,* 22 (4), 711–751.

Ranaldo, A. (2004), Order Aggresiveness in Limit Order Book Markets, Journal of Financial *Markets*, 7, 53–74.

Tsay, R. S. (2002), *Analysis of Financial Time Series*, Wiley: New York.

Verhoeven, P., Ching, S., Ng, H. (2003), Determinants of the Decision to Submit Market or Limit Orders on the ASX, *Pacific-Basin Finance Journal*, 12, 1–18.

Zhang, M. Y., Russell, J. R., Tsay, R. S. (2001), A Nonlinear Autoregressive Conditional Duration Model with Applications to Financial Transaction Data, *Journal of Econometrics* 104, 179–207

Appendix 1

***Burr distribution* with parameters** $\kappa > 0$**,** $\sigma^2 > 0$ and scale parameter is set to 1**:**

survival: $S(\varepsilon) = (1+\sigma^2\varepsilon^\kappa)^{-\frac{1}{\sigma^2}}$,

density: $f(\varepsilon) = \dfrac{\kappa\varepsilon^{\kappa-1}}{(1+\sigma^2\varepsilon^\kappa)^{1+1/\sigma^2}}$,

hazard: $h(\varepsilon) = \dfrac{\kappa\varepsilon^{\kappa-1}}{1+\sigma^2\varepsilon^\kappa}$,

expectation: $\mu = \dfrac{\Gamma\left(1+\kappa^{-1}\right)\Gamma\left(\dfrac{1}{\sigma^2}+\kappa^{-1}\right)}{\sigma^{2(1+\kappa^{-1})}\Gamma\left(\dfrac{1}{\sigma^2}+1\right)}$ if $\kappa > \sigma^2$.

Generalized gamma distribution* with parameters** $\gamma > 0$**,** $\nu > 0$ where the scale parameter is set to 1:***

survival: $S(\varepsilon) = 1 - \Gamma^i(\nu, \varepsilon^\gamma)$,

where Γ^i is the lower incomplete gamma function:

$\Gamma^i(\nu/\gamma, \varepsilon^\gamma) = \int_0^{\varepsilon^\gamma} \dfrac{t^{\nu-1}e^{-t}}{\Gamma(\nu)}dt$ and $\Gamma(\cdot)$ - gamma function,

density: $f(\varepsilon) = \dfrac{\gamma\varepsilon^{\nu\gamma-1}\exp(-\varepsilon^\gamma)}{\Gamma(\nu)}$,

hazard: $f(\varepsilon) = \dfrac{\gamma\varepsilon^{\nu\gamma-1}\exp(-\varepsilon^\gamma)}{\Gamma(\nu)\Gamma^i(\nu,\varepsilon^\gamma)}$,

expectation: $\mu = \dfrac{\Gamma\left(\nu+\gamma^{-1}\right)}{\Gamma(\nu)}$.

Wybór rozkładu składnika losowego w asymetrycznych modelach ACD

Z a r y s t r e ś c i W artykule dokonano uogólnienia asymetrycznego modelu ACD, zaproponowanego w pracy (Bauwens, Giot, 2003) w odniesieniu do nowych rozkładów składnika losowego: rozkładu Burra i uogólnionego rozkładu gamma. Wyprowadzono funkcję wiarygodności dla rozszerzonych specyfikacji i przedstawiono procedurę testowania jakości dopasowania modeli za pomocą transformat gęstości (Diebold i in., 1998). Dodatkowo, przedstawiono przykładową aplikację asymetrycznych modeli ACD w odniesieniu do odstępów czasu (tzw. czasów trwania) pomiędzy momentami, w których składane są zlecenia z limitem ceny lub zlecenia rynkowe na kierowanym zleceniami międzybankowym kasowym rynku złotego. Dokonano weryfikacji wpływu dwóch czynników mikrostruktury rynku (spreadu bid-ask i zmienności) na tempo składania wyróżnionych typów zleceń.

S ł o w a k l u c z o w e: asymetryczny model ACD, finansowe czasy trwania, transformaty gęstości, mikrostruktura rynku.

Acknowledgements

The author thanks the Thomson Reuters and the National Bank of Poland for providing the data from the Reuters Dealing 3000 Spot Matching System and Professor Małgorzata Doman for valuable comments and suggestions. The views and opinions presented in the paper are those of the author and do not reflect those of the National Bank of Poland.