Bayesian Analysis of the Box-Cox Transformation in Stochastic Volatility Models†

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Abstract. In the paper, we consider the Box-Cox transformation of financial time series in Stochastic Volatility models. Bayesian approach is applied to make inference about the Box-Cox transformation parameter (λ). Using daily data (quotations of stock indices), we show that in the Stochastic Volatility models with fat tails and correlated errors (FCSV), the posterior distribution of parameter λ strongly depends on the prior assumption about this parameter. In the majority of cases the values of λ close to 0 are more probable a posteriori than the ones close to 1.

Keywords: Box-Cox transformation, SV model, Bayesian inference.

1. Introduction

The continuously compounded rates of return (or logarithmic returns) as well as the simple rates of return are commonly used in econometric analyses of financial data. These two types of data transformation are applied arbitrarily. In the derivatives pricing literature there is the tradition of using logarithmic returns, but when the logarithmic return is modelled as a conditionally Student-t distributed random variable, the conditional expected simple rate of return is infinite. It violates the finite second moment condition for the asset payoff in call option pricing (see Duan, 1999). Duan (1999) uses the generalized error distribution (GED) for the logarithmic returns that also exhibits fat tails and includes the normal distribution as a special case. Other researchers build model with sample returns instead of log-returns and with the Student-t distribution (see e.g. Hafner, Harwartz, 1999; Härdle, Hafner, 2000; Bauwens, Lubrano, 2002). However, both the logarithmic return and simple one are variants of the well-known Box-Cox transformation of the $x_t/x_{t-1}$ ratio (where $x_t$ denotes the asset price at time $t$) with parameter 0 and 1, respectively. In the paper, we con-

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sider the Box-Cox transformation of financial time series in Stochastic Volatili-
ity (SV) models. Bayesian approach is applied to make inference about the Box-
Cox transformation parameter ($\lambda$). As parameter $\lambda$ is estimated along with other
unknown parameters, information in the data is used to determine which trans-
formation is appropriate for the data.

The structure of the article is as follows: section 2 consists of a short presenta-
tion of the Bayesian SV model with fat-tails correlated errors for the trans-
formed data, section 3 focuses on the empirical results, and finally, section 4
incorporates the conclusions.

2. Bayesian AR(1)-FCSV Model for the Transformed Data

Let $x_t$ denote the price of an asset at time $t$, $t = 0, 1, ..., T$. The Box-Cox
transformation of the $x_t/x_{t-1}$ ratio is defined as:

$$B(x_t / x_{t-1}, \lambda) = \begin{cases} 
\frac{(x_t / x_{t-1})^\lambda - 1}{\lambda} & \lambda > 0, \\
\ln(x_t / x_{t-1}) & \lambda = 0
\end{cases}$$

For $B(x_t / x_{t-1}, \lambda)$ we use an autoregressive structure\(^1\):

$$B(x_t / x_{t-1}, \lambda) - \delta_t = \rho [B(x_{t-1} / x_{t-2}, \lambda) - \delta_{t-1}] + \varepsilon_t, \quad t = 1, ..., T, \quad (1)$$

where $\{\varepsilon_t\}$ is the stochastic volatility process with fat-tails and correlated errors
(FCSV), introduced by Jacquier et. al., (2004). The discrete-time FCSV process
can be written as:

$$\varepsilon_t = u_t \sqrt{h_t} / \omega_t,$$ \hspace{1cm} (2)

$$\ln h_t = \gamma + \phi \ln h_{t-1} + \sigma \eta_t,$$ \hspace{1cm} (3)

$$\omega_t \sim \chi^2(\nu)/\nu, \quad \omega_t \perp (u_t, \eta_t), \quad t, l \in \{1, ..., T\},$$

$$(u_t, \eta_t)' \sim \text{IN} \left(0, \begin{pmatrix} 1 & \rho \\ \rho & 1 \end{pmatrix} \right), \quad t = 1, ..., T.$$

where the abbreviation "IN" denotes that the random vectors concerned are
independent and normally distributed, $\perp$ denotes stochastic independence.

In the FCSV process, when $\rho$ is equal to zero, $h_t$ is the inverse precision in the
conditional distribution, $p(\varepsilon_t|h_t)$, that is, $(\nu/\nu-2)h_t$ (for $\nu > 2$) is the conditional
variance. Thus, the FCSV model specifies a log-normal autoregressive process
for the conditional variance factor ($h_t$) with correlated innovations in the condi-
tional mean and conditional variance equations, i.e. in (2) and (3), respectively.

\(^1\) We use the autoregressive structure, because financial time series such as stock market in-
dices often present positive autocorrelation of order one of the returns (see Campbell et al., 1997).
One interpretation for the latent variable \( h_t \) is that it represents the random, uneven and autocorrelated flow of new information into financial markets (see Clark, 1973). The parameter \( \phi \) is related to the volatility persistence, and \( \sigma_h \) is the volatility of the log-volatility. The above model captures the leverage effect when the correlation \( \rho \) is negative. In fact, if \( \rho \) is negative, then a negative innovation \( u_t \) is associated with higher contemporaneous and subsequent volatilities. On the other hand, a positive innovation \( u_t \) is connected with a decrease in volatility (see Jacquier et al., 2004).

The Bayesian model is characterized by the joint probability density function of the untransformed \( x_t/x_{t-1} \) ratios (i.e. \( y = (y_1, ..., y_T)' \), where \( y_t = x_t/x_{t-1} \)), the latent variables (i.e. \( h = (h_1, ..., h_T)' \), \( \omega = (\omega_1, ..., \omega_T)' \)), and of the parameter vector \( \theta \):

\[
p(y, h, \omega, \theta | y_{(0)}) = p(y, h, \omega | \theta, y_{(0)}) p(\theta) ,
\]

where

\[
p(y, h, \omega | \theta, y_{(0)}) = p(\omega | \nu) \exp \left( -\frac{1}{2} \text{tr} \left( \Sigma^{-1} \sum_{t=1}^{T} r_t r_t' \right) \right) \times |\Sigma^*|^{-0.5} (2\pi)^{-T} \prod_{t=1}^{T} \omega_t^{0.5} h_t^{-1.5} ,
\]

\[
p(\omega | \nu) = \prod_{t=1}^{T} \left( \frac{\nu}{2} \right)^{\nu/2} \Gamma \left( \frac{\nu}{2} \right)^{-1} \omega_t^{\nu/2} \exp \left( -\frac{\nu}{2} \omega_t \right) I_{(0, \infty)}(\omega_t) ,
\]

\[
\Sigma^* = \begin{pmatrix} 1 & \rho \sigma_h \\ \rho \sigma_h & \sigma_h^2 \end{pmatrix} , \quad r_t = (u_t, \sigma_h h_t)' , \quad \theta = (\delta_1, \rho, \gamma, \phi, \sigma_h^2, \rho, \nu, \lambda)',
\]

\( y_{(0)} \) denotes initial values. The Jacobian \( J(\lambda, y) \) is \( J(\lambda, y) = \prod_{t=1}^{T} y_t^{\delta_1-1} \).

Our model specification gets completed by assuming the following prior structure:

\[
p(\delta_1, \rho, \gamma, \phi, \sigma_h^2, \rho, \nu, \lambda) = p(\delta_1) p(\rho_1) p(\gamma) p(\phi) p(\sigma_h^2) p(\nu) p(\lambda),
\]

where we use proper prior densities of the following distributions:

\( \delta_1 \sim N(0, 1) , \quad \rho_1 \sim U(-1, 1) \), \( \gamma \sim N(0, 100) \), \( \phi \sim N(0, 100) I_{(-1,1)}(\phi) \), \( \nu \sim \text{Exp}(0.1) \), \( \tau \sim IG(1, 0.005) \), \( \psi \sim N(0, \tau/2) \), \( \nu = \sigma_h^2 \rho \), \( \tau = \sigma_h^2 (1 - \rho^2) \).

The prior distribution for \( \delta_1 \) is standardized normal, \( U(-1,1) \) denotes the uniform distribution over \((-1,1)\). The prior distribution for \( \phi \) is normal, truncated by the restriction that the absolute value of \( \phi \) is less than one (\( I_{(-1,1)}(\phi) \) denotes the indicator function of the interval \((-1,1)\), which is the region of stationarity of \( \ln h_t \)).

The symbol \( IG(v_0, s_0) \) denotes the inverse Gamma distribution with mean
\(s_0/[((\nu_0 - 1)^2(\nu_0 - 2))](\text{thus, when } \rho = 0, \text{the prior mean for } \sigma_h^2 \text{ does not exist, but the precision, } \sigma_h^{-2}, \text{ has a Gamma prior with mean } 200 \text{ and standard deviation } 200). \) The symbol \(\text{Exp}(a)\) denotes the exponential distribution with mean \(1/a\) (thus the prior mean for \(v\) is equal to \(10\) with the standard deviation equals \(10\)). The prior distribution for \((\psi, \tau)\) induces a prior distribution for \((\rho, \sigma_h^2)\), which has the following form:

\[
p(\sigma^2, \rho) = s_0^{0.5} \Gamma(\nu_0)^{-1} P_0^{0.5} \frac{2^{0.5}}{(\pi)^{0.5}} (\sigma_h^{-2})^{\nu_0 + 1} e^{-\frac{\nu_0}{1 - \rho^2 - \frac{1}{\nu_0}}} e^{\frac{1}{2(1 - \rho^2)}} \frac{2^{0.5}}{(\pi)^{0.5}} \frac{\exp(\nu_0 \sigma_h^2)}{2^{0.5}} \frac{\nu_0}{1 - \rho^2 - \frac{1}{\nu_0}},
\]

\(v_0 = 1, s_0 = 0.005, \psi_0 = 0, p_0 = 2 \) (similar to Jacquier et al., 2004).

As far as the prior distribution for \(\lambda\), we assume that our prior information regarding this parameter can be represented by the following:

a) a non-standard distribution on the interval \([0, 1]\): \(p(\lambda) \propto e^{-\beta(1-x)}\), where \(\beta = 30\). This prior distribution is symmetrical and U-shaped, as shown in Figure 1.

b) the beta distribution with parameters 0.5 and 0.5;

c) the uniform distribution on the interval \([0, 1]\);

d) the exponential distribution with mean 1;

![Figure 1. Prior distributions for the Box-Cox transformation parameter (\(\lambda\)).](image)

As regards the initial condition for \(h\), i.e. \(h_0\), we assume that it is equal to 1.

The joint posterior distribution is then

\[
p(h, \omega, \theta \mid y, y_{(0)}) \propto p(\theta) p(\omega \mid \nu) \times \exp\left[-\frac{1}{2} \text{tr} \left(\Sigma^{-1} \sum_{t=1}^{T} r_t r_t^t\right)\right] J(\lambda, y) \mid \Sigma^{-0.5T} \prod_{t=1}^{T} \omega_t^{0.5} h_t^{-1.5}.
\]
The posterior probability density function is used to make inference about the parameters and latent variables.

3. Empirical Results

We consider ten international stock market indices, namely the S&P 500, NASDAQ 100, DJIA (for the US), NIKKEI (for Japan), the CAC 40 (for France), the DAX (for Germany), the FTSE 100 (for the UK), WIG 20 (for Poland), HANG SENG (for China), SPTSE 60 (for Canada).

The data set consists of the daily closing quotations of the stock market indices from January 2001 (or 2002) until February (or March) 2009 (see Table 1). Basic descriptive characteristics of the daily price ratios are presented in Table 1. All series of $x_t/x_{t-1}$ ratio exhibit strong kurtosis, and they have highly non-normal (truncated by zero) empirical distributions.

Table 1. Sample characteristics for the data sets used

<table>
<thead>
<tr>
<th>time series (x_t/x_{t-1} ratio of)</th>
<th>average</th>
<th>std. dev.</th>
<th>kurtosis</th>
<th>period from: - to:</th>
<th># obs. T</th>
</tr>
</thead>
<tbody>
<tr>
<td>WIG 20</td>
<td>1.0000</td>
<td>0.0162</td>
<td>4.9800</td>
<td>02.01.2001 – 13.02.2009</td>
<td>2035</td>
</tr>
<tr>
<td>S&amp;P 500</td>
<td>0.9998</td>
<td>0.0139</td>
<td>13.3286</td>
<td>03.01.2002 – 06.03.2009</td>
<td>1805</td>
</tr>
<tr>
<td>NIKKEI 225</td>
<td>0.9999</td>
<td>0.0163</td>
<td>11.3553</td>
<td>07.01.2002 – 06.03.2009</td>
<td>1760</td>
</tr>
<tr>
<td>FTSE 100</td>
<td>0.9999</td>
<td>0.0137</td>
<td>10.9021</td>
<td>03.01.2002 – 06.03.2009</td>
<td>1813</td>
</tr>
<tr>
<td>DAX</td>
<td>1.0000</td>
<td>0.0169</td>
<td>8.6642</td>
<td>03.01.2002 – 06.03.2009</td>
<td>1825</td>
</tr>
<tr>
<td>NASDAQ 100</td>
<td>1.0000</td>
<td>0.0178</td>
<td>7.8639</td>
<td>03.01.2002 – 06.03.2009</td>
<td>1808</td>
</tr>
<tr>
<td>CAC 40</td>
<td>0.9998</td>
<td>0.0159</td>
<td>9.7031</td>
<td>03.01.2002 – 06.03.2009</td>
<td>1838</td>
</tr>
<tr>
<td>SPTSE 60</td>
<td>1.0001</td>
<td>0.0132</td>
<td>14.2830</td>
<td>03.01.2002 – 06.03.2009</td>
<td>1798</td>
</tr>
<tr>
<td>HANG SENG</td>
<td>1.0002</td>
<td>0.0164</td>
<td>15.0382</td>
<td>03.01.2002 – 06.03.2009</td>
<td>1789</td>
</tr>
<tr>
<td>DJIA</td>
<td>0.9999</td>
<td>0.0130</td>
<td>12.5726</td>
<td>02.01.2001 – 13.02.2009</td>
<td>2039</td>
</tr>
</tbody>
</table>

Note: The data were downloaded from the website http://finance.yahoo.com.

In Table 2 we present the posterior means and standard deviations (in parenthesis) of the parameters, in the case of the AR(1)-FCSV model with the uniform prior for $\lambda$ on [0, 1]. Our posterior results are obtained in Gauss 9.0 using MCMC methods: Metropolis-Hastings within the Gibbs sampler (see, e.g. Pajor 2003 and Jacquier et al., 2004 for detail). First, for more series the autoregressive parameters seem to be insignificantly different from zero. The posterior distributions of $\delta$ and $\rho_1$ are located close to zero. Second, all indices have persistent volatility as shown by $\phi$ - the lowest posterior mean is 0.927 (for the WIG20 index), the highest one is 0.97 (for NASDAQ). It means that the half-life of shock to volatility, $HL = \ln(0.5)/\ln(\phi)$, is equal to about 9 days for the WIG20 index and 20 days for the NASDAQ index. We observed that the NASDAQ index exhibits a lower variability of volatility as shown by the precision, $\sigma_h^{-2}$. As regards the leverage effect parameter, $\rho$, the posterior means of $\rho$ are negative, from -0.15 for the WIG20 index to -0.62 for the CAC40 index.

$^2$ The results are obtained using 100 000 burnt-in and 1000 000 final Gibbs passes.
The parameter $\rho$ is estimated precisely with a standard deviation around 0.068. Almost all the posterior mass of $\rho$ is in the negative region. Thus, the leverage effect is strong for all indices excluding the WIG20 index, for which it is significantly lower. The posterior means of the degrees of freedom are between 16 (for the HANG SENG index) and 39 (for the FTSE 100 index). The HANG SENG index has the lowest posterior mean of degrees of freedom of the Student-t distribution. For the remaining indices the posterior mean of $\nu$ is above 23, indicating that the normal conditional distribution would not be strongly rejected by the data.

Table 2. Posterior means and standard deviations (in parenthesis) of the parameters of the AR(1)-FCSV model, in the case of $\lambda \sim U[0, 1]$

<table>
<thead>
<tr>
<th>parameter</th>
<th>WIG 20</th>
<th>S&amp;P 500</th>
<th>NIKKEI 225</th>
<th>FTSE 100</th>
<th>DAX 100</th>
<th>NASDAQ 100</th>
<th>CAC 40</th>
<th>SPTSE 60</th>
<th>HANG SENG</th>
<th>DJIA</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\delta_1$</td>
<td>4.74 (3.15)</td>
<td>4.74 (1.65)</td>
<td>6.52 (2.56)</td>
<td>5.43 (1.63)</td>
<td>9.70 (2.15)</td>
<td>5.77 (2.63)</td>
<td>6.59 (1.99)</td>
<td>8.68 (1.78)</td>
<td>7.20 (2.37)</td>
<td>4.14 (1.60)</td>
</tr>
<tr>
<td>$\rho$</td>
<td>0.027 (0.023)</td>
<td>-0.095 (0.024)</td>
<td>-0.034 (0.024)</td>
<td>-0.092 (0.024)</td>
<td>-0.059 (0.023)</td>
<td>-0.070 (0.024)</td>
<td>-0.080 (0.023)</td>
<td>-0.060 (0.024)</td>
<td>0.005 (0.023)</td>
<td>-0.074 (0.022)</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>-0.622 (0.119)</td>
<td>-0.332 (0.048)</td>
<td>-0.429 (0.066)</td>
<td>-0.357 (0.052)</td>
<td>-0.328 (0.049)</td>
<td>-0.297 (0.048)</td>
<td>-0.335 (0.047)</td>
<td>-0.493 (0.074)</td>
<td>-0.408 (0.074)</td>
<td>-0.348 (0.051)</td>
</tr>
<tr>
<td>$\phi$</td>
<td>0.928 (0.014)</td>
<td>0.965 (0.005)</td>
<td>0.951 (0.006)</td>
<td>0.962 (0.006)</td>
<td>0.963 (0.006)</td>
<td>0.966 (0.006)</td>
<td>0.963 (0.006)</td>
<td>0.948 (0.008)</td>
<td>0.955 (0.008)</td>
<td>0.963 (0.005)</td>
</tr>
<tr>
<td>$\sigma_n^2$</td>
<td>22.233 (5.675)</td>
<td>16.972 (2.809)</td>
<td>17.711 (3.395)</td>
<td>13.728 (2.123)</td>
<td>15.277 (2.592)</td>
<td>25.790 (5.279)</td>
<td>15.973 (2.491)</td>
<td>13.672 (2.447)</td>
<td>15.958 (3.156)</td>
<td>17.871 (2.966)</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>-0.153 (0.081)</td>
<td>-0.607 (0.063)</td>
<td>-0.55 (0.065)</td>
<td>-0.578 (0.061)</td>
<td>-0.612 (0.059)</td>
<td>-0.492 (0.081)</td>
<td>-0.62 (0.063)</td>
<td>-0.484 (0.067)</td>
<td>-0.372 (0.075)</td>
<td>-0.55 (0.064)</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>0.397 (0.255)</td>
<td>0.472 (0.265)</td>
<td>0.504 (0.266)</td>
<td>0.448 (0.263)</td>
<td>0.405 (0.256)</td>
<td>0.387 (0.253)</td>
<td>0.399 (0.255)</td>
<td>0.497 (0.266)</td>
<td>0.401 (0.256)</td>
<td>0.433 (0.262)</td>
</tr>
</tbody>
</table>

Finally, we consider the posterior evidence regarding the Box-Cox transformation parameter. Figure 2 shows the prior and posterior distributions for $\lambda$ in the case of the WIG20 index. We see from the graphs that the prior distribution for $\lambda$ strongly affects the posterior distribution for this parameter, e.g., a U-shaped prior distribution implies the U-shaped posterior distribution. In the case of the uniform prior for $\lambda$ on the interval $[0; 1]$, for most stock indices (considered here) the posterior mean is smaller than the prior mean, but the dispersion of posterior distribution is close to that of the prior distribution (in the case of $c$,...
the prior mean is equal to 0.5, the prior standard deviation is equal to 0.288). Even though the prior distribution is symmetrical, in the majority of cases the posterior distributions are asymmetrical. The values of \( \lambda \) from the interval \([0, 0.5]\) are more probable a posterior than those from \([0.5, 1]\) (see the quantiles of the posterior distributions of the Box-Cox transformation parameter in Table 4).

![Figure 2. Prior (solid line) and posterior (bars) distributions for \( \lambda \) (the WIG20 index)](image)

In the case of the non-standard prior distribution for \( \lambda \) considered in (a), except for the NIKKEI and SPTSE 60 indices, the posterior medians are below 0.1, but the probability that \( \lambda \) is less than 0.9 is not zero. In Table 5 we present the posterior probabilities that \( \lambda \) is in the interval \([0, 0.01]\) and in the interval \([0.99, 1]\). Except for the NIKKEI index, the values of \( \lambda \) from the interval \([0, 0.01]\) are more probable a posterior than those from the interval \([0.99, 1]\). Thus the data transformations which are close to the log-return are more probable a posterior than those which lead to the simple return.
It is important to stress that even though the prior distribution of $\lambda$ has a strong effect on the posterior distribution of $\lambda$, it does not affect the posterior distribution of the remaining parameters. Thus in Table 3 we present the posterior characteristics only of $\lambda$, obtained in the AR(1)-FCSV model with the exponential
distribution for the Box-Cox transformation parameter. Although the prior mean is equal to 1, for all series the posterior mean is less than 1.

Finally, in Table 6 we present the results of the formal Bayesian model comparison. We consider three AR(1)-FCSV models: with, respectively, $\lambda = 0 \ (M_1)$, $\lambda = 1 \ (M_2)$, and $\lambda \sim U(0, 1) \ (M_3)$. If $\lambda = 1$, the relation (1) is linear in the simple returns. If $\lambda = 0$, it is linear in the logarithmic returns. To obtain the marginal data densities we use the Newton and Raftery method (see Newton and Raftery 1994). The Newton and Raftery estimator is quite stable for all our models. The drawback of this method in the FCSV models is that the models differ from one another by quite a few orders of magnitude.

For all series, assuming equal prior model probabilities, the AR(1)-FCSV model with $\lambda = 0$ (log-returns) is more probable a posterior than with $\lambda = 1$ (simple returns). Only for the DJIA index, the AR(1)-FCSV model with the uniform prior distribution of $\lambda$ is quite a few orders of magnitude better than that with $\lambda = 0$.

Table 6. Posterior probabilities (under equal prior model probabilities) and marginal data densities of the observation vector $y$ in $M_i$ model (based on the Newton – Raftery method)

| Index      | $M_1: \lambda = 0$ | $M_2: \lambda = 1$ | $M_3: 0 < \lambda < 1^*$ | $p(y|M_1)$ | $p(y|M_2)$ | $p(y|M_3)$^* |
|------------|---------------------|---------------------|---------------------------|-----------|-----------|-----------|
| WIG 20     | 0.9754              | 0.0000              | 0.0246                    | 2.4·10^{-150} | 1.8·10^{-176} | 6.0·10^{-172} |
| S&P 500    | 0.9995              | 0.0000              | 0.0005                    | 2.8·10^{-176} | 2.0·10^{-186} | 1.4·10^{-179} |
| NIKKEI 225 | 0.9997              | 0.0000              | 0.0003                    | 1.5·10^{-198} | 7.5·10^{-196} | 4.2·10^{-192} |
| FTST 100   | 0.9931              | 0.0069              | 0.0000                    | 4.1·10^{-161} | 2.6·10^{-163} | 4.3·10^{-177} |
| DAX        | 1.0000              | 0.0000              | 0.0000                    | 3.4·10^{-122} | 2.4·10^{-141} | 1.3·10^{-129} |
| NASDAQ 100 | 1.0000              | 0.0000              | 0.0000                    | 9.3·10^{-193} | 1.6·10^{-198} | 1.7·10^{-113} |
| CAC 40     | 1.0000              | 0.0000              | 0.0000                    | 5.5·10^{-192} | 2.4·10^{-202} | 3.3·10^{-197} |
| SPTSE 60   | 1.0000              | 0.0000              | 0.0000                    | 3.3·10^{-45}  | 2.5·10^{-53}  | 1.1·10^{-48}  |
| HANG SENG  | 1.0000              | 0.0000              | 0.0000                    | 3.3·10^{-50}  | 5.2·10^{-56}  | 2.9·10^{-53}  |
| DJIA       | 0.0000              | 0.0000              | 1.0000                    | 2.3·10^{-202} | 7.8·10^{-254} | 2.6·10^{-199} |

Note: ^The results are obtained in the AR(1)-FCSV model with the uniform prior for $\lambda$ on the interval (0, 1).

4. Conclusions

The paper presents the stochastic volatility models with the Box-Cox transformation of financial time series. The widely used logarithmic and simple returns are nested into the Box-Cox transformation by setting $\lambda = 0$ and $\lambda = 1$, respectively. Using daily data, we show that in the stochastic volatility model with fat tails and correlated errors, the posterior distribution of the Box-Cox transformation parameter strongly depends on the prior assumption about this parameter. Our empirical results show that in the majority of cases the values of
λ close to 0 are more probable a posteriori than the ones close to 1. The formal Bayesian model comparison indicates that the Box-Cox transformation with λ = 0 (log-return) is preferred by the data in the FCSV model. However, the posterior distributions of λ show that the simple returns are not completely inappropriate.

References


Bayesowska analiza transformacji Boxa i Coxa dla w modelach o zmienności stochastycznej

Z a r y s t r e ś c i. Celem artykułu jest statystyczna analiza transformacji Boxa i Coxa ilorazu cen instrumentów finansowych w modelach FCSV. Stosowane jest podejście bayesowskie, które pozwala zbadać, w jakim stopniu dane modyfikują wstępne przekonanie o parametrze transformacji. Wyniki empiryczne pokazują, że założenia o rozkładzie a priori parametru transformacji ma istotny wpływ na kształt brzegowego rozkładu a posteriori tego parametru. Jednak w większości rozważanych przypadków rozkład te, w porównaniu z rozkładami a priori, są przesunięte w kierunku zera. Zatem transformacje ilorazu cen dające wartości bliskie logarytmicznej stopie zwrotu są bardziej prawdopodobne a posteriori niż transformacje prowadzące do prostej stopy zwrotu.

S ł o w a k l u c z o w e: transformacja Boxa i Coxa, model SV, wnioskowanie bayesowskie.