European Equity Market Integration and Optimal Investment Horizons – Evidence from Wavelet Analysis

Abstract. In the paper the process of equity market integration in Europe is examined from the wavelet perspective. The method applied is the Continuous Discrete Wavelet Transform that enables to perform global and local wavelet variance and correlation decompositions. In particular, questions about changes of the investment risk and the possibility of international portfolio diversification under different investment horizons are addressed. The study documents both convergence of the Central and Eastern European equity markets as well as their segmentation on the European market. The latter enables reduction of portfolio returns variability by an international portfolio diversification, especially for long investment horizons.

Keywords: equity market integration, time-scale analysis, wavelet variance, wavelet correlations

1. Introduction

One consequence of financial globalization are comovements of prices on different stock markets. Investigation of these processes is important due to both investors allocation decisions and policy-makers actions. From the point of view of investors the ongoing integration of capital markets increases the importance of a sectoral portfolio diversification at the expense of an international diversification. From a global perspective integration of financial markets is fundamentally related to economic growth via improvement of allocative efficiency, risk sharing and reduction of macroeconomic variability (see Kim et al., 2005). Convergence of stock markets, through the income channel, influences also the effectiveness of monetary policy and – as such – should be of considerable interest to policy-makers.

† The author acknowledges the financial support from the Polish Ministry of Science and Higher Education under the grant no. NN111 285135.
Studies by Longin and Solnik (1995) point out increasing integration of major world stock exchanges over the period 1960–1990 and find also an additional rise of correlation in the time of high volatility. Also more recent studies document convergence of stock markets, although the process is non-uniform both in time and market segments (Kim et al., 2005), asymmetric, i.e. negative shocks are more strongly transmitted via borders (Fratzscher, 2002) and leaves place for partial segmentation of certain stock markets (Bessler, Yang, 2003). Behind capital market integration stands real and nominal macroeconomic convergence, and therein reduction of currency risk and convergence of monetary policy with respect to interest rates and inflation (Fratzscher, 2002; Phengpis et al., 2004). Integration of major European equity markets and the rise in their world significance first of all results from formation of the common currency area (Fratzscher, 2002; Kim et al., 2005; Hardouvelis et al., 2006).

Only a couple of papers undertakes the task of examining the convergence of Central and Eastern European (CEE) capital markets and their integration with the world capital market (Scheicher, 2001; Voronkova, 2004; Chelley-Steeley, 2005; Gilmore et al., 2008; Harrison, Moore, 2009). Chelley-Steeley (2005) documents that for the period 1994–1999 correlations of stock index returns between CEE and developed European markets were usually below 0.3, while at the same time correlations between indices on developed European markets are often above 0.5. The majority of empirical investigations points out a certain kind of segmentation of Central-Eastern European markets and the lack of uniformly increasing integration with the rest of Europe, although CEE markets remain under a significant influence of the world capital market.

Recent theoretical studies underline the importance of agents heterogeneity in asset pricing models (see, e.g., the fractal market hypothesis of Peters, 1994). Also survey studies confirm that investors acting on financial markets have different investment strategies and different investment horizons – from one day to several years (see the discussion in Demary, 2009). Agents with long investment horizons concentrate on economic fundamentals driving trends, while speculators want to beat the market in the short run and often resort to methods of technical analysis. Heterogeneity of agents in a natural way gives rise to analyze stock prices according to different time scales (investment horizons). A method which enables investigation of stochastic processes decomposed according to scales is time-scale (wavelet) analysis. The aim of this study is to apply wavelet analysis to investigate the convergence process between Central European capital markets and developed European markets. As by design the subject of this investigation are not the directly observed results of the globalization process, and therein the so-called contagion effects on financial markets, but rather their time-scale consequences for stock investors, stock indices have been denominated in one currency (Polish Zloty) in order to compensate for foreign currency exposure.
The empirical investigation spans indices from three developed European stock markets – Frankfurt (DAX), London (FTSE1000) and Paris (CAC40) – as well as six emerging markets from the Central and Eastern Europe – Prague (PX50), Bratislava (SAX), Budapest (BUX), Sofia (SOFIX), Bucharest (BET) and Warsaw (WIG). The study concentrates on the following questions:

− Do European stock markets converge (emerging markets alone as well as both developing and developed European markets), and if so, is the process uniform in time and across investment horizons (scales)?

− For which investment horizons is international portfolio diversification most efficient?

− Is the process of the fast development of CEE stock markets accompanied by the rise of investment risk?

− Is there an EU’s effect – an increase in stock exchange comovements caused by policy coordination, the rise in trade and investment and the opening of labor markets that foster the process of real macroeconomic convergence?

The rest of the paper is organized as follows. Section 2 describes briefly the tools of wavelet analysis that are used in the study, Section 3 presents main empirical results, while the last section shortly concludes.

2. Methodology

Wavelet analysis consists in decomposing a signal into shifted and scaled versions of a basis function, \( \psi(\cdot) \), called the mother wavelet. The decomposition can be continuous or discrete depending on the kind of the wavelet transform applied. The Discrete Wavelet Transform (DWT) provides a parsimonious representation of the data and is particularly useful in noise reduction and information compression, while the Continuous Wavelet Transform (CWT) is more helpful in recognizing local features of signals, especially those that are defined over the entire real axis, although this results in excessive redundancy of information.

The Continuous Wavelet Transform of a function \( f(\cdot) \) is defined as follows:

\[
W(\lambda, t) = \int_{-\infty}^{\infty} \psi_{\lambda, t}(x) f(x) dx ,
\]

where:

\[
\psi_{\lambda, t}(x) = \frac{1}{\sqrt{\lambda}} \psi\left(\frac{x-t}{\lambda}\right), \quad \lambda > 0 .
\]

By applying the CWT we obtain a set of wavelet coefficients, \( \psi_{\lambda, t}(x) \), depending on scale \( \lambda \) and time \( t \). Let us consider a vector of length \( N = 2^J \) in the
form $x = (x_0, x_1, \ldots, x_{N-1})'$. For $j = 1, 2, \ldots, J$ and $t = 0, 1, \ldots, 2^{J-j} - 1$ we define the Discrete Wavelet Transform of vector $x$:

$$W_{j,t} = \sum_{n=0}^{N-1} x_n \psi_{j,t}(n),$$

(3)

where $\psi_{j,t}(\cdot)$ are shifted and scaled versions of the mother wavelet with dyadic shifts and scales, i.e.:

$$\psi_{j,t}(x) = 2^{-j/2} \psi \left( 2^{-j} x - t \right).$$

(4)

For a given $j$ the coefficients $W_{j,t}$ correspond to scale $\lambda_j = 2^{j-1}$. The DWT results from a critical sampling of the CWT, which means that it contains the minimal amount of wavelet coefficients for complete reconstruction of the signal.

Here in the paper, following, e.g., Percival and Walden (2002), we concentrate exclusively on the DWT considering it as a more natural way of analyzing discrete time series. Among wavelet tools based on the discrete transform are the wavelet variances and the wavelet correlations (also known as the wavelet coherences – see Sanderson et al., 2009). For a stochastic process $Y_t$ the time-dependent wavelet variance is defined as:

$$\sigma_t^2(\lambda_j) = \frac{1}{2\lambda_j} \text{Var}(W_{j,t}).$$

(5)

Assuming that (5) does not depend on time, the following variance decomposition according to time scales is obtained (see Percival, Walden, pp. 296–298):

$$\text{Var}(Y_t) = \frac{1}{2} \sum_{j=1}^{\infty} \frac{1}{\lambda_j} \text{Var}(W_{j,t}) = \sum_{j=1}^{\infty} \sigma_t^2(\lambda_j).$$

(6)

The wavelet variance at level $j$ corresponding to scale $\lambda_j = 2^{j-1}$, $\sigma^2(\lambda_j)$, informs about variation of oscillations with period lengths approximately in the

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1 Other tools based on the DWT are: multiresolution analysis taking advantage of both the DWT and its inversion, wavelet cross-correlations computed via the Continuous Discrete (Maximal Overlap) Wavelet Transform operating on discrete scale and continuous time as well as methods from complex-valued wavelet analysis, especially the discrete wavelet phase angle taking advantage of the Maximal Overlap Discrete Hilbert Wavelet Transform (MODHWT) – see Gençay et al. (2002); Whitcher, Craigmile (2004). The measures mentioned above can also be applied to only a portion of wavelet coefficients, what results in local (short-time) versions of the tools – see, e.g., Sanderson et al. (2009).

2 Such an assumption is fulfilled also for nonstationary processes provided that they are integrated of order $d$ and the width of the Daubechies wavelet filter, $L$, is sufficient to eliminate non-stationarity (i.e. $L \geq 2d$) – see Percival, Walden (2000), p. 304. In what follows we concentrate exclusively on Daubechies filters, although they are not the only ones that are interpretable in terms of generalized differences of weighted averages.
interval $2^j - 2^{j+1}$. Similarly, the wavelet covariance and wavelet correlation are introduced. For stochastic processes $Y_t^1$ and $Y_t^2$, the wavelet covariance for scale $\lambda_j$ is defined as:

$$\gamma(\lambda_j) = \frac{1}{2\sqrt{\lambda_j}} \text{Cov}(W_{1,j,t}, W_{2,j,t}).$$

(7)

As in the case of variance decomposition (6), wavelet covariances are obtained by decomposing the covariance between $Y_t^1$ and $Y_t^2$ according to different scales $\lambda_j$. Next, let us define the wavelet correlation coefficient for scale $\lambda_j$ via:

$$\rho(\lambda_j) = \frac{\gamma(\lambda_j)}{\sigma_1(\lambda_j)\sigma_2(\lambda_j)}.$$  

(8)

The quantity (8) is normalized in the interval [-1, 1] and indicates the strength and direction of a relationship between two processes for a given resolution level (i.e. for a given time scale $\lambda_j$).

In practice, when estimating the wavelet variance, covariance and correlation, instead of the DWT their modification in the form of the so-called Maximal Overlap (Continuous Discrete) DWT is used. The MODWT can be thought of as a subsampling of the CWT at dyadic scales, but leaving all times $t$ instead of only those that are multiples of powers of 2. For this reason it is called also the Non-decimated Wavelet Transform. This enables to eliminate certain artifacts produced by the DWT resulting from the lack of time-invariance, does not require data sets of length $2^L$ and – what is important from the point of view of this study – provides more statistically efficient estimators of the wavelet variance (Gençay et al., 2002, p. 135). An unbiased estimator of the wavelet variance is then defined as:

$$\tilde{\sigma}^2(\lambda_j) = \frac{1}{N_j} \sum_{t=L_{j-1}}^{N-1} \tilde{W}_{j,t}^2,$$

(9)

where $\tilde{W}_{j,t}$ are the MODWT coefficients, $L_j = (2^L - 1)(L - 1) + 1$ is the width of the wavelet filter for scale $\lambda_j$ ($L$ is the width of the basic wavelet filter at the first stage of the pyramid algorithm that computes the DWT) and $\tilde{N}_j = N - L_j + 1$ is the number of wavelet coefficients not affected by the boundary. By using (9) we assume that the wavelet filter applied eliminates all deterministic components of the process under scrutiny. If the process has constant, nonzero mean value, the formula (9) is modified appropriately. An approximate $(1-\alpha)%$ confidence interval for $\sigma^2(\lambda_j)$ is computed as follows:
\[ \tilde{\sigma}^2(\lambda_j) \pm \varsigma_{\frac{1}{2}} \left( \frac{\hat{f}(\lambda_j)(0)}{N_j} \right)^{0.5}, \]  
where \( \varsigma_{\frac{1}{2}} \) is the \((1-\alpha/2)\) quantile of the standard normal distribution and \( \hat{f}(\lambda_j)(0) \) is an estimator of the spectral density of scale \( \lambda_j \) squared wavelet coefficients at frequency 0. Estimates of wavelet covariances and wavelet correlations are computed via the following formulas:

\[ \tilde{\gamma}(\lambda_j) = \frac{1}{N_j} \sum_{t=2, \ldots, N_j} \tilde{W}_{1,j,t} \tilde{W}_{2,j,t}, \]  
(11)

\[ \tilde{\rho}(\lambda_j) = \frac{\tilde{\gamma}(\lambda_j)}{\tilde{\sigma}_1(\lambda_j)\tilde{\sigma}_2(\lambda_j)}. \]  
(12)

An approximate \((1-\alpha)\%\) CI for \(\gamma(\lambda_j)\) is obtained as previously with \(\hat{f}(\lambda_j)(0)\) being an estimate of the cross-spectrum at frequency 0. In the case of wavelet correlations Gençay et al. (2002), p. 259–260, suggest an approach taking advantage of the fact that the DWT approximately decorrelates even long memory processes. In such a case, in order to obtain confidence limits that are placed in the interval \([-1, 1]\) the Fisher \(z\)-transformation can be applied, which under the Gaussian assumption leads to the following \((1-\alpha)\%\) CI for scale \(\lambda_j\):

\[ \pm \varsigma_{\frac{1}{2}} \left( \frac{1}{\tilde{N}_j - 3} \right)^{0.5}, \]  
(13)

where \(\tilde{N}_j\) is the number of (conventional) DWT coefficients associated with scale \(\lambda_j\) that is treated here as a measure of the scale-dependent sample size.

3. Empirical Results

In the empirical examination daily closing prices of nine indices denominated in Polish Zloty at the National Bank of Poland (NBP) exchange rates have been used. We decided on examining prices instead of returns due to the fact that, firstly, using wavelets we do not need to transform data to stationarity prior to the analysis, secondly, examining indices provides better interpretation in terms of synchronization of stock market cycles and, thirdly, wavelet coefficients obtained with the Haar wavelet filter can be thought of being local estimates of multi-period returns and – as such – our results of wavelet variance and correlation analysis will have practical implications for portfolio construc-
The main advantage of using wavelets over more traditional approaches is their efficiency in data exploration resulting from optimal time-frequency resolution and the simplicity in analyzing scale-dependent phenomena.

The indices examined here are from both developed European capital markets (DAX, FTSE, CAC) and CEE markets (PX, SAX, BUX, SOFIX, BET and WIG) and span the period 2.01.2002–30.04.2009 (1912 observations), except for SAX, for which the sample starts on 2.01.2003, as well as SOFIX and BET, where the samples begin on 3.04.2007. The reason for the differences in data length is the lack of daily quotations of Slovak Koruna, Bulgarian Lev and Romanian Leu to the beginnings of our samples. After linear interpolation of missing observations the quotations have been transformed into logarithms. The computations were performed on the entire sample and in three subsamples: 01.2002–04.2004 (608 observations), 05.2004–03.2007 (761 observations), 04.2007–04.2009 (543 observations). The first subsample spans the period prior to the enlargement of the European Union in 2004, the second comprises first three years after the enlargement, while the last covers the most recent period that includes also the data on the latest financial crisis and is the only one spanning all nine indices.

The results of group unit root tests on an unbalanced panel comprising all indices and all observations performed with lag length chosen on the base of the Schwartz criterion, spectrum estimation at frequency 0 with the Bartlett kernel, Newey-West bandwidth selection, individual intercepts for the levels and no intercept for first differences are given in Table 1. The results (except for the $t^*$ statistic) allow to treat our multivariate process as integrated of order 1.

Table 1. Results of group unit root tests

<table>
<thead>
<tr>
<th>Test</th>
<th>Statistic</th>
<th>$p$-value</th>
<th>Statistic</th>
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<tr>
<td>Level</td>
<td></td>
<td></td>
<td>First difference</td>
<td></td>
</tr>
<tr>
<td>Levin, Lin, Chu $t^*$</td>
<td>-3.029</td>
<td>0.001</td>
<td>-92.054</td>
<td>0.000</td>
</tr>
<tr>
<td>Breitung $t$</td>
<td>3.199</td>
<td>0.999</td>
<td>87.593</td>
<td>0.000</td>
</tr>
<tr>
<td>Breitung $W$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>H0: no common unit root</td>
<td>Z</td>
<td>66.89</td>
<td>0.000</td>
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</tr>
</tbody>
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Depending on data length a 6- or 8-level Maximal Overlap Discrete Wavelet Transforms were executed. One of the Daubechies least asymmetric wavelet filters – LA(8) – was applied, which is nearly linear in phase and has the width 8. Besides, in the case of wavelet correlations the examination was also performed with the Haar wavelet filter$^3$. The decomposition levels 1–6 corres-

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$^3$ As they confirm results obtained with the LA(8) filter, we do not present them here.
pond to oscillations with period lengths approximately in the following intervals: 2–4, 4–8 (up to one and a half weeks), 8–16 (up to 3 weeks), 16–32 (up to 6 weeks), 32–64 (up to one quarter) and 64–128 (up to 2 quarters). When the whole samples were analyzed, two further decomposition levels were added that correspond to fluctuations with periods 128–256 (up to one year) and 256–512 (up to 2 years), respectively. The computations were executed in Matlab after modifying codes developed by B. Whitcher (the WaveCov package at www2.imperial.ac.uk/~bwhitche/software) as well as D. B. Percival and A. T. Walden (the WMTSA toolkit at www.atmos.washington.edu/~wmtsa) and supplementing them with own programs.

Figures 1–3 present decompositions of the wavelet variances with the help of LA(8) wavelet. The results point out that for the three mature markets the investment risk in the second period was significantly below its level in the first and third part of the sample. In the third period it rose for all investment horizons (for the London Stock Exchange even above its level in the first subsample). In the case of the three examined emerging markets – Prague, Budapest and Warsaw – the situation looks differently: in the pre- and post-accession periods the wavelet variances do not differ at all scales, while in the last period they rose significantly above their previous levels, except for the longest investment horizons (above 6 weeks in the case of BUX and WIG and one quarter in the case of PX). This lack of a significant increase in the investment risk for the longest horizons can be thought of as a sign that Central and Eastern European capital markets were more robust to the financial crisis at the beginning of it than the developed markets. A more detailed examination, of which we present here only the comparison between DAX and WIG (see Figure 3), performed with the help of the local wavelet variance\(^4\), shows, however, a quite similar level of volatility for medium and long horizon investments on the Polish market as compared to Frankfurt, with periods of a rapid increase of risk, except for the highest level of examination. A similar situation was observed for the other developing and developed markets.

Figures 4–10 show decompositions of the wavelet correlations. There are very strong dependences present for the developed capital markets at all investment horizons. The three markets can be thought of as a one investment possibility. Analyzing correlations between the developed and developing markets it is seen that the mature markets significantly influence the CEE stock indices, except for Bratislava and for longer horizons also Sofia. One important finding is the lack of systematic convergence of the mature and the CEE mar-

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\(^4\) The local wavelet variances were computed for windows of width 100 after aligning them with the original data and exclusion of all wavelet coefficients affected by the boundary (the circular filtering), what cut results at the beginning and the end of the period and is especially pronounced for higher resolution levels. The number of affected coefficients at the beginning (end) of the sample for the eight decomposition levels is, respectively, 3 (4), 10 (11), 24 (25), 52 (53), 108 (109), 220 (221), 444 (445), 892 (893).
kets: in the majority of cases the highest correlations occur in the last period, what can be attributed to the contagion effects on the financial markets, while the lowest are usually in the middle subsample, although they do not differ significantly from those computed for the first period. Generally, our conclusion is that we do not observe a uniformly increasing dependence between these two types of markets. This finding has been also confirmed with a more detailed examination with the help of the local wavelet correlations analysis – see Figure 9 – that was executed for data windows of length 200.

On the other hand, a systematic rise in the degree of dependence is found for three of the emerging markets: Prague, Budapest and Warsaw, what is especially pronounced for the lowest decomposition levels (see Figure 10). Besides there are also significant relationships between these indices and BET, while SAX seem to be uncorrelated with all the other indices and SOFIX is best correlated with DAX and BET.
Figure 2. Comparison of wavelet variance in subsamples together with the 95% confidence intervals using the LA(8) wavelet filter: 01.2002–04.2004 (–o–), 05.2004–03.2007 (−−−), 04.2007–04.2009(---); thick lines correspond to the later periods.
As for international portfolio diversification very promising seem to be the relationships between FTSE and WIG as well as CAC and WIG, especially for long investment horizons. Besides, we notice insignificant wavelet correlations of PX, BUX and WIG with the indices from Sofia and Bratislava. For the majority of pairs of indices the wavelet correlations seem to be homogenous across scales, except for the longest investment horizons.
Figure 5. Results of wavelet correlation decompositions (continued)

Figure 6. Results of wavelet correlation decompositions in subsamples using the LA(8) wavelet filter: 01.2002–04.2004 (–o–), 05.2004–03.2007 (–□–), 04.2007–04.2009 (–∗–)
Figure 7. Comparison of wavelet correlations in subsamples together with the 95% confidence intervals using the LA(8) wavelet filter: 01.2002–04.2004 (−○−), 05.2004–03.2007 (−□−), 04.2007–04.2009 (−∗−); thick lines correspond to the later periods.

Figure 8. Comparison of wavelet correlations in subsamples (continued)
4. Conclusions

Of the countries investigated an approximately uniform rise in integration takes place for the ‘big three’ emerging CEE equity markets: the Czech Republic, Hungary and Poland. We do not observe systematic convergence among emerging capital markets and the three mature stock exchanges (Frankfurt, London and Paris), as the rise in the wavelet correlations documented for the most recent data investigated can be explained by contagion effects on financial
markets. Our empirical results point out a certain kind of segmentation of CEE capital markets, although these markets remain under a significant influence of the developed stock exchanges. So, it seems that in the rather short post-accession period the developing capital markets do not face the EU’s effect yet. The distribution of wavelet correlations across scales is relatively homogenous. Departures from homogeneity take usually place for longer horizons and have the form of bidirectional deviations. For certain pairs of indices we found significantly negative wavelet correlations for the longest investment horizons investigated. This makes it possible substantially to reduce portfolio returns variability by international portfolio diversification. For shorter investments zero or even negative wavelet correlations have been found with the SAX and SOFIX indices. Generally, the fast development of Central and Eastern European equity markets is accompanied by a relatively moderate risk for medium-term investments, with rapid changes in volatility.

References


Integracja giełd europejskich i optymalne horyzonty inwestycyjne w świetle analizy falkowej

Z a r y s t r e ś c i. W artykule prezentuje się wyniki badania procesu integracji giełd europejskich przeprowadzonego z użyciem narzędzi Ciąglo-Dyskretnnej Transformaty Falkowej, a dokładniej globalnych i lokalnych (krótkookresowych) wariancji i korelacji falkowych. W szczególności odpowiada się na pytania o zmiany ryzyka inwestycyjnego oraz możliwość międzynarodowej dywersyfikacji portfeli przy uwzględnieniu różnych horyzontów inwestycyjnych. Badanie pokazuje, że ma miejsce proces konwergencji giełd środkowoeuropejskich, ale rynki te jako całość wykazują pewną segmentację. Daje to możliwość międzynarodowej dywersyfikacji portfeli, przede wszystkim dla dłuższych horyzontów inwestycyjnych.

S ł o w a k l u c z o w e: integracja giełd, analiza czasowo-skalowa, wariancje falkowe, korelacje falkowe.