DYNAMIC ECONOMETRIC MODELS
Vol. 10 – Nicolaus Copernicus University – Toruń – 2010

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Measuring Nonlinear Serial Dependencies Using the Mutual Information Coefficient†

A b s t r a c t: Construction, estimation and application of the mutual information measure have been presented in this paper. The simulations have been carried out to verify its usefulness to detect nonlinear serial dependencies. Moreover, the mutual information measure has been applied to the indices and the sector sub-indices of the Warsaw Stock Exchange.

K e y w o r d s: nonlinearity, mutual information coefficient, mutual information, serial dependencies.

1. Introduction

Measuring relationships between variables is an extremely important area of research in econometrics. To this end the Pearson correlation coefficient is commonly used. However, the Pearson coefficient is not a proper tool for measuring nonlinear dependencies. Therefore, in the case of nonlinearity other methods must be used. The mutual information coefficient is one of the most important tools to detect nonlinear relationships. It comes from the information theory and is based on a concept of entropy. The mutual information coefficient may be applied to measure dependencies between two time series or serial dependencies in a single time series.

2. Measuring Nonlinear Dependencies in Time Series

There are various methods to measure nonlinear dependencies in time series (cf. Granger, Terasvirta, 1993; Maasoumi, Racine, 2002; Bruzda, 2004). One of the most important is the mutual information measure (MI hereafter), given by the formula:

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† Financial support of Nicolaus Copernicus Univerity in Toruń for the project UMK 397-E is gratefully acknowledged.

$$I(X,Y)=\iint p(x,y)\log\left(\frac{p(x,y)}{p_1(x)p_2(y)}\right)dxdy, \tag{1}$$

where $p(x,y)$ is a joint probability density function and $p_1(x)$ and $p_2(y)$ are marginal densities for random variables *X* and *Y*.

It can be shown that for all *X* and *Y* the measure $I(X,Y)$ takes non-negative values and $I(X,Y)=0$ only if *X* and *Y* are independent.

It is convenient to define the mutual information coefficient, given by the expression:

$$R(X,Y)=\sqrt{1-e^{-2I(X,Y)}}. \tag{2}$$

It can be shown that the mutual information coefficient has the following properties (cf. Granger, Terasvirta, 1993; Granger, Lin, 1994):

1. $0\le R(X,Y)\le 1$,
2. $R(X,Y)=0 \Leftrightarrow X$ and Y are independent,
3. $R(X,Y)=1 \Leftrightarrow Y=f(X)$, where *f* is some invertible function,
4. *R* is unaltered if *X*, *Y* are replaced by instantaneous transformations $h_1(X),h_2(Y)$, i.e. $R(X,Y)=R(h_1(X),h_2(Y))$,
5. if (X,Y) (or $(h_1(X),h_2(Y))$, where h_1 and h_2 are instantaneous) has a joint Gaussian distribution with correlation $\rho(X,Y)$, then $R(X,Y)=|\rho(X,Y)|$.

In the literature one can find several methods for estimating a value of $I(X,Y)$. Essentially, due to the technique of estimating the probability density functions in Equation 1, they can be divided into three main groups (cf. Dionisio, Menezes, Mendes, 2003):

- histogram-based estimators,
- kernel-based estimators,
- parametric methods.

The kernel-based estimators have many adjustable parameters such as the optimal kernel width and the optimal kernel form, and a non-optimal choice of those parameters may cause a large bias in the results. For the application of parametric methods one needs to know the specific form of the generating process (Dionisio, Menezes, Mendes, 2003)). Therefore a standard way is to estimate the densities by means of histograms (cf. Darbellay, Wuertz, 2000).

One can also define auto mutual information at lag *k* for a stationary discrete-valued stochastic process $X_1,X_2,...,X_n$ as the mutual information between random variables X_t and X_{t+k}:

$$I(X_t, X_{t+k}) = \sum_{x_t} \sum_{x_{t+k}} P(x_t, x_{t+k}) \log\left(\frac{P(x_t, x_{t+k})}{P(x_t)P(x_{t+k})} \right). \tag{3}$$

Since the process is stationary, $I(X_t, X_{t+k})$ is independent of t and so we can refer to the mutual information at lag k, as $I(k)$ (Fonseca, Crovella, Salamatian, 2008).

This means that, the mutual information measure may be used to measure serial dependencies in a single time series as well. To this end, the past realizations of the investigated data X should be taken as the variable Y.

It should be emphasized that MI measures both linear and nonlinear dependencies, so to identify serial nonlinear relationships, analyzed data must be prefiltered by an estimated ARMA-type model.

3. Application of the Mutual Information Measure to Detect Serial Dependencies

3.1. Simulated Data

The aim of the simulations was to verify, if the mutual information measure may be effectively applied to detect nonlinear serial dependencies.

The time series produced from five different generating models and two different sample sizes (with each of those models) were used in the simulations. This data was generated by Barnett et al. (1998) to compare the power of some popular tests for nonlinearity and chaos[1]. Specifically, these were: five time series of 2000 observations – M1, M2, M3, M4, M5 and five time series of their first 380 observations – M1s, M2s, M3s, M4s, M5s.

The investigated series were generated from the following models[2]:

I) M1 – logistic map[3]:

$$x_t = 3.57 x_{t-1}(1 - x_{t-1}), \tag{4}$$

II) M2 – GARCH(1,1) process:

$$x_t = \sqrt{h_t} u_t, \tag{5a}$$

$$h_t = 1 + 0.1 x_{t-1}^2 + 0.8 h_{t-1}, \tag{5b}$$

where $h_0 = 1$ and $x_0 = 0$.

[1] The data was downloaded from the homepage of W.A. Barnett: http://econ.tepper.cmu.edu/ /barnett/Papers.html.

[2] In all cases, the white-noise disturbances – u_t were sampled independently from the standard normal distribution.

[3] The logistic map with the parameter equaled to 3.57 generates chaotic dynamics.

III) M3 – Nonlinear Moving Average Process (NLMA):

$$x_t = u_t + 0.8u_{t-1}u_{t-2}, \tag{6}$$

IV) M4 – ARCH(1) process:

$$x_t = \sqrt{1 + 0.5x_{t-1}^2}\,u_t, \tag{7}$$

V) M5 – ARMA(2,1) process:

$$x_t = 0.8x_{t-1} + 0.15x_{t-2} + u_t + 0.3u_{t-1}, \tag{8}$$

where $x_0 = 1$ and $x_1 = 0.7$.

In each case the mutual information measure was calculated for the raw series and for its residuals from the fitted ARMA model.

First, stationarity was verified using the Augmented Dickey-Fuller test. The null hypothesis of a unit root was strongly rejected for the all investigated data, except M5s. Thus, instead of M5s, the series of its first differences – M5s_diff was chosen for further research.

In Table 1 the ARMA models fitted to analyzed series are presented[4].

Table 1. ARMA models for the generated series

Series	ARMA model	Series	ARMA model
M1	White noise (EX=0.648)	M1s	White noise (EX=0.649)
M2	White noise (EX=0.034)	M2s	White noise (EX=0.067)
M3	White noise (EX= 0.007)	M3s	White noise (EX= 0.033)
M4	White noise (EX= 0.011)	M4s	White noise (EX= 0.018)
M5	ARMA(1,1)	M5s_diff	MA(1)

Next, the Ljung-Box test was applied to test if the residual series are white noise. The test confirmed that no investigated residuals contain linear dependencies.

To estimate the mutual information measure the method proposed by Fraser and Swinney (1986) was used[5]. This method is based on an analysis of the two-dimensional histogram. Briefly speaking, it consists in covering the two-dimensional plane containing pairs (x_t, y_t) with rectangular partitions and calculating frequencies of points in each partition. Next, Equation 1 is used, i.e. the calculated frequencies are estimators of the probability density functions and the integration is carried out numerically.

Let i_k denotes an estimated value of the mutual information measure between variables X_t and X_{t-k}. Due to a purpose of the research, the key task is to verify the hypothesis of mutual information measure's insignificance (i.e the hypothesis of independence). To this end, for each investigated series and for

[4] The models were selected based on the Schwarz criterion.

[5] In the calculations the m-file created by A. Leontitsis was used.

each $k = 1,2,\ldots,10$, the p-value was evaluated through bootstraping[6] with $10\,000$ repetitions[7]. In Tables 2-6 the calculated values of i_k and the corresponding p-values (at the bottom) are summarized. The p-values not larger than 0.005 are bolded[8].

Table 2. Values of i_k for M1s and M1

series \ *k*	1	2	3	4	5	6	7	8	9	10
M1s	1.6927	1.6963	1.6123	1.7148	1.5919	1.6849	1.5412	1.6381	1.5379	1.6560
	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
M1	2.0139	2.0090	2.0064	2.2520	1.9981	1.9991	1.9940	2.2737	1.9891	1.9891
	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

Table 3. Values of i_k for M2s and M2

series \ *k*	1	2	3	4	5	6	7	8	9	10
M2s	0.0848	0.1538	0.1191	0.1308	0.1231	0.1616	0.1701	0.1162	0.1281	0.1228
	0.9616	0.0201	0.3802	0.1786	0.3052	0.0081	0.0029	0.4412	0.2187	0.3090
M2	0.0541	0.0562	0.0477	0.0488	0.0492	0.0509	0.0541	0.0461	0.0449	0.0334
	0.0053	0.0025	0.0808	0.0536	0.0451	0.0227	0.0052	0.1303	0.1868	0.9315

Table 4. Values of i_k for M3s and M3

series \ *k*	1	2	3	4	5	6	7	8	9	10
M3s	0.1857	0.1586	0.1425	0.1469	0.1323	0.1028	0.1897	0.1600	0.1525	0.1606
	0.0492	0.3316	0.6241	0.5429	0.8032	0.9927	0.0353	0.3096	0.4389	0.2987
M3	0.0725	0.0658	0.0307	0.0429	0.0309	0.0383	0.0372	0.0404	0.0389	0.0456
	0.0000	0.0001	0.9634	0.2065	0.9599	0.5426	0.6274	0.3724	0.4868	0.0976

Table 5. Values of i_k for M4s and M4

series \ *k*	1	2	3	4	5	6	7	8	9	10
M4s	0.1365	0.1667	0.1442	0.1349	0.1198	0.1367	0.1347	0.1327	0.1435	0.1464
	0.2663	0.0205	0.1562	0.2940	0.6104	0.2613	0.2959	0.3361	0.1641	0.1303
M4	0.1053	0.0472	0.0363	0.0379	0.0286	0.0344	0.0370	0.0475	0.0368	0.0344
	0.0000	0.0051	0.3383	0.2324	0.9261	0.5058	0.2866	0.0039	0.3074	0.5059

[6] Bootstrap without replacement (i.e. permutation) was performed. Bootstrapped p-values correspond to a one-sided test.

[7] In this way, for each of the filtered series an expected distribution of MI(1) (i.e. the MI measure with k=1) was determined. Next, this distribution has led to evaluation of the p-value for each k=1,2,...,10.

[8] Note that the rejection of the null of i_k insignificance for at least one k=1,2,...,10 implies the rejection of the hypothesis of serial independence. Therefore, adopting the value 0.005 for each k implies that the probability for a type I error (in the test of serial independence) is approximately 5%.

Table 6. Values of i_k for M5s and M5

series \ k	1	2	3	4	5	6	7	8	9	10
M5s	1.4787	1.1206	0.9817	0.8640	0.7505	0.6895	0.6344	0.6310	0.6173	0.6070
	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
M5s_ diff	0.1390	0.1658	0.1288	0.1438	0.1496	0.2012	0.1642	0.1297	0.1161	0.1387
	0.5519	0.1199	0.7509	0.4542	0.3452	0.0039	0.1351	0.7340	0.9125	0.5560
M5s_ diffMA	0.1224	0.1584	0.1225	0.1242	0.1444	0.1391	0.1624	0.1510	0.1495	0.1474
	0.7971	0.1595	0.7942	0.7668	0.3745	0.4816	0.1193	0.2584	0.2821	0.3179
M5	1.7145	1.3154	1.0949	0.9504	0.8414	0.7597	0.6958	0.6449	0.5917	0.5584
	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
M5ARMA	0.0422	0.0375	0.0417	0.0412	0.0355	0.0396	0.0419	0.0486	0.0434	0.0397
	0.2714	0.6530	0.3103	0.3438	0.8012	0.4685	0.2963	0.0398	0.2030	0.4640

In Tables 7-8 the results of nonlinearity detection carried out by the MI measure are summarized.

Table 7. Results of nonlinearity detection for the long series

Series	Serial dependencies	Nonlinearity
M1	YES	YES
M2	YES	YES
M3	YES	YES
M4	YES	YES
M5	YES	NO

Table 8. Results of nonlinearity detection for the short series

Series	Serial dependencies	Nonlinearity
M1s	YES	YES
M2s	YES	YES
M3s	NO	NO
M4s	NO	NO
M5s_ diff	YES	NO

As it is clearly seen, the MI measure correctly identified each of the investigated long series. In an application to the short series it led to erroneous conclusions in the case of M3s and M4s. The obtained result is consistent with studies by other authors, i.e. it indicates that histogram-based estimators may be unreliable in a case of a small number of observations (e.g. Dionisio, Menezes, Mendes, 2003).

3.2. Stock Market Indices

In this section the indices and the sector sub-indices of the Warsaw Stock Exchange from 2.01.2001–15.04.2009 (2078 observations) were analyzed. For the each index, the three time series were investigated: daily log returns, residuals from their ARMA and ARMA-GARCH models. Investigation of the residuals from the ARMA model gives information, if dependencies are nonlinear. If so, the standardized residuals from the ARMA-GARCH model were analyzed, to verify if this class of processes can capture nonlinear dynamics found in the investigated data[9]. The results of this analysis are presented in Tables 9-20.

Table 9. Values of i_k for the WIG index

series \ k	1	2	3	4	5	6	7	8	9	10
log returns	0.0458	0.0444	0.0605	0.0612	0.0486	0.0518	0.0350	0.0365	0.0522	0.0559
	0.0000	0.0003	0.0000	0.0000	0.0000	0.0000	0.0338	0.0153	0.0000	0.0000
MA(1)	0.0412	0.0455	0.0549	0.0632	0.0427	0.0500	0.0379	0.0313	0.0552	0.0566
	0.0010	0.0000	0.0000	0.0000	0.0002	0.0000	0.0074	0.1530	0.0000	0.0000
MA(1)-	0.0458	0.0498	0.0336	0.0395	0.0359	0.0306	0.0302	0.0352	0.0328	0.0309
GARCH(3,1)	0.0225	0.0033	0.7074	0.2254	0.5142	0.8964	0.9124	0.5738	0.7700	0.8823

Table 10. Values of i_k for the WIG20 index

series \ k	1	2	3	4	5	6	7	8	9	10
log returns	0.0514	0.0415	0.0577	0.0690	0.0489	0.0509	0.0388	0.0388	0.0438	0.0537
	0.0000	0.0106	0.0000	0.0000	0.0002	0.0000	0.0381	0.0373	0.0029	0.0000
MA(1)	0.0456	0.0471	0.0579	0.0687	0.0506	0.0510	0.0402	0.0458	0.0439	0.0545
	0.0011	0.0006	0.0000	0.0000	0.0001	0.0001	0.0187	0.0009	0.0028	0.0000
MA(1)-	0.0441	0.0457	0.0337	0.0382	0.0311	0.0333	0.0307	0.0384	0.0303	0.0272
GARCH(3,1)	0.0410	0.0222	0.6723	0.2962	0.8499	0.6978	0.8683	0.2806	0.8867	0.9756

Table 11. Values of i_k for the mWIG40 index

series \ k	1	2	3	4	5	6	7	8	9	10
log returns	0.0728	0.0508	0.0630	0.0603	0.0545	0.0660	0.0508	0.0343	0.0397	0.0428
	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0058	0.0002	0.0000
AR(3)	0.0511	0.0458	0.0539	0.0569	0.0508	0.0462	0.0465	0.0376	0.0379	0.0460
	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0002	0.0002	0.0000
AR(3)-	0.0340	0.0301	0.0278	0.0404	0.0264	0.0377	0.0250	0.0309	0.0283	0.0295
GARCH(1,2)	0.0964	0.3434	0.5657	0.0039	0.6980	0.0182	0.8188	0.2750	0.5131	0.3955

Table 12. Values of i_k for the sWIG80 index

series \ k	1	2	3	4	5	6	7	8	9	10
log returns	0.0911	0.0551	0.0680	0.0579	0.0597	0.0546	0.0498	0.0416	0.0440	0.0426
	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0014	0.0003	0.0006
ARMA(1.2)	0.0478	0.0386	0.0538	0.0502	0.0397	0.0479	0.0371	0.0340	0.0369	0.0376
	0.0000	0.0031	0.0000	0.0000	0.0020	0.0000	0.0074	0.0349	0.0083	0.0056
ARMA(1,2)-	0.0268	0.0300	0.0367	0.0282	0.0258	0.0345	0.0255	0.0295	0.0278	0.0309
GARCH(1,1)	0.7878	0.5014	0.0616	0.6646	0.8545	0.1451	0.8722	0.5471	0.7014	0.4072

[9] The fit of all estimated models was positively verified using the Box-Ljung and the Engle tests.

Table 13. Values of i_k for the WIG-Banking index

series \ k	1	2	3	4	5	6	7	8	9	10
log returns	0.0429	0.0439	0.0628	0.0556	0.0485	0.0518	0.0476	0.0516	0.0602	0.0443
	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
MA(1)	0.0421	0.0469	0.0566	0.0542	0.0609	0.0530	0.0421	0.0544	0.0565	0.0496
	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
MA(1)-	0.0387	0.0346	0.0347	0.0278	0.0308	0.0320	0.0276	0.0302	0.0306	0.0354
GARCH(1,2)	0.0534	0.2279	0.2242	0.8131	0.5525	0.4328	0.8250	0.6136	0.5702	0.1790

Table 14. Values of i_k for the WIG-Construction index

series \ k	1	2	3	4	5	6	7	8	9	10
log returns	0.0525	0.0301	0.0415	0.0400	0.0365	0.0451	0.0460	0.0326	0.0321	0.0451
	0.0000	0.2070	0.0009	0.0016	0.0119	0.0001	0.0001	0.0823	0.1004	0.0001
ARMA(2,1)	0.0336	0.0386	0.0428	0.0387	0.0350	0.0365	0.0391	0.0221	0.0320	0.0481
	0.0145	0.0003	0.0000	0.0003	0.0064	0.0022	0.0002	0.7270	0.0311	0.0000
ARMA(2,1)-	0.0286	0.0289	0.0321	0.0293	0.0239	0.0305	0.0263	0.0251	0.0301	0.0338
GARCH(1,1)	0.5966	0.5661	0.2637	0.5281	0.9422	0.4099	0.8061	0.8875	0.4452	0.1569

Table 15. Values of i_k for the WIG-Developers index

series \ k	1	2	3	4	5	6	7	8	9	10
log returns	0.1392	0.1477	0.1290	0.1154	0.1859	0.1292	0.1255	0.1370	0.1699	0.1353
	0.0063	0.0013	0.0246	0.1188	0.0000	0.0240	0.0392	0.0091	0.0000	0.0118
ARMA(1,1)	0.1479	0.1562	0.1466	0.1144	0.1506	0.1515	0.1488	0.1258	0.1412	0.1484
	0.0022	0.0006	0.0028	0.1664	0.0017	0.0014	0.0021	0.0531	0.0079	0.0022
ARMA(1,1)-	0.0928	0.1147	0.0999	0.0929	0.1195	0.1153	0.1189	0.1199	0.0869	0.1241
GARCH(1,2)	0.9250	0.5124	0.8324	0.9245	0.3976	0.4980	0.4135	0.3861	0.9708	0.3014

Table 16. Values of i_k for the WIG-Food index

series \ k	1	2	3	4	5	6	7	8	9	10
log returns	0.0544	0.0361	0.0449	0.0473	0.0446	0.0311	0.0314	0.0337	0.0305	0.0342
	0.0000	0.0000	0.0000	0.0000	0.0000	0.0037	0.0028	0.0003	0.0052	0.0002
ARMA(1,1)	0.0371	0.0365	0.0418	0.0433	0.0358	0.0270	0.0310	0.0409	0.0230	0.0298
	0.0000	0.0000	0.0000	0.0000	0.0000	0.0055	0.0003	0.0000	0.0601	0.0007
ARMA(1,1)-	0.0311	0.0340	0.0338	0.0347	0.0239	0.0309	0.0281	0.0281	0.0311	0.0362
GARCH(1,1)	0.5233	0.2766	0.2873	0.2198	0.9738	0.5448	0.7944	0.7938	0.5252	0.1404

Table 17. Values of i_k for the WIG-IT index

series \ k	1	2	3	4	5	6	7	8	9	10
log returns	0.0486	0.0388	0.0466	0.0573	0.0454	0.0443	0.0335	0.0513	0.0469	0.0434
	0.0000	0.0038	0.0000	0.0000	0.0000	0.0001	0.0580	0.0000	0.0000	0.0002
AR(1)	0.0585	0.0359	0.0476	0.0619	0.0553	0.0488	0.0314	0.0499	0.0543	0.0409
	0.0000	0.0449	0.0000	0.0000	0.0000	0.0000	0.2556	0.0000	0.0000	0.0032
AR(1)-GARCH(1,1)	0.0362	0.0251	0.0270	0.0339	0.0222	0.0260	0.0282	0.0244	0.0303	0.0293
	0.0778	0.8876	0.7622	0.1810	0.9796	0.8343	0.6611	0.9176	0.4641	0.5646

Table 18. Values of i_k for the WIG-Media index

series \ k	1	2	3	4	5	6	7	8	9	10
log returns	0.0481	0.0560	0.0448	0.0562	0.0475	0.0456	0.0350	0.0422	0.0304	0.0393
	0.0555	0.0049	0.1229	0.0047	0.0642	0.1020	0.6139	0.2144	0.8539	0.3531
MA(1)	0.0484	0.0571	0.0516	0.0529	0.0519	0.0398	0.0464	0.0450	0.0446	0.0426
	0.1063	0.0097	0.0473	0.0333	0.0432	0.4922	0.1644	0.2159	0.2322	0.3298
MA(1)-GARCH(1,1)	0.0484	0.0414	0.0427	0.0481	0.0510	0.0352	0.0397	0.0363	0.0465	0.0370
	0.1380	0.4673	0.3925	0.1451	0.0735	0.8260	0.5785	0.7691	0.2020	0.7320

Table 19. Values of i_k for the WIG-Oil&Gas index

series \ k	1	2	3	4	5	6	7	8	9	10
log returns	0.0825	0.0780	0.0761	0.0711	0.0862	0.0658	0.0562	0.0685	0.0829	0.0667
	0.0185	0.0503	0.0733	0.1660	0.0076	0.3335	0.7510	0.2422	0.0166	0.3009
AR(2)	0.0816	0.0652	0.0619	0.0824	0.0820	0.0863	0.0573	0.0823	0.0878	0.0771
	0.0203	0.3279	0.4671	0.0183	0.0195	0.0062	0.6711	0.0184	0.0043	0.0493
AR(2)-GARCH(1,1)	0.0451	0.0611	0.0493	0.0573	0.0716	0.0573	0.0524	0.0652	0.0592	0.0472
	0.8837	0.2362	0.7406	0.3809	0.0368	0.3787	0.6043	0.1240	0.3055	0.8178

Table 20. Values of i_k for the WIG-Telecom index

series \ k	1	2	3	4	5	6	7	8	9	10
log returns	0.0467	0.0395	0.0429	0.0687	0.0440	0.0393	0.0417	0.0469	0.0419	0.0514
	0.0072	0.1307	0.0369	0.0000	0.0234	0.1405	0.0579	0.0062	0.0518	0.0007
GARCH(1.3)	0.0311	0.0340	0.0338	0.0347	0.0239	0.0309	0.0281	0.0281	0.0311	0.0362
	0.5186	0.2752	0.2834	0.2234	0.9693	0.5411	0.7826	0.7821	0.5207	0.1448

The results summarized in Tables 9-20 indicate that evidence of serial dependencies was found for the most investigated indices[10]. The same conclusion may be drawn for the residuals from the ARMA models, which means that the detected dependencies are nonlinear. In most cases the estimated ARMA-GARCH models were able to capture these nonlinearities. Only in the case

[10] The exception is the WIG-Oil&Gas index. In this case the obtained result is rather unusual, i.e. filtering data by the ARMA model caused the appearance of significance of the MI measure.

of WIG and mWIG40 indices there are reasons to believe that identified nonlinearity is not caused by an ARCH effect.

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Współczynnik informacji wzajemnej jako miara zależności nieliniowych w szeregach czasowych

Z a r y s t r e ś c i. W artykule scharakteryzowano konstrukcję, estymację oraz możliwości zastosowania współczynnika informacji wzajemnej. Przedstawiono wyniki symulacji, prowadzących do weryfikacji jego przydatności w procesie identyfikacji zależności nieliniowych w szeregach czasowych. Ponadto zaprezentowano wyniki zastosowania tego współczynnika do analizy indeksów Giełdy Papierów Wartościowych w Warszawie.

S ł o w a k l u c z o w e: nieliniowość, współczynnik informacji wzajemnej, mutual information, identyfikacja zależności.